Verdet Constant Dispersion in Annealed Optical Fiber Current Sensors

A. H. Rose, S. M. Etzel, and C. M. Wang

***Abstract*—The Verdet constant in annealed optical fiber current sensors has been measured at wavelengths from 636 to1320 nm. The measurements are fitted to two models, one classical and the other an expansion of the classical model that includes a nonlinear term. These measurements and models are compared to previous measurements made in optical fiber and bulk SiO_2. Our measurements have an average accuracy of ±0.6% and an average measurement uncertainty of ±0.5% over the 636 to 1320 nm range.**

I. Introduction

THIS paper reports the measurement of the Verdet constant in annealed optical fiber and compares those measurements to previous works in optical fiber and bulk SiO_2 from 633 to 1550 nm [1]–[10]. The strength of the Faraday effect in a material is quantified by the Verdet constant V and is usually given the units of rad/$(\mathrm{m}\cdot\mathrm{T})$. For current sensing with diamagnetic materials it is convenient to use the value μV, which has the units of rad/A, where μ is the permeability of free space.

From the work of Garn *et al.* and Lassing *et al.* the value of μV at 633 nm was known to ±0.4% [1], [7]. The value of μV, at 633 nm, given in those works, can be used to estimate the Verdet constant at other wavelengths λ from a classical model, $\mu V = \mu V_o(\lambda/\lambda)^2$. [11]. The work of Noda *et al.* showed that this model, where λ is converted to frequency ν, fitted to measured data over the 633 to 1520 nm range, agrees with the data to less than ±5% [6]. In this paper, we compare the classical model to our measurements of μV in annealed single mode fiber.

Having accurate values of μV would enable the calibration and evaluation of optical current sensors for electric power metering applications [12]–[14]. Many previous measurements of μV in optical fiber had uncertainties of approximately ±2% or no stated uncertainty [2]–[6], [8], [10]. This large uncertainty is due to the presence of linear birefringence in the fiber or the difficulty in calibrating magnetic fields over the length of the fiber [2], [3], [9], [10].

Linear birefringence Δn or retardance $(\delta = 2\pi\Delta n/\lambda)$ in fiber is difficult to measure accurately to less than 1° and is unstable with movement and temperature. Linear birefringence directly affects the measurement of the Faraday rotation or a measure of μV [12]–[16]. Also, the Faraday effect in fiber is often small compared to the retardance usually present and dominates or masks the measurement of the rotation. Typically, μV is determined with the use of a solenoid magnet, where the profile of the fringing field and fiber interaction is difficult to determine with high accuracy. With the development of fiber annealing another method of determining μV in fiber is possible [15]. Annealed coils of fiber with low birefringence and a nearly closed optical path make a true current sensor that can be used to determine μV.

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II. Theory and Measurement Apparatus

The Faraday effect, used in optical fiber current sensing, is a magnetically induced circular birefringence. A beam of light flowing through a fiber with a magnetic field present will have its linear polarization state rotated through an angle θ_F equal to

$$\theta_F = V \int B \cdot dl \tag{1}$$

where B is the magnetic field parallel to the direction of the light flow, and l is the length of the light and field interaction in the material. In annealed fiber the light path is nearly closed around a current carrying conductor and Ampere's law applies so that (1) is simplified to

$$\theta_F = \mu V N I \tag{2}$$

where N is the number of times the path closes around the conductor and I is the current [15]. If linear birefringence is present in the fiber, a more complicated expression is needed to describe the effective rotation θ of a current sensor [16].

To measure the Faraday rotation of our annealed fiber sensors we used the apparatus shown in Fig. 1. The polarizer is aligned with the axis of δ, if present, and the analyzer is aligned at ±45° to the polarizer. Two separate intensities are recorded for the ±45° analyzer positions [15], [16]. From these two intensities, a set of amplifiers and a computer produce a signal equal to the difference divided by the sum. If θ_F is small then the effective rotation is equal to this signal or [15], [16]

$$\theta = \frac{\Delta}{\Sigma} = 2\mu V N I \frac{\sin\delta}{\delta}. \tag{3}$$

Taking the difference divided by the sum doubles the effective rotation and normalizes out any laser power or coupling fluctuations [2], [15], [16]. From (3) the current sensitivity of a coil can be expressed as

$$\frac{\theta}{I} = \frac{\Delta}{\Sigma I} = 2\mu V N \frac{\sin\delta}{\delta} \tag{4}$$

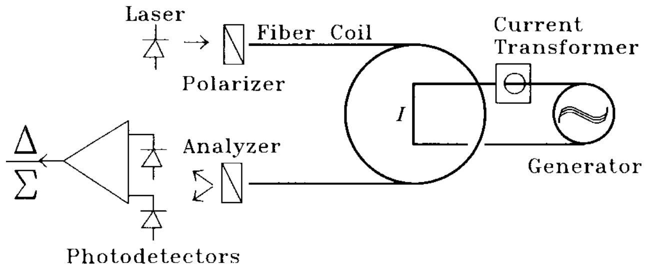

Fig. 1. Experimental apparatus used to measure and μV.

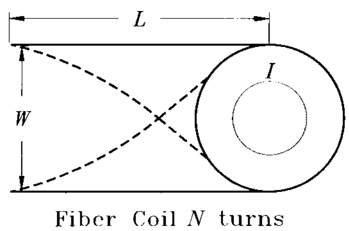

Fig. 2. Fiber coil configurations.

and is determined by measuring a coil's rotation for a given current [15]. Equations (3) and (4) are valid when δ is uniform and the axis position is constant along the length of the fiber [16].

When determining θ or the value of μV for annealed fiber coils the effect of the leads or a partial turn in the coil must also be considered. The coils used for these measurements were prepared as shown in Fig. 2. The dashed leads show an alternate form some fiber coils assumed. Fig. 2 shows W equal to the diameter of the coil ϕ. W could be larger or smaller than the coil diameter, to accommodate fiber positioning stages.

A good measure of the remaining turn in the coil can be made by measuring the length L and separation W of leads and computing the line integral of (1) in cylindrical coordinates, as shown in Appendix A. The number of turns N from this integral is

$$N = N_o + \left(1 \pm \frac{\arctan(W/2L)}{\pi}\right) \tag{5}$$

where N_o is the number of complete turns. The "+" is for coils with the leads crossed an odd number of times, as shown with the dashed leads in Fig. 2, and the "−" is for coils with the leads open or crossing an even number of times. For this work W ranged from 5 cm to 15 cm, L ranged from 10 cm to 30 cm, and N_o ranged from 1 to 50 turns, depending on the particular coil.

A good measure of μV is possible with annealed fiber coils where $\delta \approx 0$ and the current I, the number of turns N, and the effective rotation θ are all well known. Annealing fiber removes all the stress induced birefringence, but does not remove birefringence from core ellipticity. A core ellipticity of 1% can produce a birefringence of about 100°/m. This residual birefringence in annealed fiber can be averaged to near zero by twisting the fiber before annealing [17], [18]. For most optical fiber, twisting and annealing will effectively remove the residual linear birefringence so that $\sin\ \delta/\delta \approx 1$ [17], [18]. Also, in the annealing process, the fiber can be formed

TABLE I
OPTICAL FIBER PARAMETERS

Fiber Type	λ_o (nm) ±0.5 nm	λ_c (nm)	NA	GeO_2 (%)	δ_R (°/m)
A	636.0	580	0.11	4	n/a
B	787.6	750	0.13	5.4	95 ± 5 [18]
C	787.6	780	0.11	4	53 ± 3 [18]
C	831.9	—	—	—	—
D	829.0	780	0.12	4	36 ± 21
E	1322.3	1250	0.13	4.3	10 ± 1 [18]

into multi-turn coils to produce a larger effective rotation and reduce any errors due to the leads or the measurement of the leads. The value μV can be determined by inverting (3) when $\delta \approx 0$ so that

$$\mu V = \frac{\Delta}{2\Sigma N I}. \tag{6}$$

We measured μV with the apparatus shown in Fig. 1, calibrated amplifiers, voltmeters, a current transformer and a computer. The amplifiers and voltmeters were calibrated with the use of a calibrated signal analyzer and source. The lock-in amplifier and voltmeter (used to measure the ac difference signal Δ) and sum amplifier Σ were calibrated with 0.1% uncertainty. The difference amplifier Δ was calibrated with a 0.5% uncertainty. The current transformer was calibrated with a 0.2% uncertainty. A voltmeter with a 0.1% uncertainty was used to read the current transformer. A computer digitally read all the voltmeters. The uncertainty on N is less than or equal to 0.1% for the coils used. The root sum of squares of these uncertainties give our measurement of μV a total possible uncertainty of 0.6%.

The fiber coils measured a signal current between 10 and 50 A. Annealed fiber coils were handled with care so that birefringence was not reintroduced. The leads were placed in index matching gel to remove any cladding light (cladding light would bias the measured Σ signal) and held in place by a soft clay so that no additional birefringence could be introduced.

We selected several step index single mode fiber types and multimode diode laser light sources (to reduce coherence effects) to use in measuring μV from 636 to 1320 nm. Table I shows some of the optical parameters of each fiber. The operating wavelength reported λ_o is the mean wavelength of the laser used. The cutoff wavelength λ_c and numerical aperture NA of each fiber were taken from the manufacturer. All the fibers were SiO_2 based with a GeO_2-doped core.

The reported residual retardance δ_R (after annealing) was measured for some of these fibers to determine the number of twist per meter (1 twist $= 2\pi$ of rotation) needed to make the twisted and annealed fiber coil isotropic [17], [18]. Fiber D had a δ_R that varied a significant amount, making the measurement of μV difficult and requiring many annealed fiber coils to obtain only a few that were isotropic.

δ_R was determined by annealing (but not twisting) coils of various numbers of turns, or length. The current sensitivity

TABLE II
VERDET CONSTANTS

Fiber Type	ϕ (cm)	λ_o (nm) ±0.5 nm	$\mu V \pm 3\sigma$ (μrad/A)	N	# of measurements
A	7	636.0	4.58 ± 0.01	5.86	10
B	7	787.6	2.90 ± 0.01	20.94	10
C	7	787.6	2.89 ± 0.02	10.92	10
C	7	831.9	2.58 ± 0.02	10.92	10
D coil 1	3	829.0	2.517 ± 0.002	1.86	5
D coil 2	3	829.0	2.575 ± 0.001	2.85	5
D coil 3	3	829.0	2.52 ± 0.03	4.87	4
D coil 4	7	829.0	2.56 ± 0.03	25.91	3
D 1-4	—	829.0	2.54 ± 0.08	—	17
E	7	1322.3	0.992 ± 0.004	50.93	11

(4) of these coils of annealed (not twisted) fiber was measured and normalized to the number of turns. A least-squares fit of the current sensitivity data was fit to the $\sin\,\delta_R/\delta_R$ function to determine a value for δ_R [17], [18].

III. μV Dispersion Measurements

To determine μV, we used only twisted and annealed fiber coils with δ_R near zero, or with the highest θ [17], [18]. Table II summarizes our μV measurements with fibers A through E. Because of the large variation of δ_R in fiber D, several coil lengths and diameters were used to obtain a confident value for μV. Table II shows the mean and 3σ for all the measurements on fiber D.

Figs. 3–6 show graphically our measurement of μV and the measurements of others. In Figs. 3–5 four models are plotted. The small dashed line represents the classical model for the dispersion of $\mu V = \mu V_o(\lambda_o/\lambda)^2$ where $V_o = 4.61$ μrad/A, is the Verdet constant of SiO_2 at 633 nm as measured by Garn *et al.* [1], [11]. (They report only a ±2% accuracy.) The dash-dot-dash line represents the fitted model, $\mu V = V_H\nu^2$, of Noda *et al.* where ν is the optical frequency and V_H is 2.0×10^{-35} μrad/(A·Hz^2) [6]. (Noda *et al.* do not report an uncertainty for V_H.) The model $\mu V = V_H\nu^2$ is mathematically equivalent to the classical model $\mu V = \mu V_o(\lambda_o/\lambda)^2$. We fitted Noda *et al.* model to our six measurements of μV using a least-squares fit, designated NIST1, and to the model $\mu V = V_H\nu^2 + V_{H2}\nu^4$, designated NIST2. The second term in NIST2 is a simple Taylor series expansion of the classical model [11]. Table III shows the least-squares fit values for V_H and V_{H2}, their uncertainties, and the residual mean squares. The weighted least squares with weight on the inverse of (σ^2/n) was used, where n is the number of measurements. The nonlinear NIST2 model has a substantially smaller residual than the classical NIST1 model. The best way to see the impact of the nonlinear term in NIST2 is by examining the average difference between the data and the model. Figs. 3–5 show that NIST2 provides an average difference of 0.01 μrad/A over the 636 to 1320 nm range as compared to NIST1 with an average difference of 0.03 μrad/A.

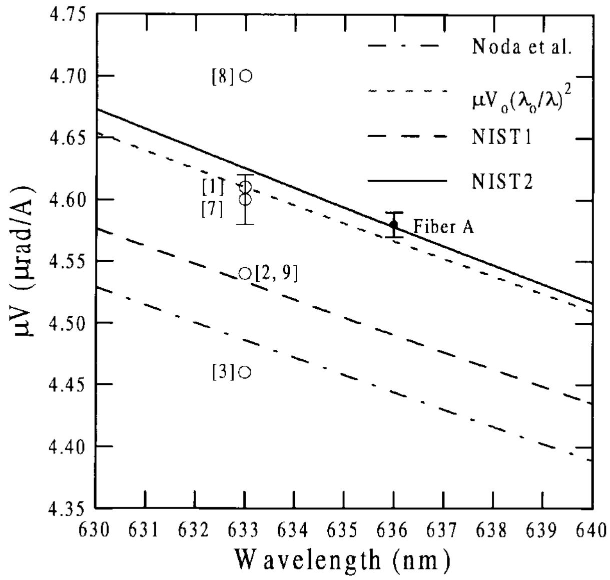

Fig. 3. Verdet constant measurements from 630 to 640 nm. The • mark our work with fiber A and o the mark the work of others.

Our fitted V_H value for the NIST1 model is comparable to the value of Noda *et al.* even though they made their measurements with high birefringent fiber [6]. The fiber used was a PANDA type with a Ge-doped silica core. Also, to obtain the dispersion of μV they relied on the work of others for the short wavelength regions [6].

As shown in Fig. 3, the value of Lassing *et al.* at 633 nm compares well with our value using fiber A at 636 nm, the value of Garn *et al.* in bulk SiO_2, and with the NIST2 model [1], [7]. The Verdet value of Lassing *et al.*, 4.60±0.02 μrad/A, was checked against a Rogowski coil on a large current pulse-producing machine [7]. They observed less than a 0.6% difference between the optical fiber current sensor and the Rogowski coil.

Fig. 4 shows the measurements from 785 to 850 nm. The difference between fibers B and C could be related to the difference in the GeO_2 doping. However, from our measurement at 636 nm on fiber A, the GeO_2 doping appears to have little effect on the Verdet constant in optical fibers. Noda *et al.* also observed that the GeO_2 doping had little effect in their measurements [6].

Fiber D, used at 829 nm, was the same fiber used by Los Alamos National Laboratories (LANL) to measure a high current pulse [19]. The LANL measurements produced a 6.1% discrepancy between the current measured with a Rogowski coil and a fiber current sensor using their estimate of $\mu V = 2.72$ μrad/A [19]. With their estimate of μV, the fiber sensor recorded currents lower than the Rogowski coil. In those tests large currents, greater than 100 kA, and a twisted version of the same fiber was used to reduce the effects of δ to near zero [19]. With the twisted fiber the Faraday rotation at those large currents was much greater than the linear birefringence of the fiber so δ would have a negligible effect on the current measurement. From the LANL work, $\mu V = 2.55$ μrad/A can be determined when the estimated μV is corrected with the Rogowski coil currents.

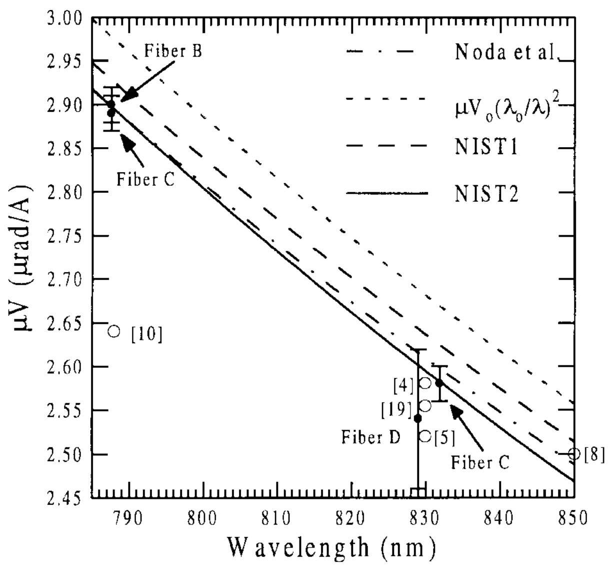

Fig. 4. Verdet constant measurements from 785 to 850 nm. The • mark our measurements with fibers B, C, and D, and the ∘ mark the work of others.

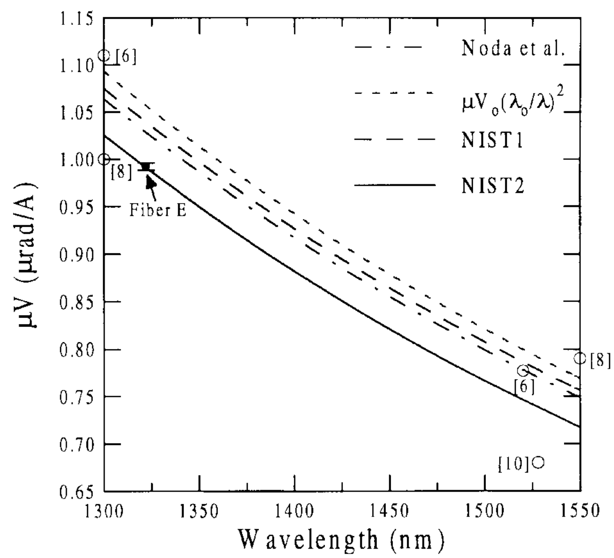

Fig. 5. Verdet constant measurement from 1300 to 1550 nm. The • mark our work with fiber E and the ∘ mark the work of others.

With our measured value for μV at 2.54±0.02 μrad/A ($\pm 3\sigma/\sqrt{n}$), the LANL fiber current sensor measurements would increase by about 6.6%. This would reduce the measurement discrepancy between the Rogowski coil and the fiber sensor to about 0.5%, which is within our uncertainty (0.8%) for μV at this wavelength with fiber D.

Fiber C was measured at a second wavelength, 831.9 nm, to compare with fiber D. The two Verdet constants compare well with each other, as shown in Fig. 4. The difference between these two fibers may be due to the GeO_2 doping or more likely due to the effects of residual retardance in fiber D.

Fig. 5 shows the four models (classical, Noda *et al.*, NIST1, and NIST2) with our Verdet value at 1322.3 nm and the values of Cruz *et al.*, Donati *et al.*, and Noda *et al.* from 1300 to 1550 nm [6], [8], [10]. Fig. 6 shows all the measured values, both ours and others, for μV from 600 to 1550 nm with the two models NIST1 and NIST2. The Verdet values at 1150 nm were measured by Donati *et al.*, and Noda *et al.* [6], [8].

IV. Conclusion

We have measured μV in optical fiber with an average accuracy of ±0.6% and an average measurement uncertainty of ±0.5% over the 636 to 1320 nm range. We compared the classical model, tied to one value of μV_o at 633 nm, to our experimental values. The model predicts larger values for μV, on average by about 3% in the 800 nm range and about 10% in the 1300 nm range. Noda *et al.* and NIST1 are better fits to the data than the single-value-classical model. We confirmed the work of Noda *et al.* by performing a similar fit to our data and obtained a value for V_H very near 2.0 × 10^{-35} μrad/(A·Hz^2) [6]. However, we added greater precision and a parameter standard error shown in Table III. GeO_2-doping effects appear minimal and agree with the conclusions of Noda *et al.* [6].

NIST2 adds a nonlinear component, the first term in a Taylor series expansion of the classical model, and improves the fit to the experimental data as measured by the residual-mean-squares error. With these improved Verdet constant values and model, optical fiber current sensors could be evaluated for performance or calibrated to 0.6%.

The majority of our work was in the 780 to 850 nm range due to the popularity of diode laser light sources, fiber, and other optical components at these wavelengths. For longer

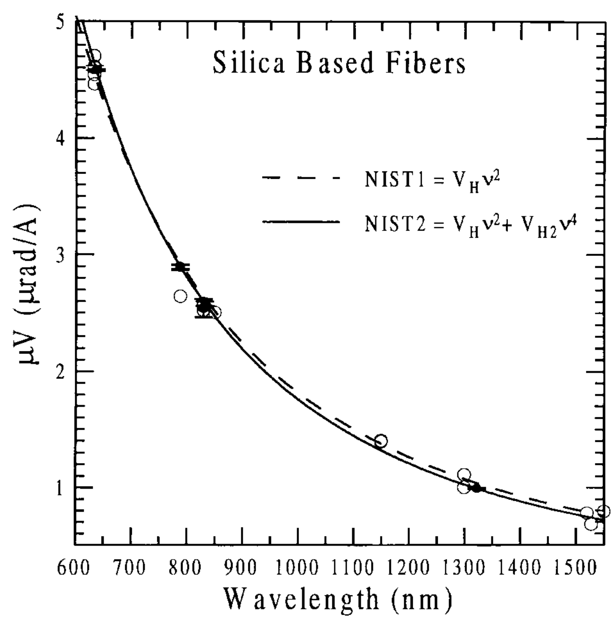

Fig. 6. Verdet constant measurement from 600 to 1550 nm. The • mark our work with fibers A, B, C, D, and E. The ∘ mark the work of others.

TABLE III
Verdet Constant Dispersion Models

Model	Parameter Estimate V_H or V_{H2}	Standard Error of Estimate	Residual Mean Squares
$\mu V = V_H \nu^2$	2.021 x 10^{-35} μrad/(A•Hz^2)	0.36%	5 x 10^{-16}
$\mu V = V_H \nu^2$	1.887 x 10^{-35} μrad/(A•Hz^2)	0.07%	3 x 10^{-18}
$+V_{H2} \nu^4$	7.80 x 10^{-66} μrad/(A•Hz^4)	0.9%	

wavelengths, 1300 to 1550 nm, we recommend more work than the present study, to give confidence to Verdet values in this wavelength region.

APPENDIX A

The remaining turn in an open fiber coil can be determined by knowing the length L, separation W of the coil leads, as shown in Fig. 2, and computing the normalized line integral of (1). Cylindrical coordinates will be used assuming an infinitely long wire running through the center and perpendicular to the plane of the coil. The magnetic flux produced by current in the wire is $B = \mu I/2\pi r$ and $dl = Nr\, d\theta$ where r is the fiber coil or light path radius and N is the complete number of turns. Making these substitutions for B and dl in (1) produce

$$\theta_F = \frac{\mu VNI}{2\pi}\left(\int_0^{\pi\pm\alpha} d\theta + \int_{\pi\mp\alpha}^{2\pi} d\theta\right) \tag{A1}$$

where $\alpha = \arctan(W/2L)$ and is the angle made by the open turn with respect to the center of the coil. The $\pm$ in the limits of the line integral allow for the leads crossing an even or odd number of times. The $+$ is for coils with the leads crossed an odd number of times, as shown with the dashed leads in Fig. 2, and the $-$ is for coils with the leads open or crossing an even number of times. Completing the integration in (A1) and making a substitution for α produce

$$\theta_F = \mu VNI\left(1 \pm \frac{\arctan(W/2L)}{\pi}\right). \tag{A2}$$

Normalizing (A2) to a perfectly closed fiber coil gives the partial turn produced by the fiber coil leads. The total number of turns N is determined by adding the partial turn to the complete number of turns N_o

$$N = N_o + \left(1 \pm \frac{\arctan(W/2L)}{\pi}\right). \tag{A3}$$

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