$N = 2$ $\quad$ $C(N)$ = set of all words formed by 1 & 2

$I = \{1, 2\}$

$$\beta = 22222\,1222\,\hat{2}$$

$I = \{1, 11, 2\}$

secure but not tight:

$$\beta = 11\,22\ldots$$

Observe: If I is secure, then so is $I \cup J$ for any J

(a) $\Rightarrow$ (b) $\quad$ S'pose $\hat{I} = C(N)$. $\quad$ WTS $C(N) = \bigcup_{\alpha \in I} \alpha^*$

$\subseteq$ S'pose $\beta \in C(N)$. Because $C(N) = \hat{I}$, $\exists\ \alpha_1, \alpha_2, \ldots$ with each $\alpha_j \in I$ such that

$$\beta = \alpha_1 \alpha_2 \alpha_3 \ldots$$

Thus $\beta \in \alpha_1^* \Rightarrow \beta \in \bigcup_{\alpha \in I} \alpha^*$.

$\supseteq$ S'pose $\beta \in \bigcup_{\alpha \in I} \alpha^* \subseteq C(N) = \hat{I}$.

$\boxed{\text{HW: Finish this. Email it to us by Tuesday. (Ben \& Xander)}}$

2.2. Maps in metric spaces. If $F: X \to X$, then we define the *Lipschitz constant* of F by

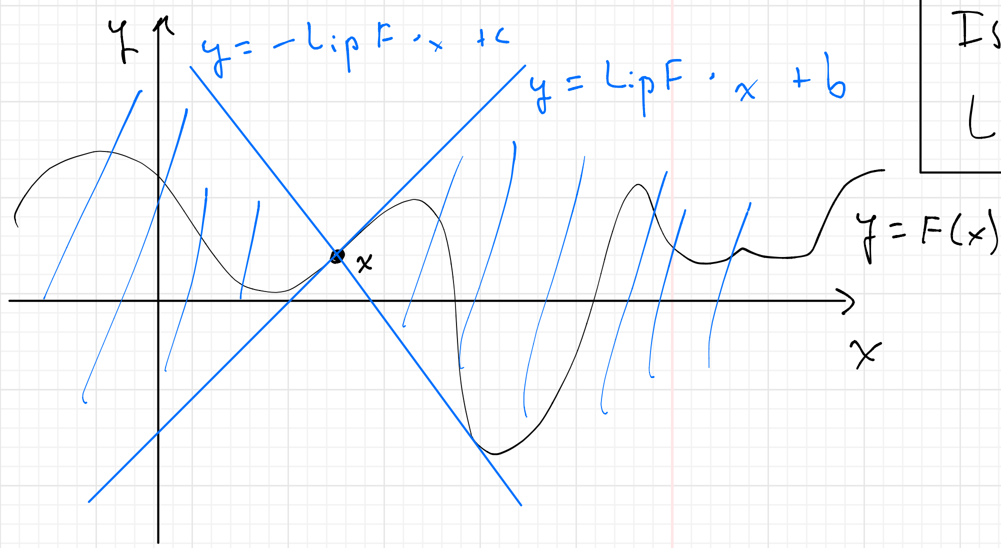
$$\operatorname{Lip} F = \sup_{x \neq y} \frac{d(F(x),F(y))}{d(x,y)}.$$

Of course if $\operatorname{Lip} F = \lambda$, then $d(F(x),F(y)) \leq \lambda d(x,y)$ for all $x, y \in X$, and moreover $\operatorname{Lip} F$ is the least such λ. We say F is *Lipschitz* if $\operatorname{Lip} F < \infty$ and F is a *contraction* if $\operatorname{Lip} F < 1$.

Take $X = \mathbb{R}$, $F : \mathbb{R} \to \mathbb{R}$

$$\operatorname{Lip} F = \sup_{x \neq y} \frac{|F(x) - F(y)|}{|x-y|}$$

HW: Is F diff iff $\operatorname{Lip} F < \infty$?
Is F cts iff $\operatorname{Lip} F < \infty$?



A Function with $\operatorname{Lip} F = \infty$:

- topologist's sine curve
- $x \mapsto \frac{1}{x}$
- $x \mapsto x^2$

$x \mapsto |x|$ is Lipschitz (i.e. $\operatorname{Lip} |\cdot| < \infty$).

$$\sup_{x \neq y} \frac{\big||x| - |y|\big|}{|x-y|} = 1 \qquad \operatorname{Lip} |\cdot| = 1$$

Google: Banach fixed point theorem
Contraction mapping principle

$\boxed{\text{prove this}}$

In Prop 2.3.1, Hutchinson says that the "only if" direction is clear. $\boxed{\text{Prove it}}$