Ex $\{ s_n = \sum_{j=1}^{n} \frac{1}{j} \}$ is not Cauchy

$m < n \qquad |s_n - s_m| = s_n - s_m$

$s_n = \cancel{1} + \cancel{\frac{1}{2}} + \dots + \cancel{\frac{1}{m}} + \frac{1}{m+1} + \dots + \frac{1}{n}$

$-s_m = \cancel{1} + \cancel{\frac{1}{2}} + \cancel{\frac{1}{3}} + \dots + \cancel{\frac{1}{m}}$

$\frac{1}{m+1} + \frac{1}{m+2} + \dots + \frac{1}{n-1} + \frac{1}{n}$

$= \sum_{j=m+1}^{n} \frac{1}{j}$

$$= \sum_{j=m+1}^{n} \frac{1}{j} \geq \sum_{j=m+1}^{n} \frac{1}{n} = \frac{n-m}{n}$$

Choose $n > 2m$.

Then $\frac{n-m}{n} > \frac{m}{2m} = \frac{1}{2}$.

* If $\{s_n\}$ is Cauchy, then we can make $|s_n - s_m|$ small.
* WTS $\exists\, \varepsilon > 0$ s.t. for any $N > 0$ $\exists\, m, n > N$ such that

$$|s_n - s_m| > \varepsilon$$

$$s_{2^n} = 1 + \frac{1}{2} + \underbrace{\frac{1}{3} + \frac{1}{4}}_{> \frac{1}{2}} + \underbrace{\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}}_{> \frac{1}{2}} + \dots$$

$$\dots + \underbrace{\frac{1}{2^{n-1}} + \dots + \frac{1}{2^n}}_{> \frac{1}{2}}$$

$$\approx n \tfrac{1}{2}$$

$$a_n = \sum_{j=0}^{n} \frac{1}{10^{10^n}}$$

$$0.1\underbrace{00\cdots00}_{10^{10}}1\underbrace{00\cdots00}_{10^{100}}100\cdots$$

C(2)

$$C(2) = \prod_{n=1}^{\infty} \{1,2\} = \{ \alpha_1 \alpha_2 \alpha_3 \dots \mid \alpha_j \in \{1,2\} \}$$

$$\prod_{n=1}^{2} \{1,2\} = \{1,2\} \times \{1,2\} = \{(1,1),(1,2),(2,1),(2,2)\}$$

$$\prod_{n=1}^{3} \{1,2\} = \{ (1,1,1), (1,1,2), (1,2,1), (1,2,2), (2,1,1), (2,1,2), (2,2,1), (2,2,2) \}$$

$$= \{ (\alpha_1, \alpha_2, \alpha_3) \mid \alpha_j \in \{1,2\} \}$$

$$= \{ \alpha_1 \alpha_2 \alpha_3 \mid \alpha_j \in \{1,2\} \}$$

$\omega = 1 2 1 1 1 2 1 1 2 1 \dots \in C(2)$

what is $\sigma_1 \omega$?

$\sigma_1 \omega = 1\omega = 1 1 2 1 1 1 2 1 1 2 1 \dots$

$$\boxed{\begin{array}{l}\text{Finish § 2.1} \\ \text{Prove Prop 8}\end{array}}$$