Agenda for Math 005.003 (3 June 2020, 11–11:50 am):

1. Q&A—Have questions ready!

S2. (a) It takes five workers three hours to shovel 400 cubic feet of sand. How many hours are required by seven workers to shovel 800 cubic feet of sand?

$$\text{Work } (\text{ft}^3 \text{ of sand}) = C \times (\# \text{ of workers}) \times (\text{time})$$

$$\rightsquigarrow \quad W \text{ ft}^3 = C \cdot (P \text{ workers})(T \text{ hrs})$$

Given: $400 \text{ ft}^3 = C\,(5 \text{ workers})\,(3 \text{ hrs})$

$$\Rightarrow C = \frac{400 \text{ ft}^3}{15 \text{ workers}\cdot\text{hrs}} = \frac{80}{3}\ \frac{\text{ft}^3}{\text{worker}\cdot\text{hr}}$$

Question: $800 \text{ ft}^3 = C\,(7 \text{ workers})\,(T \text{ hrs})$

$$= \left(\frac{80}{3}\ \frac{\cancel{\text{ft}^3}}{\cancel{\text{worker}}\ \text{hr}}\right)(7\ \cancel{\text{workers}})(T \text{ hrs})$$

$$\Rightarrow T \text{ hrs} = \frac{10\ \cancel{800}}{\frac{\cancel{80}}{3}\cdot 7\ \frac{1}{\text{hr}}} = \boxed{\frac{30}{7} \text{ hrs}}$$

(b) You require two ounces of paint (2 oz) to spray paint a certain sphere. How much paint do you require to spray paint a sphere with 27 times the volume?

Volume of Paint Needed (in oz)

= Thickness (in) × Surface Area being Painted (in^2)

(1) $V \text{ oz} = T \cdot (A \text{ in}^2)$

thickness + some conversion factor to go from in^3 to oz = constant

(2a) vol of a sphere $= \frac{4}{3}\pi\, r^3 \text{ in}^3 = C_1\,(r \text{ in})^3 \Rightarrow r \text{ in} = \sqrt[3]{\frac{Vol}{C_1}}$

C_1 ; radius

(2b) surface area of a sphere $= C_2\,(r \text{ in})^2$

Suppose that the original sphere has volume 1 in^3. The new sphere has volume 27 in^3.

$$V \text{ oz} = T \cdot (A \text{ in}^2) = T\ C_2 (r \text{ in})^2$$

$$= T\ C_2 \left(\sqrt[3]{\frac{Vol}{C_1}} \text{ in}\right)^2$$

$$= \left(T\, C_2 \frac{1}{\sqrt[3]{C_1}}\right) \sqrt[3]{Vol}^{\,2}$$

$$= C\ \sqrt[3]{Vol}^{\,2}$$

$$\Rightarrow \boxed{V \text{ oz} = C\ \sqrt[3]{Vol}^{\,2}}$$

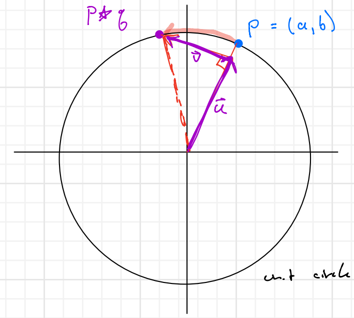
Given: $2 \text{ oz} = C\left(\sqrt[3]{1}^{\,2} \text{ in}^2\right) \Rightarrow C = 2\ \frac{\text{oz}}{\text{in}^2}$

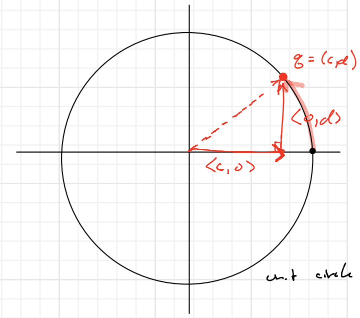
Question: $V \text{ oz} = C\left(\sqrt[3]{27}^{\,2} \text{ in}^2\right) = \left(2\ \frac{\text{oz}}{\cancel{\text{in}^2}}\right)\left(9\ \cancel{\text{in}^2}\right) = \boxed{18 \text{ oz}}$

Facts: Area $= C_A\, \ell^2$

Volume $= C_V\, \ell^3$

constants of proportionality





$$q = \langle c,0\rangle + \left(\langle 0,d\rangle + (0,0)\right)$$
$$= \langle c,d\rangle + (0,0)$$

$$p \star q = \vec{v} + \left(\vec{u} + (0,0)\right)$$

$\vec{u}$: is a vector in the direction of p with length $\|\langle c,0\rangle\| = c$

$\Rightarrow \vec{u} = c\ \underbrace{\langle a,b\rangle}_{\text{length } 1}$

$\vec{v}$: is a vector perp to $\vec{u}$ with length $\|\langle 0,d\rangle\| = d$

$\Rightarrow \vec{v} = d\,\langle -b, a\rangle$

$$p \star q = d\langle -b,a\rangle + \left(c\langle a,b\rangle + (0,0)\right)$$
$$= \langle -db, da\rangle + \left(\langle ca, cb\rangle + (0,0)\right)$$
$$= (ac - bd,\ bc + ad)$$

$$\boxed{(a,b) \star (c,d) = (ac - bd,\ bc + ad)}$$