Agenda for Math 005.003 (1 June 2020, 11—11:50 am)

1. Midterm 2 ✓
2. Worksheet 10

Continuously compounding interest and unrestricted population growth are both examples of quantities modeled by geometric growth. Suppose that you eat something contaminated with pathogenic *E. coli*. You ingest only 1,000 of these bacteria. Their population doubles every twenty minutes and undergoes unrestricted growth.

1. How many bacteria will there be in 20 minutes?

2. How many bacteria will there be in 40 minutes?

3. Set up an equation that describes the number of bacteria you will have at time t. What is the growth rate of this population?

(1) 2000

(2) 4000

(3)

t	$P(t)$ (pop @ time t)	Exponent
0	$1000 = 1000 \cdot 2^0$	$0 = 1/20 \cdot t$
20	$2000 = 1000 \cdot 2^1$	$1 = \frac{1}{20} \cdot t$
40	$4000 = 1000 \cdot 2^2$	$2 = \frac{1}{20} \cdot t$
60	$8000 = 1000 \cdot 2^3$	$3 = 1/20 \cdot t$
80	$16000 = 1000 \cdot 2^4$	$4 = 1/20 \cdot t$

$$\Rightarrow \boxed{P(t) = 1000 \cdot 2^{\frac{1}{20} t}}$$

If $f(t) = c e^{kt}$, then $k =$ (intrinsic) growth rate.

$$P(t) = \cancel{1000} \cdot 2^{\frac{1}{20} t} = \cancel{1000} \cdot e^{kt} \quad \text{(solve for } k\text{)}$$

$$\Rightarrow 2^{\frac{1}{20} t} = e^{kt}$$

$$\Rightarrow \ln\left(2^{\frac{1}{20} t}\right) = \ln\left(e^{kt}\right) = kt$$

$$\Rightarrow k = \frac{\ln\left(2^{t/20}\right)}{t} \underset{t=1}{=} \ln\left(2^{1/20}\right) = \frac{1}{20}\ln(2)$$

$$\underset{t=4}{=} \frac{\ln\left(2^{4/20}\right)}{4} = \frac{\frac{4}{20}\ln(2)}{4}$$

$$= \frac{1}{20}\ln(2)$$

$$\boxed{\begin{array}{l} x^n = y \iff x = \sqrt[n]{y} \\ e^x = y \iff x = \ln(y) \end{array}}$$

(4) $P(50) = \ldots$

(5) $P(T) = 5{,}000{,}000 = 1000 \cdot 2^{T/20}$ (solve for T)