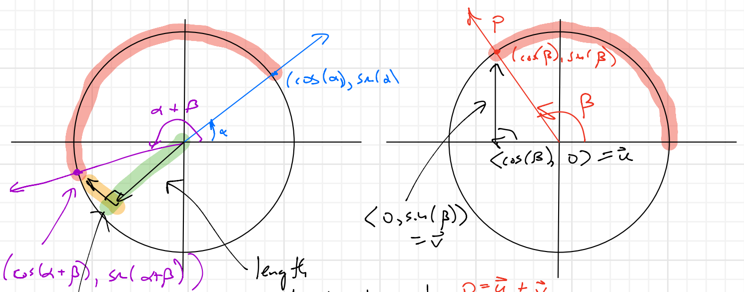
Agenda for Math 005.003 (20 May 2020, 11–11:50 am):

1. The angle addition formula for cosine
2. Q&A



length $= \|\vec u\| = |\cos(\beta)|$ $\rightsquigarrow$ $\langle \cos(\alpha), \sin(\alpha)\rangle \cdot \cos(\beta)$

$P = \vec u + \vec v$

length $= \|\vec v\| = |\sin(\beta)|$ $\rightsquigarrow$ $\langle -\sin(\alpha), \cos(\alpha)\rangle \sin(\beta)$

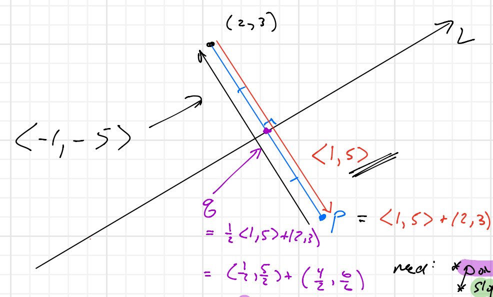
$$(\cos(\alpha+\beta), \sin(\alpha+\beta)) = \cos(\beta)\langle \cos(\alpha), \sin(\alpha)\rangle + \sin(\beta)\langle -\sin(\alpha), \cos(\alpha)\rangle$$

$$= \langle \cos(\alpha)\cos(\beta), \sin(\alpha)\cos(\beta)\rangle + \langle -\sin(\alpha)\sin(\beta), \cos(\alpha)\sin(\beta)\rangle$$

$$= \langle \underbrace{\cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)}_{\cos(\alpha+\beta)}, \underbrace{\sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)}_{\sin(\alpha+\beta)} \rangle$$

II.5.1 #4

Problem 4. Find a line L such that reflection of the point $(2,3)$ across L is equivalent to adding the vector $\langle 1,5\rangle$ to $(2,3)$. Which points in the plane will have the property that reflection across L will be equivalent to translation by $\langle 1,5\rangle$? What translation corresponds to the reflection of $\langle 1,5\rangle + (2,3)$ across L?



$Q = \frac12\langle 1,5\rangle + (2,3)$

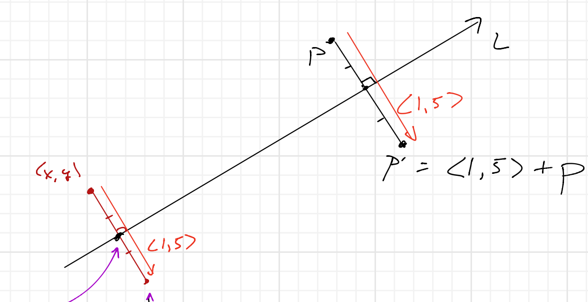
$= \langle \frac12, \frac52\rangle + (\frac42, \frac62)$

$= (\frac52, \frac{11}{2})$

need: * point on L * slope of L

$L \perp \langle 1,5\rangle$ which means that the slope of L is $-\frac15$

$L: \quad y - \frac{11}{2} = -\frac15\left(x - \frac52\right)$



$\underbrace{\frac12\langle 1,5\rangle + (x,y)}$ is on L $\qquad \langle 1,5\rangle + (x_0, y_0)$

$(\frac12 + x_0, \frac52 + y_0) \leftarrow y - \frac{11}{2} = -\frac15\left(x - \frac52\right)$

$\Longrightarrow \left(\frac52 + y_0\right) - \frac{11}{2} = -\frac15\left(\left(\frac12 + x_0\right) - \frac52\right)$

solve for y_0 $\Longrightarrow$ $y_0 - \frac62 = -\frac{1}{10} - \frac{x_0}{5} + \frac12$

$= -\frac{x_0}{5} + \frac{4}{10}$

$\Longrightarrow \quad y_0 = -\frac{x_0}{5} + \frac{4}{10} + \frac62 \; \frac{30}{10}$

$= -\frac{x_0}{5} + \frac{34}{10}$

$= -\frac15 x_0 + \frac{17}{5}$

The collection of points with the desired property is all of the points which satisfy

$$\underbrace{y = -\frac15 x + \frac{17}{5}.}_{\text{line } \parallel \text{ to } L \text{ thru } (2,3)}$$