Agenda for Math 005.003 (18 May 2020, 11–11:50 am):

1. Worksheet 8

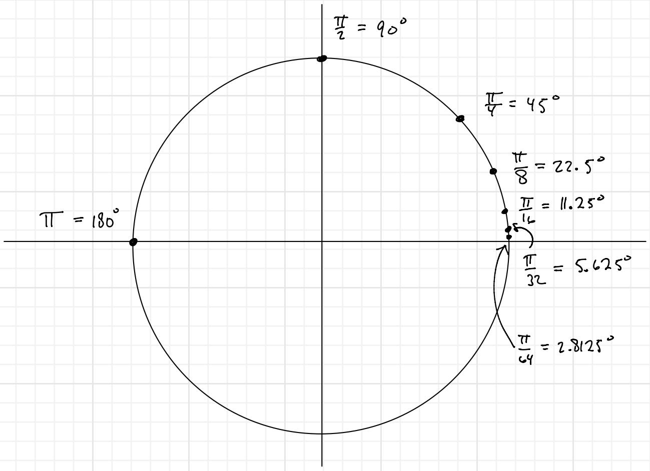
???

1. A point p on the unit circle is said to be a principle dyadic angle if the radian measure of p is zero or $\frac{2\pi}{2^n}$ for some natural number n. Draw a unit circle. Then mark and label the first seven non-zero principle dyadic angles on it, giving both degree and radian measures of the labeled angles.

Examples: $0, \frac{2\pi}{4}$ $(n=2)$, $\cancel{\frac{2\pi}{6}\ (n=}$ $\frac{2\pi}{8}$ $(n=3)$ $\qquad n \geq 1$

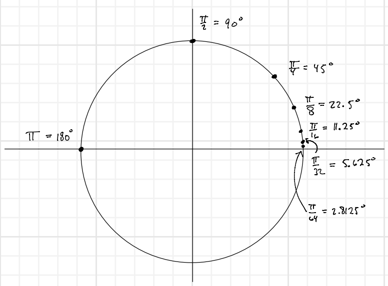
$0, \frac{2\pi}{2}, \frac{2\pi}{4}, \frac{2\pi}{8}, \frac{2\pi}{16}, \frac{2\pi}{32}, \frac{2\pi}{64}, \frac{2\pi}{128}$

$(n=1)$ $(n=2)$ $(n=3)$ $(n=4)$ $(n=5)$ $(n=6)$ $(n=7)$



Solution: The first seven nonzero principle dyadic angles are $\pi, \frac{\pi}{2}, \frac{\pi}{4}, \frac{\pi}{8}, \frac{\pi}{16}, \frac{\pi}{32}$, and $\frac{\pi}{64}$.

These are shown in the picture below.

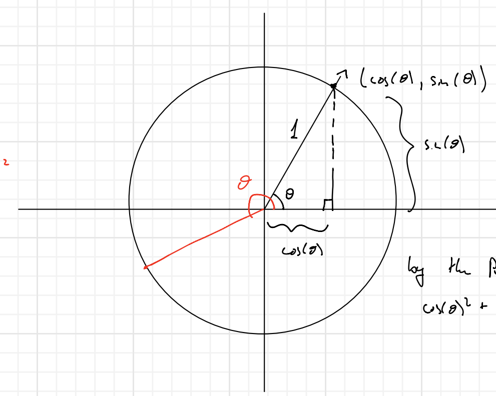


2. Use the half-angle formula for the cosine function to calculate cos(90°), cos(45°), cos(22.5°), cos(11.25°), and cos(5.625°).

1) Pythagorean Identity: for any angle θ, $\sin(\theta)^2 + \cos(\theta)^2 = 1$. $\longrightarrow \sin(\theta)^2 = 1 - \cos(\theta)^2$

2) Angle Addition for Cosine: for any two angles θ & φ,

$$\cos(\theta + \varphi) = \cos(\theta)\cos(\varphi) - \sin(\theta)\sin(\varphi).$$



by the Pythagorean Thm, $\cos(\theta)^2 + \sin(\theta)^2 = 1$

WTF a formula which gives $\cos(\alpha/2)$ in terms of things I already know. If I take $\theta = \varphi = \alpha/2$, then (2) becomes

α - alpha
θ - theta
φ - phi

$$\cos(\theta + \varphi) = \cos\left(\tfrac{\alpha}{2} + \tfrac{\alpha}{2}\right) = \cos(\alpha)$$

$$\begin{aligned} &\parallel \\ &\cos(\theta)\cos(\varphi) - \sin(\theta)\sin(\varphi) \\ &= \cos(\tfrac{\alpha}{2})\cos(\tfrac{\alpha}{2}) - \sin(\tfrac{\alpha}{2})\sin(\tfrac{\alpha}{2}) \\ &= \cos(\tfrac{\alpha}{2})^2 - \sin(\tfrac{\alpha}{2})^2 \end{aligned}$$

$$\Rightarrow \cos(\alpha) = \cos(\tfrac{\alpha}{2})^2 - \underbrace{\sin(\tfrac{\alpha}{2})^2}_{???} = \cos(\tfrac{\alpha}{2})^2 - \left(1 - \cos(\tfrac{\alpha}{2})^2\right)$$

$$= 2\cos(\tfrac{\alpha}{2})^2 - 1$$

$$\Rightarrow \cos(\tfrac{\alpha}{2}) = \pm\sqrt{\frac{\cos(\alpha) + 1}{2}}$$

(sign depends on quadrant) $\longleftarrow$ HALF ANGLE FORMULA FOR COSINE

$$\cos\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{\cos(\alpha)+1}{2}}$$ for any α

- $\cos(180^\circ) = {}^-1$
- $\cos(90^\circ) = \pm\sqrt{\frac{\cos(180^\circ)+1}{2}} = \pm\sqrt{\frac{0}{2}}$ $\left(\begin{array}{l}90^\circ = 180^\circ/2 \\ \text{set } \alpha = 180^\circ\end{array}\right)$

  $= 0$

- $\cos(45^\circ) = \pm\sqrt{\frac{\cos(90^\circ)+1}{2}} = \sqrt{\frac{1\cdot 2}{2\cdot 2}} = \frac{\sqrt{2}}{2}$ (set $\alpha = 90^\circ$)

  $\hookrightarrow \sqrt{\frac{2}{4}} = \frac{\sqrt{2}}{\sqrt{4}} = \frac{\sqrt{2}}{2}$

- $\cos(22.5^\circ) = \pm\sqrt{\frac{\cos(45^\circ)+1}{2}} = \sqrt{\frac{\left(\frac{\sqrt{2}}{2}+1\right)2}{2\cdot 2}} = \sqrt{\frac{\sqrt{2}+2}{4}}$

  $= \frac{\sqrt{2+\sqrt{2}}}{2}$

KEEP GOING...