Agenda for Math 005.003 (13 May 2020, 11–11:50 am):

1. Q&A

4. Can you change the speed of Boat 2 to ensure that the boats collide but maintain the same path of motion? If so how fast should Boat 2 go and what will be its new velocity vector? Hint: It may be helpful to consider the distance between the boats and scale the velocity vector of Boat 2 by a constant.

Know: $p(t) = t\vec{v} + p_0$

"position at time t is t times velocity plus an initial position"

displacement

$p_1(t) = t\langle 10, 4\rangle + \langle 2, 3\rangle = (10t+2, 4t+3)$

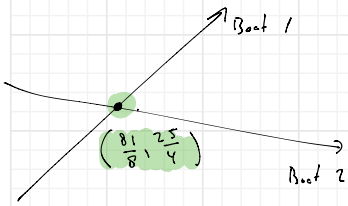
$p_2(t) = t\langle 1, 2\rangle + \langle 9, 4\rangle = (t+9, 2t+4)$

$y - 3 = \frac{2}{5}(x-2) \Rightarrow 5y - 15 = 2x - 4 \Rightarrow 5y = 2x + 11$

$y - 4 = 2(x-9) \Rightarrow y - 4 = 2x - 18 \Rightarrow -y = -2x + 14$

$4y = 25$

$y = \frac{25}{4}$



$\frac{25}{4} = y = 2x - 14$

$\Rightarrow 25 = 8x - 56$

$\Rightarrow 8x = 81$

$\Rightarrow x = \frac{81}{8}$

$(10t+2, 4t+3) = (\frac{81}{8}, \frac{25}{4})$?

$\Leftrightarrow 10t + 2 = \frac{81}{8}$

$\Rightarrow 80t + 16 = 81$

$\Rightarrow 80t = 65$

$\Rightarrow t = \frac{65}{80} = \frac{13}{16}$

for now, assume that this is correct

$\Leftrightarrow 4t + 3 = \frac{25}{4}$

$\Rightarrow 16t + 12 = 25$

$\Rightarrow 16t = 13$

$\Rightarrow t = \frac{13}{16}$

MISTAKE Fix this

HW!!

* The boats both pass through some point $(\frac{81}{8}, \frac{25}{4})$ ← WRONG

* Boat 1 passes through that point at time $t = \frac{13}{16}$ ← WRONG

To change the speed of Boat 2, we need to change the length of the velocity vector. This can be done by multiplying by a scalar (i.e. a real number).

$\vec{v}_{new} = k\vec{v}_{old} = k\langle 1, 2\rangle$ (where k is some yet-unknown number)

$\Rightarrow p_2(t) = t\vec{v}_{new} + (9, 4)$

$= tk\langle 1, 2\rangle + (9, 4)$

$= (tk + 9, 2tk + 4)$ ← position of boat 2 at time t if the velocity is scaled by k

Goal: find k such that $p_1(t) = p_2(t)$ when $t = \frac{13}{16}$

$\Leftrightarrow (\frac{81}{8}, \frac{25}{4}) = (tk + 9, 2tk + 4)$

$\Leftrightarrow \begin{cases} \frac{81}{8} = \frac{13}{16}k + 9 \\ \frac{25}{4} = 2 \cdot \frac{13}{16}k + 4 \end{cases}$ Solve for k