Agenda for Math 005.003 (11 May 2020, 11–11:50 am):

1. Discuss Worksheet 7

For the following problems, you will assume the following: Initially at time zero, Boat 1 is at $(2,3)$ and Boat 2 is at $(9,4)$. Boat 1 is traveling four miles per hour North (the positive y direction) and ten miles per hour East. Boat 2 is traveling two miles per hour North and one mile per hour East.

1. What are the vector equations for the positions of Boat 1 and Boat 2 as functions of time? How fast is each boat moving?

2. Independent of time, describe the paths of motion of Boat 1 and Boat 2. Do the lines that the two boats travel along intersect?

3. Will the boats crash into each other?

Do #1. When you have an answer, hit the thumbs up button in the participants window.

1) $\vec{v}_1 = \langle 10, 4 \rangle$, $P_1(0) = (2,3)$

$P_1(t) = t\vec{v}_1 + P_1(0) = t\langle 10,4\rangle + (2,3) = (10t+2, 4t+3)$

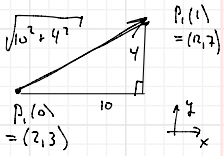
$P_1(t)$: position of boat 1 at time t; $t\vec{v}_1$: displacement; $P_1(0)$: initial position

$P_2(t) = t\langle 1,2\rangle + (9,4) = (t+9, 2t+4)$

$\text{speed} = \frac{\text{miles}}{\text{hour}} = \frac{d(P_1(0), P_1(1))}{1 \text{ hr}}$ ← distance from $t=0$ to $t=1$ hr

$= \frac{\sqrt{10^2+4^2} \text{ miles}}{1 \text{ hour}}$

$= \sqrt{116}$ miles/hr



Note: $\sqrt{116} = \sqrt{10^2+4^2} = \|\langle 10,4\rangle\| = \|\vec{v}_1\|$

In general, speed = ||velocity||. Thus boat 2's speed

$\|\vec{v}_2\| = \|\langle 1,2\rangle\| = \sqrt{1^2+2^2} = \sqrt{5}$ mph.

Solution: Position at time t is given by

$p(t) = t\,(\text{velocity}) + (\text{initial position})$.

Let $P_1(t)$ and $P_2(t)$ denote the positions of boats 1 and 2 at time t, respectively. Then

$P_1(t) = t\langle 10,4\rangle + (2,3) = (10t+2, 4t+3)$, and

$P_2(t) = t\langle 1,2\rangle + (9,4) = (t+9, 2t+4)$.

Speed is the length of the velocity vector, so

speed of boat 1 $= \|\langle 10,4\rangle\|$ mph $= \sqrt{116}$ mph, and

speed of boat 2 $= \|\langle 1,2\rangle\|$ mph $= \sqrt{5}$ mph. □

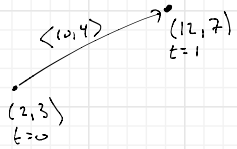
2) What is the path of motion of boat 1?

$P_1(t) = (10t+2, 4t+3)$

slope $= \frac{4}{10}$ ← related to $\langle 10,4\rangle$

point $= (2,3)$

$\Rightarrow y - 3 = \frac{4}{10}(x-2)$



$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{3-7}{2-12} = \frac{-4}{-10} = \frac{4}{10}$

Path for boat 1 is $y-3 = \frac{4}{10}(x-2)$

Path for boat 2 is $y-4 = \frac{2}{1}(x-9)$

Intersect? Yes. Their slopes are not equal.

3) $P_1(t) = (10t+2, 4t+3)$

$P_2(t) = (t+9, 2t+4)$

Then two boats crash if they are in the same place at the same time.

$\Leftrightarrow$ there is some time t such that $P_1(t) = P_2(t)$.

$\Leftrightarrow (10t+2, 4t+3) = (t+9, 2t+4)$

$\Leftrightarrow \begin{cases} 10t+2 = t+9 \\ 4t+3 = 2t+4 \end{cases}$

$9t = 7 \Rightarrow t = 7/9$

$2t = 1 \Rightarrow t = 1/2$

Therefore the boats crash if and only if $\frac{7}{9} = t = \frac{1}{2}$.

$\frac{7}{9} \neq \frac{1}{2}$, Therefore the boats don't crash.