Agenda for Math 005.003 (4 May 2020, 11–11:50 am):

1. May the fourth be with you.
2. Worksheet 6.

1. Find the minimum y value of a point on the graph given by the set of points satisfying

$$y = x^2 + x + 1.$$

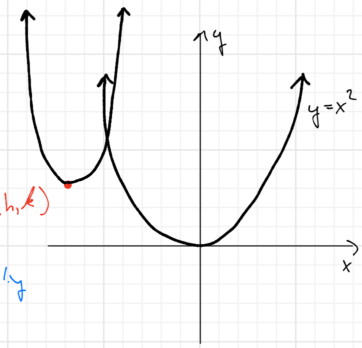
2. Find the maximum y value of a point on the graph given by the set of points satisfying

$$y = -x^2 + 2x + 4.$$

Parabola:

$$y = a(x-h)^2 + k$$

a accounts for:
* horizontal & vertical scaling
* reflections



For prob 1, the graph of x^2+x+1 is a parabola, and the min is at the lowest point, which is the vertex.

$$x^2 + x + 1 = a(x-h)^2 + k$$

where a is some scaling factor, and (h,k) is the vertex

the minimum

$$1x^2 + 1x + 1 = ax^2 - 2ahx + ah^2 + k$$

$$\begin{cases} 1 = a \\ 1 = -2ah = -2h \Rightarrow h = -\frac{1}{2} \\ 1 = ah^2 + k \end{cases}$$

$$= 1\left(-\tfrac{1}{2}\right)^2 + k$$

$$= \frac{1}{4} + k \Rightarrow \boxed{k = \frac{3}{4}}$$

The minimum y-value is $\frac{3}{4}$.

Prob 2

$$-x^2 + 2x + 4 = a(x-h)^2 + k,$$

(h,k) is the vertex
k = max y-value

If we know what the graph of f looks like, then
* $f(x-h)$ is translated horizontally
* $f(x)+k$ is translated vertically
* $f(-x)$ is reflected across the y-axis
* $-f(x)$ is reflected across the x-axis
* $f(\frac{x}{a})$ is a horizontal scaling
* $b\,f(x)$ is a vertical scaling

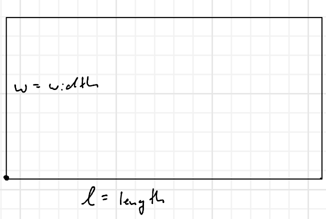
$$\rightsquigarrow f(x) = \frac{a^2}{a^2} b\left(\frac{x}{a} - h\right)^2 + k$$

$$= \frac{b}{a^2} \cdot \left(a^2\left(\frac{x}{a} - h\right)^2\right) + k$$

$$= \frac{b}{a^2}\left(\cancel{a}\frac{x}{\cancel{a}} - ha\right)^2 + k$$

rename this "a" rename this "h"

3. You have 400 feet of fencing. You will fence in a rectangular region of your backyard. What should the side lengths of the rectangle be so that the area is maximal? What shape will the yard be?



goal: to maximize area

$$A = \ell w$$

$$400 \text{ ft} = 2\ell + 2w$$

$$\Rightarrow \ell = 200 - w$$

$$A = \ell w$$

$$= (200 - w)w$$

$$A = -w^2 + 200w$$

Find the vertex:

$$-w^2 + 200w = a(w-h)^2 + k \qquad \text{find } k$$

$$= aw^2 - 2ahw + ah^2 + k$$

$a = -1$; $-2h = 200 \Rightarrow h = 100$; $0 = ah^2 + k = -100^2 + k \Rightarrow k = 10{,}000$

In this problem, (h,k) is the vertex.
* h is a width
* k is an area

The maximum area is $k = 10{,}000 \text{ ft}^2$ which happens when the rectangle is $h = 100$ ft wide. The rectangle will be

$$\ell = 200 - h = 100 \text{ ft}$$

long.