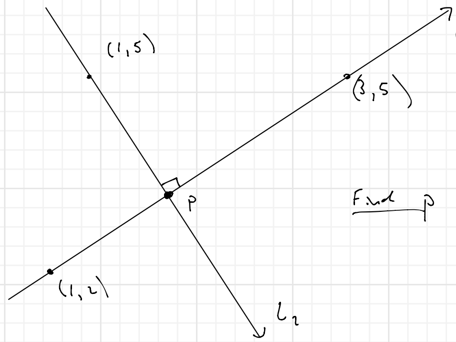
Agenda for Math 005.003 (29 April 2020, 11–11:50 am

1. Q&A (Exam review, worksheet 5, etc.)

A2. Let L_1 be the line passing through $(1,2)$ and $(3,5)$. Let L_2 be the line passing through $(1,5)$ and perpendicular to L_2. Where do L_1 and L_2 intersect?



If I want to find the intersection of two lines, I can solve the system of equations given by equations for the lines.

① Find an equation for L_1.

② Find an equation for L_2.

③ Solve the system.

✓ ① The slope of L_1 is $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{5-2}{3-1} = \dfrac{3}{2}$.

$\rightsquigarrow$ $y = \frac{3}{2}x + b$ is solved by $(x,y) = (1,2)$

$\Rightarrow 2 = \frac{3}{2}(1) + b \Rightarrow b = \frac{1}{2}$

$\Rightarrow L_1$ is given by $\boxed{y = \frac{3}{2}x + \frac{1}{2}}$

OR $\rightsquigarrow$ $y - y_1 = m(x - x_1)$ (pt-slope)

$\Rightarrow y - 2 = \frac{3}{2}(x-1)$

✓ ② L_2 is given by $y - 5 = \boxed{-\frac{2}{3}}(x-1)$

$L_2 \perp L_1$ and the slopes of $\perp$ lines are negative reciprocals

$\Rightarrow \boxed{y = -\frac{2}{3}x + \frac{17}{3}}$

③ We are looking for a point P which is on both lines. So if $P = (x,y)$ then

$$\begin{cases} ① & y = \frac{3}{2}x + \frac{1}{2} \quad \text{AND} \\ ② & y = -\frac{2}{3}x + \frac{17}{3} \end{cases}$$

$$\Rightarrow \begin{cases} (2y = 3x + 1)\,2 & \Rightarrow 4y = \cancel{6x} + 2 \\ (3y = -2x + 17)\,3 & \Rightarrow 9y = \cancel{-6x} + 51 \end{cases}$$

$13y = 53$

$\boxed{\Rightarrow y = \frac{53}{13}}$

$2y = 3x + 1 \Rightarrow 3x = 2y - 1$

$\Rightarrow x = \dfrac{2y-1}{3} = \dfrac{2\left(\frac{53}{13}\right) - 1}{3}$

Final Answer: The point of intersection is

$$\boxed{\left(\frac{2\left(\frac{53}{13}\right)-1}{3}, \frac{53}{13}\right)}$$

S2. One yard is three feet. You build a box that has a volume of 100 cubic feet. It takes two people ten minutes to spray paint this box. How many minutes does it take three people to spray paint a box that has a volume of 500 cubic feet? Assume that the area of surface to be covered is proportionate to both the number of people working and the time spent spray painting.

- (Area being painted) $= \boxed{C_1 \cdot (\text{number of people})}$
  $= C_2 \cdot (\text{time spent painting})$

- How long does it take one person to paint some fixed area, say $1\ ft^3$?

$$\left(\text{Time} = \frac{\text{Area}}{C_2} = \frac{1}{C_2}\left(C_1 \cdot (\text{number of people})\right)\right)$$

rough idea

If a box is $100\ ft^3$, what is its surface area?

$$\underbrace{A = C_3 \sqrt[3]{Vol}^{\,2}}$$

Area is proportional to the square of the cube root of volume

$$A = \underbrace{C_1 \cdot N_{people} = C_4 \cdot N_{people} \cdot T_{min}}_{=A}$$

Area is proportional to the # of workers & time

$C_3 \sqrt[3]{V}^{\,2}\ ft^2 = C_4\ N\,T$ people·min

$\Rightarrow \boxed{\sqrt[3]{V}^{\,2}\ ft^2 = C\ N\,T \text{ people}\cdot\text{min}}$

① $\sqrt[3]{100}^{\,2}\ ft^2 = C \cdot 2 \cdot 10$ people·min

$\Rightarrow C = \dfrac{\sqrt[3]{100}^{\,2}}{20} \dfrac{ft^2}{\text{people}\cdot\text{min}}$

② $\sqrt[3]{500}^{\,2}\ ft^2 = C \cdot N \cdot T$ people·min

$= \left(\dfrac{\sqrt[3]{100}^{\,2}}{20} \dfrac{ft^2}{\cancel{\text{people}\cdot\text{min}}}\right)\left(3\ \cancel{\text{people}}\right)\left(T\ \cancel{\text{min}}\right)$

$\Rightarrow T_{min} = \dfrac{\sqrt[3]{500}^{\,2}}{\sqrt[3]{100}^{\,2}} \cdot \dfrac{20}{3}$ min