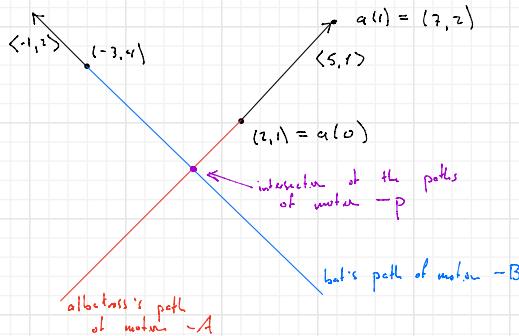Agenda for Math 005.003 (22 April 2020, 11–11:50 am):

1. Discuss Worksheet 4
2. Discuss any and all questions about the material

Albatross $\quad\begin{pmatrix}2,1\\-3,4\end{pmatrix}\quad\begin{array}{l}\langle5,1\rangle\\\langle-1,2\rangle\end{array}$
Bat

Do the albatross and bat ever meet (or did they meet in the past)?

$a(1) = (7,2)$
$\langle 5,1\rangle$
$(2,1) = a(0)$
intersection of the paths of motion — p
bat's path of motion — B
albatross's path of motion — A
$\langle -1,2\rangle \quad (-3,4)$

If the albatross and bat meet, they meet at p.

$A: \quad y - 1 = \frac{1}{5}(x-2)$
$B: \quad y - 4 = -2(x+3)$

$\begin{cases} y = \frac{1}{5}x + \frac{3}{5} \\ y = -2x - 2 \end{cases} \implies 0 = \left(\frac{1}{5}+2\right)x + \left(\frac{3}{5}+2\right)$

$= \frac{11}{5}x + \frac{13}{5}$

$\implies \frac{11}{5}x = \frac{-13}{5}$

$\implies \boxed{x = -\frac{13}{11}}$

$\implies y = -2\left(-\frac{13}{11}\right) - 2 = \frac{26}{11} - 2$

$\boxed{y = \frac{4}{11}}$

$\boxed{p = \left(-\frac{13}{11}, \frac{4}{11}\right)}$

Let $a(t)$ denote the albatross's location at time t, and $b(t)$ denote the bat's location.

$\boxed{\begin{aligned} a(t) &= t\langle 5,1\rangle + (2,1) = (5t+2, t+1) \\ b(t) &= t\langle -1,2\rangle + (-3,4) = (-t-3, 2t+4) \end{aligned}}$

When is the albatross at $p = \left(-\frac{13}{11}, \frac{4}{11}\right)$? That is, when is $a(t)$ equal to p?

$a(t) = p \iff (5t+2, t+1) = \left(-\frac{13}{11}, \frac{4}{11}\right)$

$\iff \begin{cases} 5t+2 = -\frac{13}{11} \to 5t = -\frac{35}{11} \implies t = -\frac{7}{11} \\ t+1 = \frac{4}{11} \to t = -\frac{7}{11} \end{cases}$

In English, the albatross is at p $\boxed{\frac{7}{11} \text{ ths}}$ of an hour before we started observing.

The bat is at p:

$b(t) = (-t-3, 2t+4) = \left(-\frac{13}{11}, \frac{4}{11}\right)$

$\iff -t-3 = -\frac{13}{11}$

$\iff -t = \frac{20}{11}$

$\iff t = -\frac{20}{11}$

The bat is at p $\boxed{\frac{20}{11} \text{ ths}}$ of an hour before we start observing.

Because the albatross and the bat are not at p at the same time, they never meet. ∎

$\boxed{\begin{aligned} a(t) &= t\langle 5,1\rangle + (2,1) = (5t+2, t+1) \\ b(t) &= t\langle -1,2\rangle + (-3,4) = (-t-3, 2t+4) \end{aligned}}$

Alternately, the albatross & bat meet if $a(t) = b(t)$.

$\implies (5t+2, t+1) = (-t-3, 2t+4)$

$\implies \circledast \begin{cases} 5t+2 = -t-3 & \text{①} \\ t+1 = 2t+4 & \text{②} \end{cases}$

$\implies$ ① $\to 6t = -5 \implies t = -\frac{5}{6}$

② $\to t = -3$

Therefore $-\frac{5}{6} = t = -3 \implies -\frac{5}{6} = -3$ NONSENSE!!

Therefore the system $\circledast$ has no solution, thus the albatross and bat don't meet. ∎