Agenda for Math005.005 (20 April 2020, 11–11:50 am)

1. Discussion of Worksheet 4

(1) $\frac{8-2}{5-1} = \frac{6}{4} = \frac{3}{2} = \text{slope}$

$(1,2)$
$(5,8)$
$\uparrow x \quad \uparrow y$

$y - \underbrace{8}_{y_1} = \underbrace{\tfrac{3}{2}}_{m}(x - \underbrace{5}_{x_1})$

$y - 2 = \frac{3}{2}(x-1)$

Solution: The point-slope equation for a line is given by

$$y - y_1 = m(x - x_1)$$

where (x_1, y_1) is any point on the line, and m is the slope of the line. The slope is

$$m = \frac{y_1 - y_2}{x_1 - x_2},$$

where (x_1, y_1) and (x_2, y_2) are any two points on the line. In the given problem, take

$$(x_1, y_1) = (1,2) \quad \text{and} \quad (x_2, y_2) = (5,8).$$

An equation through these points is then

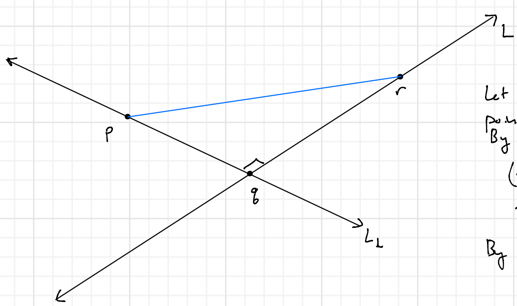
$$y - 2 = \frac{3}{2}(x-1).$$

(2) $y - \underbrace{7}_{y_1} = \underbrace{-\tfrac{2}{3}}_{m}(x - \underbrace{2}_{x_1})$

neg reciprocal

(3) **3.** Draw a line in the plane. Call this line L. Draw a point, p, that is not on L. Let $L_\perp$ be the line perpendicular to L that intersects p. The line $L_\perp$ will intersect L at a point q. Use basic geometry to argue that q is the point on L that is closest to p.

$\perp$ = perp $\rightsquigarrow$ $L_\perp$ is read "L-perp"



Let r be any other point on L ($r \neq q$). By Pythagoras

$(pq)^2 + (rq)^2 = (pr)^2$

$\Rightarrow (pq)^2 < (pr)^2.$

By the fact,

$pq < pr.$

This means that if r is any point on L other than q, then the distance from r to p is greater than the distance from q to p. Therefore q is closest to p. $\blacksquare$

WTS: $\underbrace{pq < pr}$

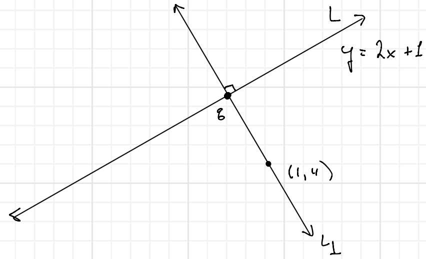
this means that

$pr = pq + \text{error}$ where error is positive

In general,

$a < b \iff$ there is some positive ε such that $a + \varepsilon = b$

Fact: $a < b \Leftarrow a^2 < b^2$ and both $a, b > 0$



Is $(1,4)$ on this line?
NO: $4 \neq 2(1) + 1 = 3$

By (3), q is closer to $(1,4)$ than any other point on L.

$$\begin{cases} L_\perp: & y - 4 = -\frac{1}{2}(x-1) \\ L: & y = 2x + 1 \end{cases}$$

q is the solution to this system //