Problem 1. Suppose that

$$f(x) = \begin{cases} 2x & \text{if } x < 1 \\ 4x - 7 & \text{if } x \geq 1 \end{cases} \quad \text{and} \quad g(x) = \begin{cases} x - 1 & \text{if } x < 3 \\ 3x + 5 & \text{if } x \geq 3. \end{cases}$$

Write using interval notation and then graph on a real number line all solutions to the inequality

$$f(x) \geq g(x).$$

$$f(x) = \begin{cases} 2x & \text{if } x < 1 \\ 4x - 7 & \text{if } 1 \leq x < 3 \\ 4x - 7 & \text{if } x \geq 3 \end{cases}$$

$$g(x) = \begin{cases} x - 1 & \text{if } x < 1 \\ x - 1 & \text{if } 1 \leq x < 3 \\ 3x + 5 & \text{if } x \geq 3 \end{cases}$$

$x < 1$: $\quad f(x) = 2x \quad$ and $\quad g(x) = x - 1$

WTF $\;x\;$ such that $\;f(x) \geq g(x)$

want to find $\qquad \Longleftrightarrow \quad 2x \geq x - 1$

$\qquad\qquad\qquad \Longleftrightarrow \quad x \geq -1$

Solutions: $\quad x < 1 \;$ AND $\; x \geq -1 \;\Longleftrightarrow\; x \in (-\infty, 1) \cap [-1, \infty)$

$\qquad\qquad$ "is an element of" $\qquad\qquad = [-1, 1)$

$1 \leq x < 3$: $\quad f(x) = 4x - 7 \geq x - 1 \quad g(x)$

$\qquad\qquad \Longleftrightarrow \quad 3x \geq 6$

$\qquad\qquad \Longleftrightarrow \quad x \geq 2$

$\qquad\qquad \Longleftrightarrow \quad x \in [2, 3)$

$x \geq 3$: $\quad f(x) = 4x - 7 \geq 3x + 5 = g(x)$

$\qquad\qquad \Longleftrightarrow \quad x \geq 12$

$\qquad\qquad \Longleftrightarrow \quad x \in [12, \infty)$

$f(x) \geq g(x) \quad$ whenever $\quad x \in [-1, 1) \cup [2, 3) \cup [12, \infty)$

https://www.desmos.com/calculator/o8hfs7vj8l

Problem 5. Find all solutions to the system of linear equations

$$\begin{cases} x + y + z = 1 & \text{(i)} \\ x - y + z = 3 & \text{(ii)} \\ 2x + 3z = -5. & \text{(iii)} \end{cases}$$

(1) $\quad$ (iii) $\Longrightarrow \quad 2x = -3z - 5 \quad \Longrightarrow \quad x = -\tfrac{3}{2}z - \tfrac{5}{2}$

$\Longrightarrow$ (ii) $\equiv \left(-\tfrac{3}{2}z - \tfrac{5}{2}\right) - y + z = 3 = \tfrac{6}{2}$

$\qquad\qquad\qquad\qquad \underbrace{\qquad}_{x}$

$\Longrightarrow -\tfrac{1}{2}z - y = \tfrac{11}{2}$

$\Longrightarrow -y = \tfrac{1}{2}z + \tfrac{11}{2}$

$\Longrightarrow y = -\tfrac{1}{2}z - \tfrac{11}{2}$

$\Longrightarrow$ (i) $\equiv \left(-\tfrac{3}{2}z - \tfrac{5}{2}\right) + \left(-\tfrac{1}{2}z - \tfrac{11}{2}\right) + z = 1$

$\Longrightarrow -z - 8 = 1$

$\Longrightarrow -z = 9$

$\Longrightarrow \boxed{z = -9}$

$\Longrightarrow x = -\tfrac{3}{2}z - \tfrac{5}{2} = -\tfrac{3}{2}(-9) - \tfrac{5}{2} = \boxed{11 = x}$

$\Longrightarrow y = -\tfrac{1}{2}z - \tfrac{11}{2} = -\tfrac{1}{2}(-9) - \tfrac{11}{2} = \boxed{-1 = y}$

$$\boxed{\begin{array}{l} \text{The system is solved by} \\[4pt] x = 11, \quad y = -1, \quad \text{and} \quad z = -9. \end{array}}$$

Problem 5. Find all solutions to the system of linear equations

$$\begin{cases} x + y + z = 1 & \text{(i)} \\ x - y + z = 3 & \text{(ii)} \\ 2x + 3z = -5. & \text{(iii)} \end{cases}$$

(2) $\quad$ (i) + (ii) $\equiv 2x + 2z = 4 \;\Longleftrightarrow\; x + z = 2$

(iii) + (-2)(x + z = 2)

$\Longrightarrow \quad \begin{array}{r} 2x + 3z = -5 \\ + \; -2x - 2z = -4 \\ \hline z = -9 \end{array}$

$\Longrightarrow \quad x + z = x - 9 = 2 \quad \Longrightarrow \quad \boxed{x = 11}$

$1 = x + y + z = 11 + y - 9 = 2 + y \quad \Longrightarrow \quad \boxed{y = -1}$