Agenda for Math 005.003 (8 April 2020, 11-11:50 am):

1. How was the homework graded?
2. Q&A

$f(x) = \frac{3x+2}{x-1}$. Discuss the domain of f.

$$f(x) = \frac{g(x)}{h(x)} \quad \text{where} \quad g(x) = 3x+2, \quad h(x) = x-1$$

The domain of f is

$$\mathrm{Dom}(f) = \big(\mathrm{Dom}(g) \cap \mathrm{Dom}(h)\big) \setminus \{x \mid h(x) = 0\}$$

$$\boxed{\begin{aligned} \mathrm{Dom}(g) &= \mathbb{R} = (-\infty, \infty) \\ \mathrm{Dom}(h) &= \mathbb{R} = (-\infty, \infty) \\ \{h(x)=0\} &= \{1\} \end{aligned}}$$

$$\begin{aligned} \mathrm{Dom}(f) &= (\mathbb{R} \cap \mathbb{R}) \setminus \{1\} \\ &= \boxed{\mathbb{R} \setminus \{1\}} \\ &= \boxed{(-\infty, 1) \cup (1, \infty)} \end{aligned}$$

Solution: The domain of f consists of all real numbers where both the numerator and the denominator are defined, and the denominator is not zero. The numerator & denominator are always defined, and the denominator is zero only when $x=1$. Therefore

$\mathrm{Dom}(f) = \mathbb{R} \setminus \{1\}$. ∎

3. The piecewise defined function h is defined by

$$h(x) = \begin{cases} x+5 & \text{if } x < 2 \\ 10-x & \text{if } x \geq 2. \end{cases}$$

Find all x such that

$$h(x) < 0.$$

Find all x such that

$$h(x) \geq 0.$$

~~**4.** The function h is defined by a formula on each of the intervals $(-\infty, 2)$ and $[2, \infty)$. Intersect these intervals with the sets you found in the previous problem.~~

5. For the function h above, write the function F given by

$$F(x) = |h(x)|$$

as a piecewise defined function.

GOAL: Find $|h(x)|$.

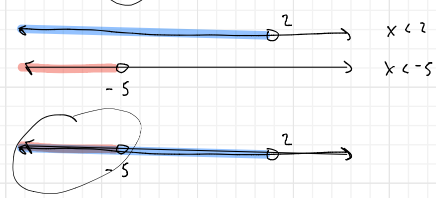
$$|x| = \begin{cases} x & \text{if } x \geq 0, \\ -x & \text{if } x < 0. \end{cases}$$

⊛ Find the set of x s.t. $h(x) < 0$.

Case 1: $x < 2 \Rightarrow h(x) = x+5$

$h(x) < 0 \Rightarrow x+5 < 0 \Rightarrow x < -5$

$x < 2$ AND $x < -5$



$\{x < 2\} \cap \{x < -5\} = \{x < -5\}$

$(-\infty, 2) \cap (-\infty, -5) = \boxed{(-\infty, -5)}$

If $x < -5$, then $h(x) < 0$.

Case 2: $x \geq 2 \Rightarrow h(x) = 10 - x$

$0 > h(x) = 10 - x \Rightarrow x > 10$

$\Rightarrow \boxed{x > 10}$

If $x > 10$, then $h(x) < 0$.

⊛ Find the set of x s.t. $h(x) \geq 0$.

It turns out that if $h(x)$ is not $<$ zero, then $h(x) \geq 0$. $\Rightarrow$ If $-5 \leq x \leq 10$, then $h(x) \geq 0$

⊛

$$|h(x)| = \begin{cases} -(x+5) & x < 2 \text{ and } x < -5 \quad (h(x) < 0) \\ x+5 & x < 2 \text{ and } x \geq -5 \quad (h(x) \geq 0) \\ 10-x & x \geq 2 \text{ and } x \leq 10 \quad (h(x) \geq 0) \\ -(10-x) & x \geq 2 \text{ and } x > 10 \quad (h(x) < 0) \end{cases}$$

$$= \begin{cases} -x-5 & \text{if } x < -5, \\ x+5 & \text{if } -5 \leq x < 2, \\ 10-x & \text{if } 2 \leq x \leq 10, \\ x-10 & \text{if } x > 10. \end{cases}$$