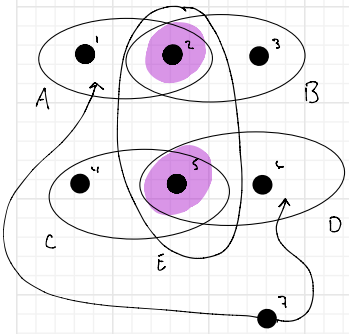
Agenda for Math 005.003 (31 March 2020, 11-11:50 am):

1. Answer questions.

6. Is the following statement true? Suppose that A, B, C, D and E are sets. If $A \cap B$ is a subset of E and if $C \cap D$ is a subset of E, then $(A \cup C) \cap (B \cup D)$ is a subset of E.



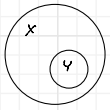
2 must be in E

5 must be in E

7 is not in E, but it is in $(A \cup C) \cap (B \cup D)$

"is a subset of"

$X \subseteq Y$ means that if x is an element of X (i.e. if x is in X) then x is also an element of Y.



6) Show that the statement "$A \cap B \subseteq E$ and $C \cap D \subseteq E \Rightarrow (A \cup C) \cap (B \cup D) \subseteq E$" is false.

Solution: Let

$A = \{1, 2, 7\}$,
$B = \{2, 3\}$,
$C = \{4, 5\}$,
$D = \{5, 6, 7\}$, and
$E = \{2, 5\}$.

Then

$A \cup C = \{1, 2, 7, 4, 5\}$ and $B \cup D = \{2, 3, 5, 6, 7\}$,

so

$(A \cup C) \cap (B \cup D) = \{2, 7, 5\}$.

But $E = \{2, 5\}$. So the part 7 is in $(A \cup C) \cap (B \cup D)$, but it is not in E. Therefore

$(A \cup C) \cap (B \cup D)$ is not a subset of E. ∎