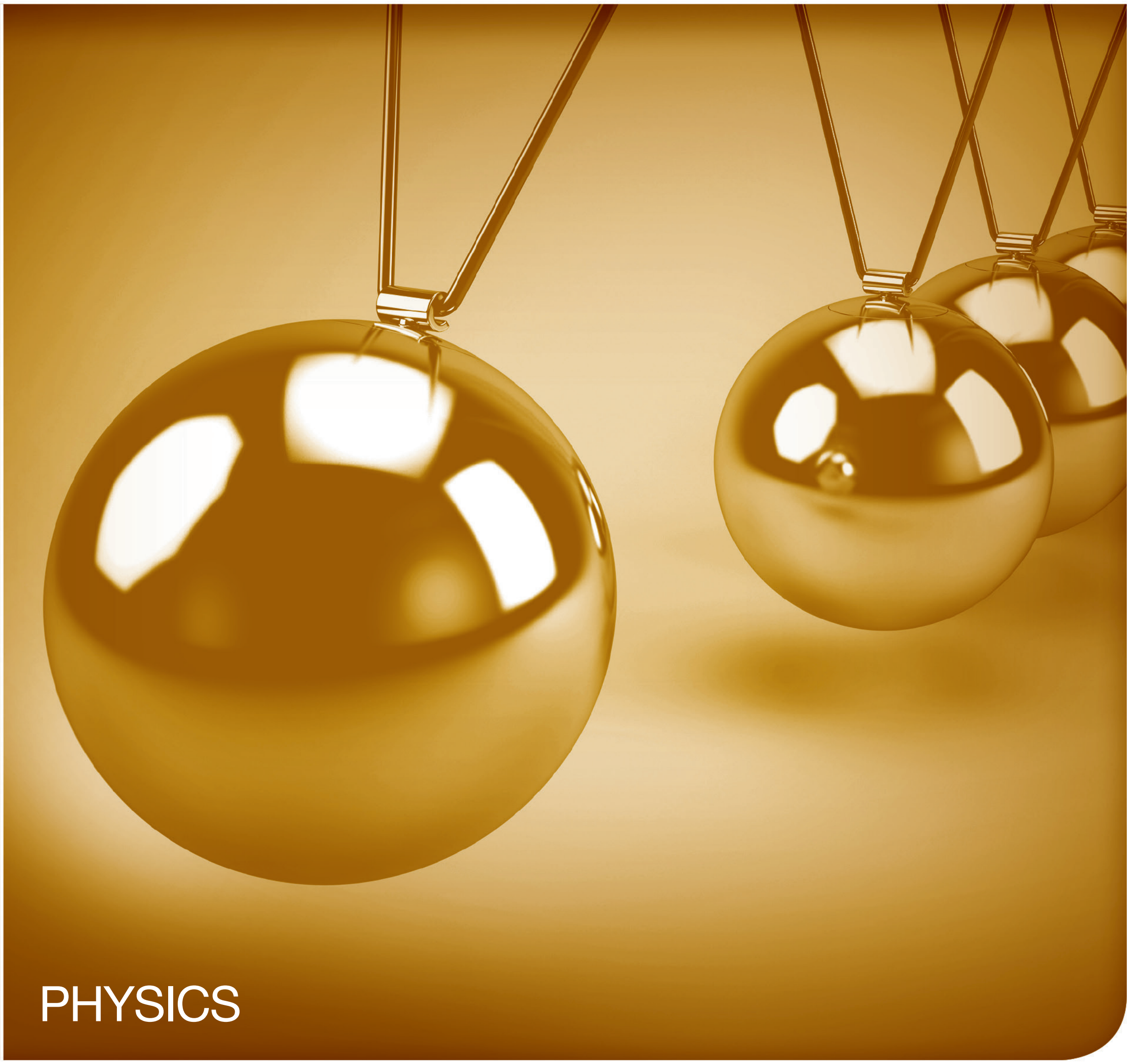

Kinematics

Investigation Manual

CAROLINA
DISTANCE LEARNING

KINEMATICS

Table of Contents

Overview

Kinematics is the branch of physics that deals with the analysis of the motion of objects without concern for the forces causing the motion. Scientists have developed equations that describe the movement of objects within certain parameters, such as objects moving with a constant velocity or a constant acceleration. Using these equations, it is possible to predict the future position and velocity of an object. This investigation focuses on objects moving with a constant velocity or a constant acceleration. Data will be collected on these objects, and the motion of the objects will be analyzed through graphing these data.

Outcomes

- Explain linear motion for objects traveling with a constant velocity or constant acceleration.
- Utilize vector quantities, such as displacement and acceleration, and scalar quantities, such as distance and speed.
- Analyze graphs that depict the motion of objects moving at a constant velocity or constant acceleration.
- Analyze and predict the motion of objects moving at a constant velocity or constant acceleration.

Time Requirements

Preparation .... 5 minutes

Activity 1: Graph and Interpret Motion Data of a Moving Object .... 20 minutes

Activity 2: Calculate the Velocity of a Moving Object..... 30 minutes

Activity 3: Graph the Motion of an Object Traveling under Constant Acceleration……................. 30 minutes

Activity 4: Predict the Time for a Steel Sphere to Roll Down an Incline….. .... 20 minutes

Activity 5: Demonstrate that a Sphere Rolling Down an Incline Is Moving under Constant Acceleration.. 30 minutes

Background

Mechanics is the branch of physics that studies the motion of objects and the forces and energies that affect those motions. Classical mechanics refers to the motion of objects that are large (compared to subatomic particles) and slow (compared to the speed of light). The effects of quantum mechanics and relativity are negligible in classical mechanics. Classical mechanics can describe most objects and forces encountered in daily life, such as the motion of a baseball, a train, or even a bullet or the planets. Engineers and other scientists apply the principles of physics in many scenarios. Physicists and engineers often collect data about an object and use graphs of the data to describe the motion of objects.

Kinematics is a specific branch of mechanics that describes the motion of objects without reference to the forces causing the motion. Examples of kinematics include describing the motion of a racecar moving on a track or an apple falling from a tree, but only in terms of the object's position, velocity, acceleration, and time without describing the force from the engine of the car, the friction between the tires and the track, or the gravity pulling the apple. For example, it is possible to predict the time it would take for an object dropped from the roof of a building to fall to the ground using the following kinematic equation:

$$\Delta \boldsymbol{x} = \frac{1}{2} \boldsymbol{a} t^2$$

In this equation, $\Delta \boldsymbol{x}$ is the displacement from the starting position at a given time, $\boldsymbol{a}$ is the acceleration of the object, and t is the time after the object is dropped. The equation does not include any variables for the forces acting on the object or the mass or energy of the object. As long as some initial conditions are known—such as the object's position, acceleration, and velocity at a given time—the motion or position of the object at any future or previous time can be calculated by applying kinematics. This method has many useful applications. One could calculate the path of a projectile such as a golf ball or an artillery shell, the time or distance for an object to come to rest, or the speed an object would be traveling after falling a given distance.

Early scientists such as Galileo Galilee (1564–1642), Isaac Newton (1642–1727), and Johannes Kepler (1571–1630) studied the motion of objects and developed mathematical laws to describe and predict their motion. Until the

Key

Personal protective equipment (PPE)

goggles gloves apron

follow link to video

photograph results and submit

stopwatch required

warning corrosion flammable toxic environment health hazard

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Background continued

late sixteenth century, the idea that heavier objects fell faster than lighter objects was widely accepted. This idea had been proposed by the Greek philosopher Aristotle, who lived during the fourth century BC. Because the idea seemed to be supported by experience, it was generally accepted. A person watching a feather and a hammer dropped simultaneously from the same height would certainly observe the hammer falling faster than the feather. According to legend, Galilee, an Italian physicist and mathematician, disproved this idea in a dramatic demonstration by dropping objects of different mass from the tower of Pisa to demonstrate that they fell at the same rate. In later experiments, Galilee rolled spheres down inclined planes to slow down the motion and get more accurate data. By analyzing the ordinary motion of objects and graphing the results, it is possible to derive some simple equations that predict their motion.

To study the motion of objects, a few definitions should be understood.

- A **vector** refers to a number with a direction and magnitude (or size).
- Numbers that have a magnitude but not a direction are referred to as **scalar**.

Velocity and acceleration are examples of vectors, and time and temperature demonstrate scalar quantities. In kinematics, vectors are important because the goal is to calculate the location and direction of movement of the object at any time in the future or past. For example, if an object is described as being 100 miles from a given position traveling at a speed of 50 miles per hour, that could mean the object will reach its final position in 2 hours. It could also mean the object could be located up to 100 miles farther away in 1 hour or somewhere between 100 and 200 miles away depending on the direction. The quantity **speed**, which refers to the rate of change in position of an object, is a scalar quantity because no direction of travel is defined. The quantity **velocity**, or the rate of change of the position of an object, refers to both the speed and direction of an object. Velocity is a vector quantity.

Distance, or the amount of space between two objects, is a scalar quantity. **Displacement**, which is distance in a given direction, is a vector quantity. If a bus travels from Washington, DC, to New York City, the distance the bus travels is approximately 230 miles. The displacement of the bus is (roughly) 230 miles northeast. If the bus travels from Washington, DC, to New York City and back, the distance traveled is roughly 460 miles, but the displacement is zero because the bus begins and ends at the same point.

It is important to define the units of scalar and vector quantities when studying mechanics. A person giving directions from Washington, DC, to New York City might describe the distance as being approximately 4 hours. This may be close to the actual travel time, but this does not indicate actual distance.

To illustrate the difference between distance and displacement, consider the diagrams in Figures 1–3.

In the number line in Figure 1, the displacement from zero represented by the arrowhead on the number line is –3, indicating both direction and

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magnitude. The distance from zero indicated by the point on the number line equals 3, which is the magnitude of the displacement. For motion in one dimension, the plus or minus sign is sufficient to represent the direction of the vector.

Figure 1.

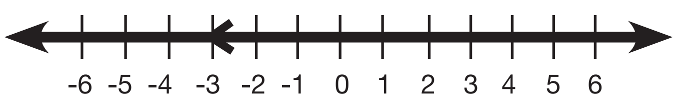


Figure 2.

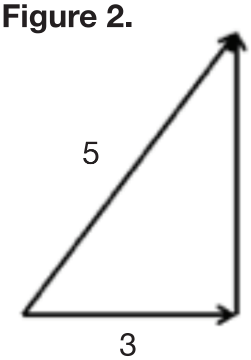
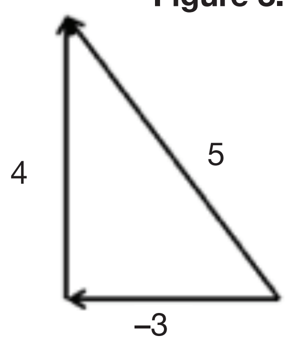


Figure 3.



The arrows in Figures 2 and 3 represent displacement vectors for an object. The long lines represent a displacement with a magnitude of 5. This displacement vector can be resolved into two component vectors along the *x* and *y* axes. In all three figures, the object is moved some distance in either the positive or negative *x* direction and then some distance in the positive *y* direction; however, the final position of the object is different in each diagram. The total distance between the object's initial and final position in each instance is 5 meters (m); however, more information is needed to describe the displacement ($\Delta \mathbf{x}$) from the initial position.

In Figure 2, the displacement vector can be given by 5 m at 53.1°. This vector is found by the addition of the two component vectors, 3 m at 0° and 4 m at 90°, using conventional polar coordinates that assign 0° to the positive *x* direction and progress counterclockwise toward 360°. The displacement in Figure 3, using the same convention, is 5 m at 143.1°. In each case, the magnitude of the vector is the length of the arrow, that is, the displacement that the object travels. Most texts indicate that a variable represents a vector quantity by placing an arrow over the variable or placing the variable in bold.

To indicate the magnitude of a vector, absolute value bars are used. For example, the magnitude of the displacement vector in each diagram is 5 m. In Figure 2, the displacement is given by:

$$\Delta \mathbf{x} = 5 \text{ m at } 51.3°$$

The magnitude of this vector may be written as:

$$|\ \Delta \mathbf{x}\ | = d = 5 \text{ m}$$

The displacement vector in Figure 2 ($\Delta \mathbf{x}$ = 5 m at 53.1°) can be resolved into the component vectors 3 m at 0° and 4 m at 90°.

Two more terms that are critical for the study of kinematics are velocity and **acceleration**. Both terms are vector quantities.

The displacement vector in Figure 2 ($\Delta \mathbf{x}$ = 5 m at 53.1°) can be resolved into the component vectors 3 m at 0° and 4 m at 90°.

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Background continued

- Recall that velocity (***v***) is the rate of change of the position of an object. For an object moving in the *x* direction, the magnitude of the velocity (speed) may be described as:

$$\boldsymbol{v} = \frac{\boldsymbol{x}_2 - \boldsymbol{x}_1}{\Delta t}$$

In this equation, x_2 is the position at time t_2 and x_1 is the position of the object at time t_1. The variable Δt represents the time interval $t_2 - t_1$. The symbol Δ is the Greek symbol delta and refers to a change or difference. Δt is read "delta *t*." Time in the following example is provided in seconds (s).

If an object is located at a position designated x_1 = 2 m and moves to position x_2 = 8 m over a time interval Δt = 2 s, then the average speed can be calculated:

$$\frac{8\text{ m} - 2\text{ m}}{2\text{ s}} = 3\text{ m/s}$$

The velocity for this object can be indicated as:

$$\boldsymbol{v} = 3\text{ m/s}$$

Because velocity is a vector quantity, the positive sign indicates that the object was traveling in the positive *x* direction at a speed of 3 m/s.

- Acceleration is the rate of change of velocity. The magnitude of acceleration may be described as:

$$\boldsymbol{a} = \frac{\boldsymbol{v}_2 - \boldsymbol{v}_1}{\Delta t}$$

For example, an object with an initial velocity $\boldsymbol{v}_1$ = 10 m/s slows to a final velocity of $\boldsymbol{v}_2$ = 1 m/s over an interval of 3 s.

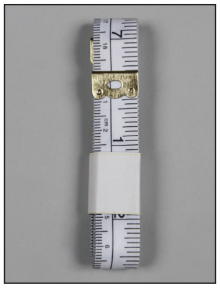
$$\frac{1\,{}^{\text{m}}/_{\text{s}} - 10\,{}^{\text{m}}/_{\text{s}}}{3\text{s}} = -3\ {}^{\text{m}}/_{\text{s}}/_{\text{s}}$$

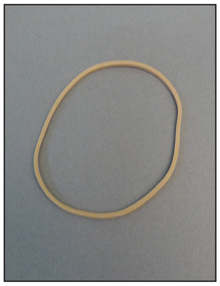
The object has an average acceleration of –3 meters per second per second, which can also be written as –3 meters per second squared, or –3 m/s^2. The negative means the object could be slowing down in a positive direction and/or that the acceleration vector is pointing in the opposite direction as the velocity vector.

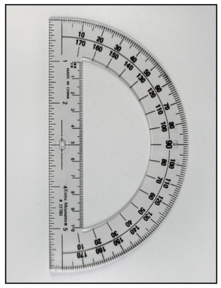
Because only the initial and final positions or velocities over a given time interval are used in these equations, the calculated values indicate the average velocity or acceleration. Calculating the instantaneous velocity or acceleration of an object requires the application of calculus. Only average velocity and acceleration are considered in this investigation.

Materials

Included in the central materials set:


Tape measure


Rubber bands


Protractor

Needed but not supplied:

- Scientific or graphing calculator, or a computer with spreadsheet software (optional)
- Permanent marker
- Masking tape
- Digital camera or mobile device capable of recording video
- Timer or stopwatch
- Books
- Pencil

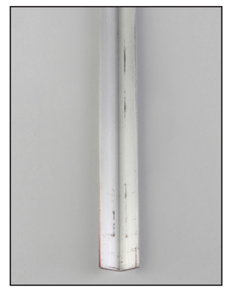
Included in the mechanics module kit:


Constant velocity car

2 C batteries


Metal sphere


Acrylic sphere


Angle bar


Foam board


Clay

Reorder Information: Replacement supplies for the Kinematics investigation (Conceptual Physics Mechanics Module kit, item number 580404) can be ordered from Carolina Biological Supply Company.

Call: 800.334.5551 to order.

KINEMATICS

Safety

Safety goggles should be worn while conducting this investigation.

Read all the instructions for this laboratory activity before beginning. Follow the instructions closely, and observe established laboratory safety practices.

Do not eat, drink, or chew gum while performing this activity. Wash your hands with soap and water before and after performing the activity. Clean the work area with soap and water after completing the investigation. Keep pets and children away from lab materials and equipment.

Preparation

1. Read through the assigned activities.
2. Collect materials needed for this investigation.
3. Locate and clear an area of level floor space to conduct the constant velocity experiment. The space should be free of obstruction and 3 to 4 meters long with a surface that will allow the vehicle to maintain traction but not impede the vehicle.

Technology

Alternate Methods for Collecting Data Using Digital Devices

Much of the uncertainty in physics experiments arises from human reaction time error in measuring the times of events. Some of the time intervals are very short, which increases the effect of human error due to reaction time.

Observing the experiment from a good vantage point that removes parallax errors and recording measurements for multiple trials helps to minimize error; using a digital device as an alternate method of data collection may further minimize error. Many digital devices, such as smartphones and tablets, have cameras and software that allow the user to pause or slow down the video. If you film the activity against a scale such as a tape measure, you can use your video playback program to record position and time data. This can provide more accurate data and may eliminate the need for multiple trials.

If the time on your device's playback program is not sufficiently accurate, you may download an app such as the following free apps:

- **Hudl Technique**
  iOS and Android (https://www.hudl.com/products/technique)

- **SloPro**
  iOS (https://itunes.apple.com/us/app/slopro-1000fps-slow-motion/id507232505?mt=8)

  Android (https://slopro.en.uptodown.com/android)

Or you may upload the video to your computer. Your operating system or software suite may include video playback programs, or these programs may be available for download.

ACTIVITY

ACTIVITY 1

A Graph and Interpret Motion Data of a Moving Object

One way to analyze the motion of an object is to graph the position and time data. The graph of an object's motion can be interpreted and used to predict the object's position at a future time or to calculate an object's position at a previous time.

Table 1 represents the position of a train on a track. The train can move in only one dimension: either forward (the positive *y* direction) or in reverse (the negative *y* direction).

Table 1.

Time = *x*-axis (seconds)	Position = *y*-axis (meters)
0	0
5	20
10	40
15	50
20	55
25	57
30	60
35	70
40	70
45	70
50	55

1. Plot the data from Table 1 on a graph. Use the *y*-axis to represent the displacement from the starting position ($y = 0$), and plot the time coordinate on the *x*-axis. Graph the data on graph paper (page 19), or use a graphing calculator or spreadsheet to graph the data.

2. Connect all the coordinates on the graph with straight lines.

ACTIVITY 2

A Calculate the Velocity of a Moving Object

In this activity, you will graph the motion of an object moving with a constant velocity. You will calculate the speed of the object by allowing the constant velocity car to travel a given distance and measuring the time it took to move this distance. As seen in Activity 1, this measurement will only provide the average speed. In Activity 2, you will collect time data at several travel distances, plot these data, and analyze the graph.

1. Find and clear a straight path approximately 3 meters long.

2. Install the batteries, and test the vehicle.

Note: The vehicle should be able to travel 2 meters in a generally straight path. If the vehicle veers significantly to one side, you may need to allow the vehicle to travel next to a wall. The friction will affect the vehicle's speed, but the effect will be uniform for each trial.

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ACTIVITY 2 continued

3. Use a tape measure or ruler to measure a track 2 meters long. The track should be level and smooth with no obstructions. Make sure the surface of the track provides enough traction for the wheels to turn without slipping. Place masking tape across the track at 25-centimeter intervals.
4. Set the car on the floor approximately 5 centimeters behind the start point of the track.

Note: Starting the car a short distance before the start point allows the vehicle to reach its top speed before the timing starts and prevents the short period of acceleration from affecting the data.

5. Set the stopwatch to the timing mode, and reset the time to zero.
6. Start the car, and allow it to move along the track.
7. Start the stopwatch when the front edge of the car crosses the start point.
8. Stop the stopwatch when the front edge of the car crosses the first 25-centimeter point.
9. Recover the car, and switch the power off. Record the time by the appropriate vehicle position in **Data Table 1**.
10. Repeat Steps 5–9 for each 25-centimeter interval marked. Each trial will have a distance that is 25 centimeters longer than the previous trial.
11. Graph the time and displacement data points.
12. Draw a line of best fit through the data points. The points should generally fall in a straight line. (If you have access to a graphing calculator or a computer with spreadsheet software, the calculator or spreadsheet can be programmed to draw the line of best fit, or trend line.)
13. Calculate the slope of the line. Based on the equation of a line that crosses the y-axis at $y = 0$, the slope of the line, m, will be the velocity of the object.

$$y = mx$$

$$\mathbf{x} = \mathbf{v}\Delta t$$

14. Repeat the activity to indicate the velocity of the car at any time as noted in **Data Table 2**.

Note: Because the battery-powered car moves with a constant speed, all the values for the velocity of the car in Data Table 2 should be the same. The value of the velocity for the car should be the slope of the line in the previous graph.

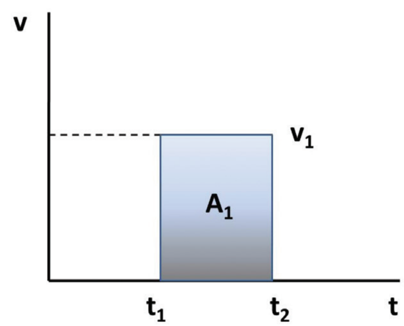
15. Graph the data points from **Data Table 2** on a second graph. Label the y-axis "Velocity" and the x-axis "Time." The motion of the car should generate a horizontal line. (On a velocity vs. time graph, an object moving with a constant speed is represented by a horizontal line.)
16. Draw a vertical line from the x-axis at the point time = 2 seconds so that it intersects the line representing the velocity of the car.

continued on next page

17. Draw a second vertical line from the *x*-axis at the point time = 4 seconds so that it intersects the line representing the velocity of the car.

18. Calculate the area represented by the rectangle enclosed by the two vertical lines you just drew, the line for the velocity of the car, and the *x*-axis. An example is shown as the blue shaded area in Figure 4.

Figure 4.



Note: To calculate the area of the rectangle, multiply the value for the time interval between time $t = 2$ s and time $t = 4$ s by the velocity of the car. This area represents the displacement traveled by the object during this time interval.

This technique is often referred to as calculating the area "under the curve." The graph of velocity vs. time for an object that is traveling with a constant acceleration will yield a horizontal line, but using the same method of graphing the velocity vs. time and finding the area under the curve in a given time interval can allow you to calculate the distance traveled by the object.

Displacement = velocity • time

In this equation, the time units (s) cancel out when velocity and time are multiplied, leaving the displacement unit in meters.

ACTIVITY 3

A Graph the Motion of an Object Traveling under Constant Acceleration

Collecting data on freefalling objects requires accurate timing instruments or access to a building with heights of several meters where you can safely drop objects over heights large enough to allow accurate measurement with a stopwatch. To collect usable data in this activity, you will record the time for objects rolling down an incline. This reduces acceleration to make it easier to record accurate data on the distance that an object moves.

1. Collect the steel and acrylic spheres, angle bar, clay, tape measure, timing device, permanent marker, and protractor.
2. Use the permanent marker and tape measure to mark the inside of the angle bar at 1-centimeter increments.
3. Set the angle bar at an incline of between 5° to 10°. Use the clay to set the higher end of the angle bar and to stabilize the system (see Figure 5).

Figure 5.

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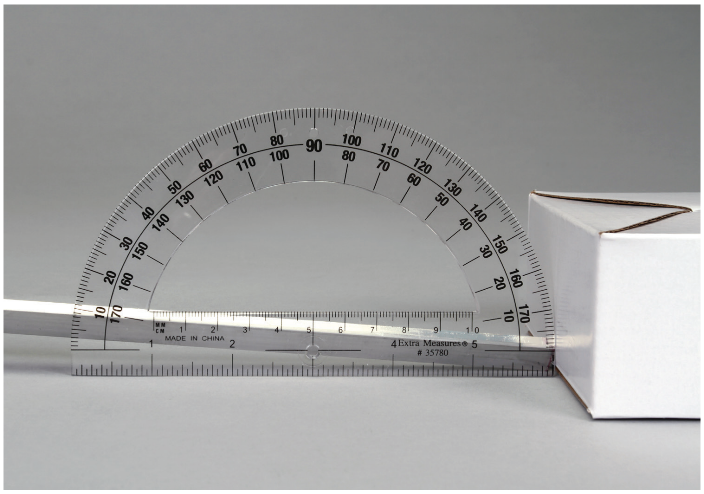
ACTIVITY 3 continued

4. Set the angle bar so that the lower end terminates against a box, book, or wall to stop the motion of the sphere (see Figure 6).

Figure 6.



5. Place the steel sphere 10 centimeters from the lower end of the track.
6. Release the steel sphere, and record the time it takes for the sphere to reach the end of the track in **Data Table 3**.
7. Repeat Steps 5 and 6 two more times for a total of 3 measurements at a starting point of 10 cm. Calculate the average time, and record it in **Data Table 3**.
8. Repeat Steps 5–7 seven more times, increasing the distance between the starting point and the end of the track by 10 centimeters each time (up to 80 centimeters). You are recording the time it takes for the sphere to accelerate over an increasing displacement.
9. Using the data from Steps 6–8, create a graph of displacement vs. time.
10. Complete **Data Table 3** by calculating the square of the average time for each displacement.
11. Create a graph of displacement vs. time squared.

Graphing the displacement vs. time data from Data Table 3 will generate a parabola, which means the *y* value is proportional to the square of the *x* value, or:

$$y \infty x^2$$

So the equation for a line that fits all the data points is:

$$y = Ax^2 + Bx + C$$

In this activity, the *y*-axis is displacement and the *x*-axis is time; therefore, displacement is proportional to the time squared, or:

$$\Delta \mathbf{x} \infty t^2$$

Exchanging the *y* in the equation with displacement ($\Delta \mathbf{x}$) gives a formula that looks like:

$$\Delta \mathbf{x} = At^2 + Bt + C$$

Knowing the displacement $\Delta \mathbf{x}$ at any time *t*, you just need to find the constants: *A*, *B*, and *C*.

The equation that describes the displacement of an object moving with a constant acceleration is one of the kinematic equations:

$$\Delta \mathbf{x} = \frac{1}{2}\mathbf{a}\Delta t^2 + \mathbf{v}_1\Delta t$$

The following section describes how to find this equation using the same method of finding the area under the curve discussed in Activity 2 (see the "Note" on page 11).

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B Finding an Equation for the Motion of an Object with Constant Acceleration (optional)

1. The general form of a line is:

$$y = mx + b$$

In this equation, *m* is the slope of the line, and *b* is the *y*-intercept, or the point where the line crosses the *y*-axis. Because the first data point represents time zero and displacement zero, the *y*-intercept is zero and the equation for the line simplifies to:

$$y = mx$$

2. The data collected in this activity showed that:

$$\Delta \mathbf{x} \propto t^2$$

This means that the displacement for the object that rolls down an inclined plane is represented mathematically as:

$$\Delta \mathbf{x} = kt^2 + c$$

In this equation, *k* is an unknown constant representing the slope of the line, and *c* is an unknown constant representing the *y*-intercept. The displacement of the sphere as it rolls down the incline can be calculated using this equation if the constants *k* and *c* can be found.

3. Further experimentation indicates that the constant *k* for an object in freefall is one-half the acceleration. If the object is released from rest, the constant *c* will be zero. So for an object that is released from rest and is falling under the constant acceleration due to gravity, the displacement from the point of release is given by:

$$\Delta \mathbf{x} = \frac{1}{2}\mathbf{a}t^2$$

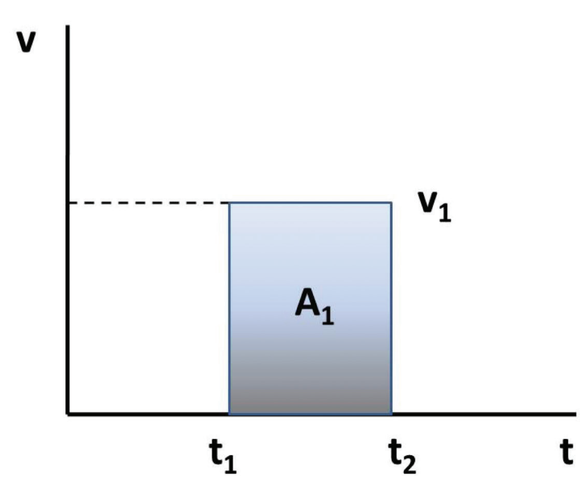
$\Delta \mathbf{x}$ is the displacement, *t* is the time of freefall, and ***a*** is the acceleration. For objects in freefall near Earth's surface, the acceleration due to gravity has a value of 9.8 m/s^2.

4. Another way to derive this equation and find the values for *k* and *c* is to consider the velocity vs. time graph for an object moving with a constant acceleration. Recall the velocity vs. time graph for the object moving with constant velocity from Activity 2. If velocity is constant, the equation of that graph is:

$$\mathbf{v} = k$$

In this equation, ***v*** represents the velocity plotted on the *y*-axis and *k* is the constant value of the velocity. Plotted against time on the *x*-axis, this graph is a horizontal line, as depicted in Figure 7.

Figure 7.



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ACTIVITY 3 continued

By definition, the shaded area is the displacement traveled by the object during the time interval:

$$\Delta t = t_2 - t_1$$

$$\boldsymbol{v} = \frac{\textbf{\textit{displacement}}}{time} = \frac{\Delta \boldsymbol{x}}{\Delta t}$$

$$\Delta \boldsymbol{x} = \boldsymbol{v}\Delta t$$

5. If an object has a constant acceleration, then by definition:

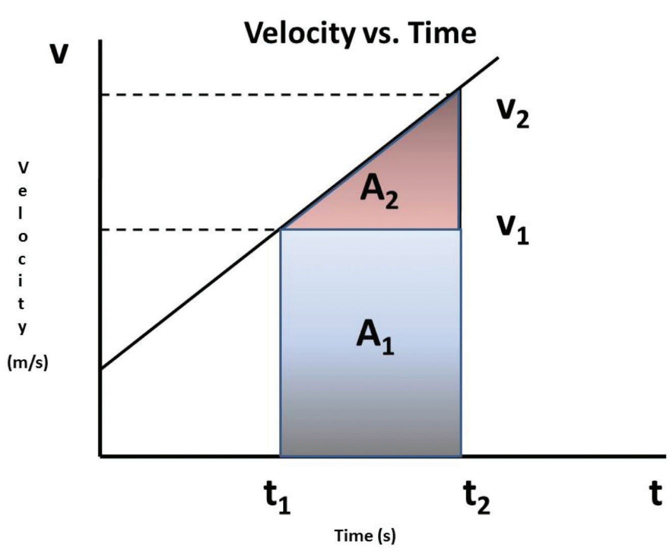
$$\boldsymbol{a} = \frac{\boldsymbol{v}_2 - \boldsymbol{v}_1}{\Delta t}$$

or

$$\boldsymbol{v}_2 = \boldsymbol{a}\Delta t + \boldsymbol{v}_1$$

6. This equation is in the general form of a line $y = mx + b$, with velocity on the y-axis and time on the x-axis. The graph of this equation would look like the graph in Figure 8.

Figure 8.



Similar to how the shaded area A_1 in Figure 7 represents the displacement traveled by the object during the time interval $\Delta t = t_2 - t_1$, the shaded area A_2 combined with A_1 equals the displacement traveled by the object undergoing constant acceleration.

The area A_1 can be given by:

$$A_1 = \boldsymbol{v}_1\Delta t$$

The area A_2 can be given by:

$$A_2 = \frac{1}{2}(\boldsymbol{v}_2 - \boldsymbol{v}_1)\Delta t$$

7. Because this is the area of the triangle where the length of the base is Δt and the height of the triangle is $(\boldsymbol{v}_2 - \boldsymbol{v}_1)$, add these two expressions and rearrange:

$$\Delta \boldsymbol{x} = \frac{1}{2}(\boldsymbol{v}_2 - \boldsymbol{v}_1)\Delta t$$

Then substitute:

$$\boldsymbol{v}_2 = \boldsymbol{a}\Delta t + \boldsymbol{v}_1$$

To yield this equation:

$$\Delta \boldsymbol{x} = \frac{1}{2}(\boldsymbol{a}\Delta t + \boldsymbol{v}_1 + \boldsymbol{v}_2\,\Delta t + \boldsymbol{v}_1\Delta t)$$

8. Simplify to this equation:

$$\Delta \boldsymbol{x} = \frac{1}{2}\boldsymbol{a}\Delta t^2 + \boldsymbol{v}_1\Delta t$$

This equation gives the theoretical displacement for an object undergoing a constant acceleration, $\boldsymbol{a}$, at any time t, where $\Delta \boldsymbol{x}$ is the displacement during the time interval, Δt, and $\boldsymbol{v}_1$ is the initial velocity.

9. If the object is released from rest, as in the activity, $\boldsymbol{v}_1 = 0$ and the equation simplifies to:

$$\Delta \boldsymbol{x} = \frac{1}{2}\boldsymbol{a}\Delta t^2$$

continued on next page

ACTIVITY 4

A Predict the Time for a Steel Sphere to Roll Down an Incline

Read the following information carefully.

In this activity, you will use this kinematic equation to predict how long the sphere will take to roll down the inclined track:

$$\Delta \boldsymbol{x} = \frac{1}{2} \boldsymbol{a} \Delta t^2$$

First, solve the equation for time:

$$t = \sqrt{\frac{2\Delta \boldsymbol{x}}{\boldsymbol{a}}}$$

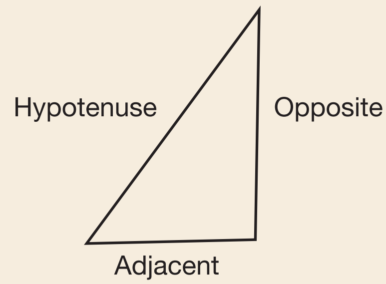
If the object in the activity was in freefall, you would just need to substitute the distance it was falling for $\Delta \boldsymbol{x}$ and substitute the acceleration due to Earth's gravity for $\boldsymbol{a}$, which is $\boldsymbol{g} = 9.8 \text{ m/s}^2$. The acceleration of an object *sliding*, without friction, down an incline is given by $\boldsymbol{a} = \boldsymbol{g} \sin \theta$, where θ is the angle between the horizontal plane (the surface of your table) and the inclined plane (the track), and $\boldsymbol{g}$ is the acceleration due to Earth's gravity.

In this activity, however, a **solid sphere** is *rolling down an incline*, so the acceleration is given by:

$$\boldsymbol{a} = 0.71 \, \boldsymbol{g} \sin \theta$$

Find the trigonometric sin of an angle by measuring the angle with a protractor and using the "sin" function on a calculator or by simply dividing the length of the side opposite the angle (the height from which the sphere starts) by the length of the hypotenuse of the right triangle (the length of the track). (See Figure 9.)

Figure 9.



$$\sin \theta = \frac{\textit{opposite}}{\textit{hypotenuse}}$$

1. Set up the angle bar as a track. Measure the length of the track and the angle of elevation between the track and the table.
2. Rearrange the kinematic equation to solve for time (second equation), and substitute the value 0.71 $\boldsymbol{g} \sin \theta$ for $\boldsymbol{a}$ (third equation). Use a distance of 80 centimeters for $\Delta \boldsymbol{x}$.

$$\Delta \boldsymbol{x} = \frac{1}{2} \boldsymbol{a} \Delta t^2$$

$$t = \sqrt{\frac{2\Delta \boldsymbol{x}}{\boldsymbol{a}}}$$

$$t = \sqrt{\frac{2\Delta \boldsymbol{x}}{(0.71 \boldsymbol{g} \sin \theta)}}$$

3. Release the steel sphere from the start point at the elevated end of the track, and measure the time it takes for the sphere to roll from position $\boldsymbol{x0} = 0$ to a final position $\boldsymbol{xf} = 80$ cm.
4. Compare the measured value with the value predicted in Step 2. Calculate the percent difference between these two numbers.
5. Repeat Steps 3 and 4 with the acrylic sphere. What effect does the mass of the sphere have on the acceleration of the object due to gravity?

ACTIVITY 5

A Demonstrate that a Sphere Rolling Down an Incline Is Moving under Constant Acceleration

1. Obtain the piece of foam board. Use a ruler and a pencil to draw lines across the short dimension (width) of the board at 5-centimeter increments.
2. Collect rubber bands from the central materials set. Wrap the rubber bands around the width of the foam board so they align with the pencil marks made in Step 1 (see Figure 10).

Figure 10.

3. Use a book to prop up the foam board as an inclined plane at an angle from 5° to 10° above the horizontal.
4. Place the steel sphere at the top of the ramp, and allow the sphere to roll down the foam board.

Note: The sound as the steel sphere crosses the rubber bands will increase in frequency as the sphere rolls down the ramp, indicating that the sphere is accelerating. As the sphere continues to roll down the incline, it takes less time to travel the same displacement.

If the steel sphere is released from rest and is moving under a constant acceleration, then the displacement of the sphere from the initial position is given by:

$$\Delta \boldsymbol{x} = \frac{1}{2} \boldsymbol{a} \Delta t^2$$

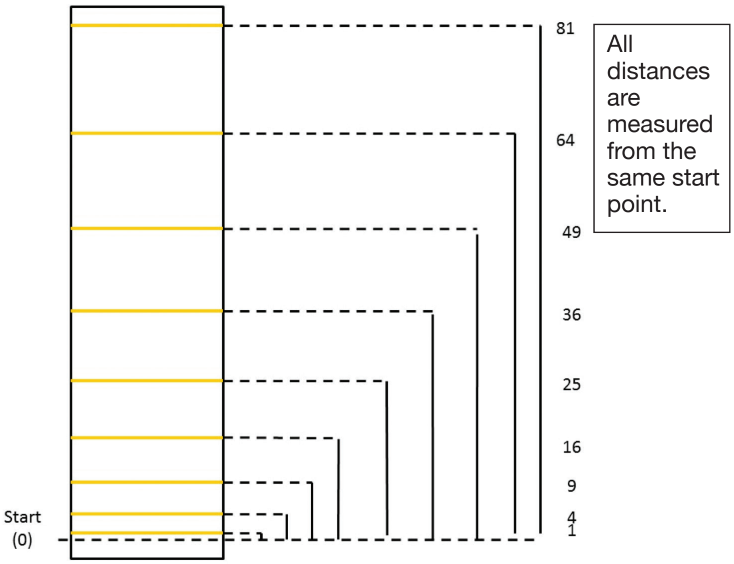
5. Remove the rubber bands from the foam board.
6. On the reverse side of the foam board, use a pencil to mark a line across the short dimension of the board 2 centimeters from the end. Label this line “0” (zero). Mark lines at the distances listed in Table 2. Each measurement should be made from the zero line (see Figure 11).

Table 2.

Displacement (cm)
1
4
9
16
25
36
49
64
81

continued on next page

Figure 11.



7. Place rubber bands on the foam board, covering the pencil lines you just made.
8. Set the foam board up at the same angle as in Step 3.
9. Roll the steel sphere down the foam board.

Note: The sounds made as the sphere crosses the rubber bands on the foam board in the second trial should be at equal intervals. The sphere is traveling a greater displacement each time it crosses a rubber band, but the time interval remains constant, meaning the sphere is moving with a constant acceleration.

Disposal and Cleanup

Return the materials to the appropriate kits.

Note: For more information on trigonometry, kinematic equations, and rotational motion exercises, see the following links:

Basic Right Triangle Trigonometry
https://www.carolina.com/teacher-resources/Interactive/basic-right-triangle-trigonometry/tr31008.tr?question=right%20triangle

Derivation of the Kinematics Equation
https://www.carolina.com/teacher-resources/Interactive/derivation-of-the-kinematics-equation/tr32615.tr?question=kinematics

Ring and Disc Demonstration
https://www.carolina.com/teacher-resources/Document/ring-and-disc-demonstration/tr32424.tr

ACTIVITY

Observations

Data Table 1.

Time vs. Displacement	
Time (s)	**Displacement (m)**
	0.00
	0.25
	0.50
	0.75
	1.00
	1.25
	1.50
	1.75
	2.00

Data Table 2.

Velocity vs. Time	
Time (s)	**Velocity (m/s)**
1	
2	
3	
4	
5	
6	
7	
8	

Data Table 3.

Acceleration over Increasing Displacement			
Time (s)	**Average time (s)**	**Average Time^2 (s^2)**	**Displacement (m)**
Trial 1 =			0.1
Trial 2 =			
Trial 3 =			
Trial 1 =			0.2
Trial 2 =			
Trial 3 =			
Trial 1 =			0.3
Trial 2 =			
Trial 3 =			
Trial 1 =			0.4
Trial 2 =			
Trial 3 =			
Trial 1 =			0.5
Trial 2 =			
Trial 3 =			
Trial 1 =			0.6
Trial 2 =			
Trial 3 =			
Trial 1 =			0.7
Trial 2 =			
Trial 3 =			
Trial 1 =			0.8
Trial 2 =			
Trial 3 =			

Title: ______________________________

Label (*y*-axis): ______________________________

Label (*x*-axis): ______________________________

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