

Master Internship/Stage de Fin d'Études

“Axiomization of Information Measures”

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Location: The student will be working at the COMELEC Department of Telecom Paris.

Related Fields: Information Theory and Probability.

Project Description: Basic information measures such as entropy and mutual information can be defined in two ways (see e.g., [1, 2]): 1.) An *operational characterization* reveals the information measures as the solution to a suitably defined operational problem (for example, entropy as the quantity characterizing the least number of bits required to reliably compress a stochastic source.) 2.) An *axiomatic characterization* reveals the information measure as the functional satisfying a set of suitably chosen properties (axioms). While operational definitions are extremely relevant to practical applications, axiomatic definitions may play a crucial role in revealing the fundamental nature of information measures and the interplay and connections between them.

In the first part of this project, we shall analyze existing axiomatic definitions of information measures and seek to devise a systematic approach to arrive at a provably minimal and consistent axiomization of said measures. In the second part of the project we will then derive new axiomatic definitions for information measures that so far only have operational characterizations. An example for such an information measure is Wyner’s common information [3], which will be a central focus in our project. It bears multiple operational characterizations, e.g., the minimum amount of common communication rate required in a distributed compression problem to achieve optimal overall performance, or the minimum amount of common randomness required to coordinate two remotely generated sequences according to a given probability distribution. In contrast to various operational characterizations, no axiomatic characterization of common information is known to date.

Expected Skills: The candidate is expected to have good analytic skills and a solid background in probability and information theory. The project is of fundamental and theoretical nature.

References

- [1] C. E. Shannon, “A mathematical theory of communication,” *The Bell System Technical Journal*, vol. 27, no. 3, pp. 379–423, 1948.
- [2] I. Csiszár, “Axiomatic characterizations of information measures,” *Entropy*, vol. 10, pp. 261–273, 2008.
- [3] A. Wyner, “The common information of two dependent random variables,” *IEEE Transactions on Information Theory*, vol. 21, no. 2, pp. 163–179, 1975.