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The IS-LM Model

This set of notes expands the Keynesian model with effects to incorporate the monetary sector. This model is popularly known as the IS-LM model, and allows for both fiscal and monetary policy multipliers. The notation follows approximately that in Mishkin's text.

1 Derivation

To allow for a role for money, let's first modify the model. On the real side of the economy, everything is the same, except for equation (5), for investment.

(1)	$Y = Y^{AD}$	Output equals aggregate demand, an equilibrium condition
(2)	$Y^{AD} \equiv C + I + G + NX$	Definition of aggregate demand
(3)	$C = a_0 + cY_D$	Cons. Fn., *c* is the marginal propensity to consume, *mpc*
(4)	$Y_D = (Y - T)$	Definition of Disposable Income
(5)	$T = TA_0 + tY$	Tax function; $TA_0$is lump sum taxes, *t* is tax rate.
(6)	$I = IN_0 - bi$	Investment function
(8)	$G = GO_0$	Government spending on goods and services, a constant
(9)	$NX = NX_0$	Net Export spending, a constant

The only essential difference is that investment spending now depends on the interest rate. The coefficient *b* is the interest sensitivity of investment. Since income now depends on interest rates, which is endogenous, then solving equations (1)-(9) yields an equation of a line.

(10) $Y = \alpha[A_0 - bi]$ <IS curve>

(10') $i = \frac{A_0}{b} - \frac{(1-c(1-t))}{b}Y$ <IS curve>

Where $A_0 \equiv a_0 - c(TA_0) + IN_0 + GO_0 + NX_0$ and $\alpha \equiv \frac{1}{1-c(1-t)}$. The expression in (10) and (10') means that for lower levels of interest rates, investment, a component of aggregate demand, is higher, and thus income is also higher.

We introduce the monetary sector by setting out money supply and money demand.

Eq.No.	Equation	Description
(11)	$\frac{M^s}{P} = \frac{M^d}{P}$	Equilibrium condition
(12)	$\frac{M^s}{P} = \frac{M_0}{P_0}$	Money supply
(13)	$\frac{M^d}{P} = \mu_0 + kY - hi$	Money demand

Substitute (12) and (13) into (11), to obtain:

(14) $\frac{M_0}{P_0} = \mu_0 + kY - hi$ -h (k) is the interest (income) sensitivity of real money demand

Solving for the interest rate yields:

(15) $i = \frac{\mu_0}{h} - \left(\frac{1}{h}\right)\left(\frac{M_0}{P_0}\right) + \left(\frac{k}{h}\right)Y$ <LM curve>

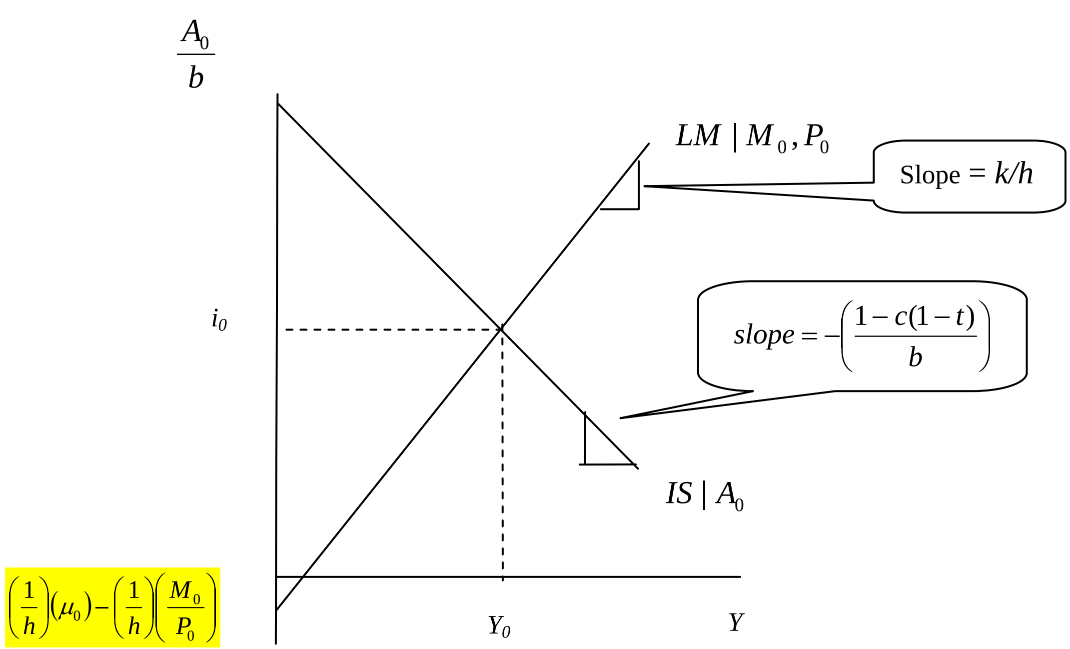
There are two unknowns, and two equations. To figure out equilibrium income and equilibrium interest rates, one would need to solve the system. This is shown in the appendix. For now, I'll merely show the answer, and relate it to the graphical depiction.

(16) $Y_0 = \hat{\alpha}[A_0 + \left(\frac{b}{h}\right)\left(\frac{M_0}{P_0}\right) - \left(\frac{b}{h}\right)\mu_0\ ]$

where $\hat{\alpha} \equiv \frac{1}{1-c(1-t)+(bk/h)}$

Notice that equilibrium income now depends on the level of autonomous spending, the real exchange rate, and the money stock (in real terms). The equilibrium interest rate is a complicated function of autonomous spending, real exchange rate and the money stock. To obtain this value, one would substitute (5) into (14). This is done in the appendix.

The equilibrium income level and interest rate is depicted in the figure below:



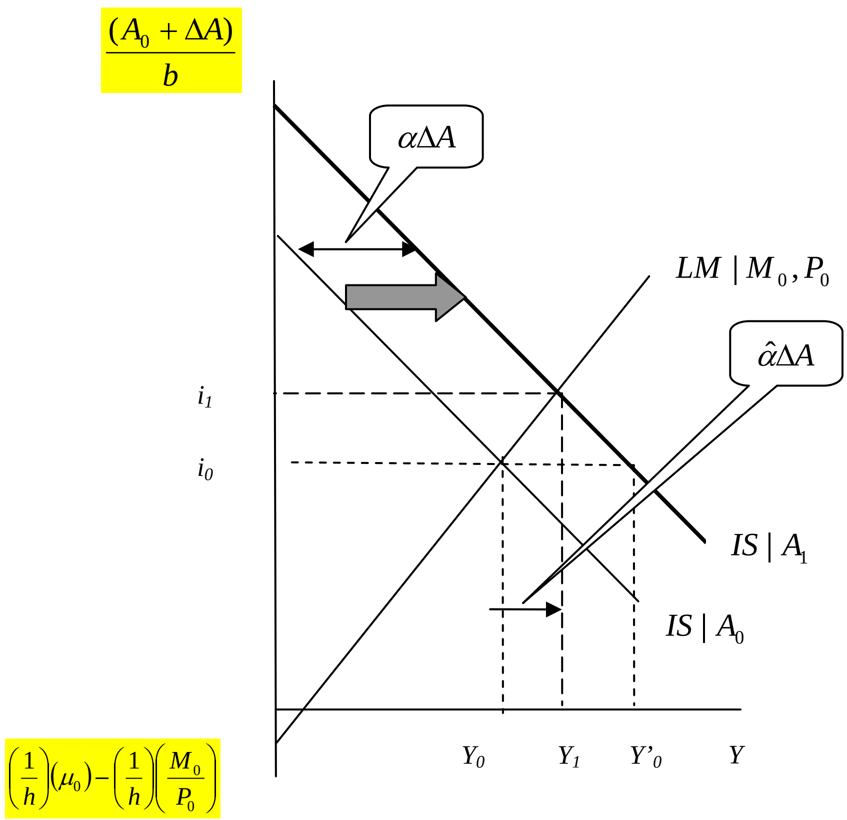
One can see that the IS curve has a slope that depends upon the parameters by solving (10) with i on the left hand side, as in equation (10'). Note:

- The position of the IS curve depends upon A_0.
- The position of the LM curve depends upon the real money stock, (M_0/P_0).

Only at the combination of i_0 and Y_0 is it true that both the goods market and the money market are in equilibrium ($Y=Y^{AD}$ and $M^s=M^d$ respectively).

2 Policy in the IS-LM Model

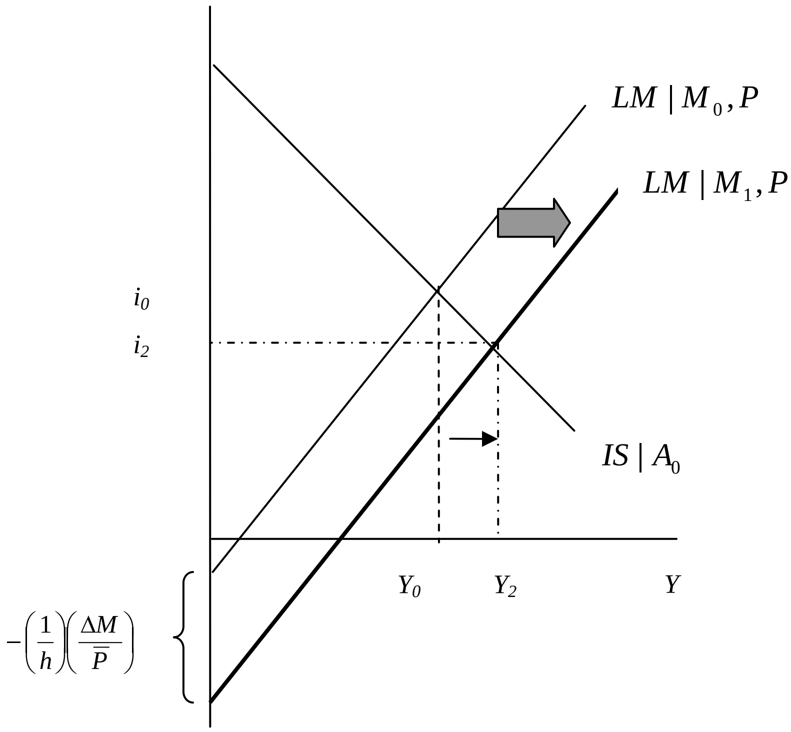
Notice that if one increases A, EXP, or q, then the vertical intercept increases, which is the same as the IS curve shifting out rightward. If one increases M when P is constant, then M/P rises, and the LM vertical intercept shifts down, which is the same as the LM curve shifting out rightward. In the former case, output increases, and interest rates rise. In the latter, interest rates fall, and output rises.



Notation: $A_1 = A_0 + \Delta A$ and $Y_1 - Y_0 = \hat{\alpha}\Delta A$

Notice that the increase in output $\hat{\alpha}\Delta A$ is smaller than that which would have been implied by the simple Keynesian multiplier ($\alpha\Delta A$).

The reason the increase in output is less in this system is because of "crowding out". Higher output leads to higher money demand which, given a constant money supply, results in a higher equilibrating interest rate. The higher interest rate depresses investment, thus offsetting in part the increase in output.



Note: $\Delta\left(\frac{M}{P}\right) = \frac{M_1}{P_0} - \frac{M_0}{P_0}$ and $Y_2 - Y_0 = \hat{\alpha}(b/h)\Delta(M/P)$

Interest rates fall, resulting in a higher level of investment, thus a higher level of aggregate demand and hence output.

How can one solve for the change in income resulting from changes in policy variables analytically? Take equation (16), and break it up into the constituent changes (i.e., take a total differential):

(17) Y $\Delta Y = \hat{\alpha}[\Delta A + \left(\frac{b}{h}\right)\Delta\left(\frac{M}{P}\right) - \left(\frac{b}{h}\right)\Delta(\mu)]$

For changes in government spending *only*:

(18) $\Delta Y = \hat{\alpha}\Delta GO \rightarrow \dfrac{\Delta Y}{\Delta GO} = \hat{\alpha}$

For changes in money *only* (with prices constant):

(19) $\dfrac{M^d}{P} = \dfrac{M^s}{P} \rightarrow \dfrac{\Delta Y}{\Delta(M/P)} = \hat{\alpha}\left(\dfrac{b}{h}\right)$

Appendix
Solving the IS-LM System Algebraically

To solve for equilibrium income, substitute the equation (15) <LM> into (10) <IS>:

(10) $Y = \alpha[A_0 - bi]$ <IS curve>

(15) $i = \frac{\mu_0}{h} - \left(\frac{1}{h}\right)\left(\frac{M_0}{P_0}\right) + \left(\frac{k}{h}\right)Y$ <LM curve>

(A1) $Y = \alpha\left[A_0 - b\left(\frac{1}{h}\mu_0 - \frac{1}{h}\frac{M_0}{P_0} + \frac{k}{h}Y\right)\right]$

Bring the multiplier and the Y term to the left hand side.

(A2) $Y(1 - c(1-t) + \frac{bk}{h}) = \left[A_0 - b\left(\frac{1}{h}\mu_0 - \frac{1}{h}\frac{M_0}{P_0}\right)\right]$

Divide both sides by the term in parentheses to obtain equation (31):

(16) $Y_0 = \hat{\alpha}\left[A_0 + \left(\frac{b}{h}\right)\left(\frac{M_0}{P_0}\right) - \left(\frac{b}{h}\right)\mu_0\right]$

where $\hat{\alpha} \equiv \frac{1}{1-c(1-t)+(bk/h)}$

To obtain the equilibrium interest rate, substitute (16) into (10’) or (15). The equilibrium interest rate will depend upon all the variables that output depended upon.

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