ILLUSION IN REALITY: Visual Perception in Displays

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ABSTRACT

Research into visual perception ultimately affects display design. Advances in display technology affects, in turn, our study of perception. Although this statement is too general to provoke controversy, this paper presents a real-life example that may prompt display engineers to make greater use of basic knowledge of visual perception, and encourage those who study perception to track more closely leading edge display technology. Our real-life example deals with an ancient problem, the moon illusion: Why does the horizon moon appear so large while the elevated moon look so small? This was a puzzle for many centuries. Physical explanations, such as refraction by the atmosphere, are incorrect. The difference in apparent size may be classified as a misperception, so the answer must lie in the general principles of visual perception. The factors underlying the moon illusion must be the same factors as those that enable us to perceive the sizes of ordinary objects in visual space. Progress toward solving the problem has been irregular, since methods for actually measuring the illusion under a wide range of conditions were lacking. An advance in display technology made possible a serious and methodologically controlled study of the illusion. This technology was the first heads-up display. In this paper we will describe how the heads-up display concept made it possible to test several competing theories of the moon illusion, and how it led to an explanation that stood for nearly 40 years. We also consider the criticisms of that explanation and how the optics of the heads-up display also played a role in providing data for the critics. Finally, we will describe our own advance on the original methodology. This advance was motivated by previously unrelated principles of space perception. We used a stereoscopic heads up display to test alternative hypotheses about the illusion and to discriminate between two classes of mutually contradictory theories. At its core, the explanation for the moon illusion has implications for the design of virtual reality displays. How do we scale disparity at great distances to reflect depth between points at those distances? We conjecture that one yardstick involved in that scaling is provided by oculomotor cues operating at near distances. Without the presence of such a yardstick it is not possible to account for depth at long distances. As we shall explain, size and depth constancy should both fail in virtual reality displays where all of the visual information is optically in one plane. We suggest ways to study this problem, and also means by which displays may be designed to present information at different optical distances.

Keywords; Augmented Reality, Stereoscopic Displays, Moon Illusion, Visual Perception.

1. INTRODUCTION

To ensure that display elements will be visible, legible, and distinctive, designers rely on basic data related to visual acuity, contrast sensitivity, and color discrimination. Such psychophysical data reflect “early” stages in the processing of visual stimuli. However, in many applications “later” stages of perceptual processing are equally important. For example, simulated visual scenes may enable the user to perceive the position, orientation, size and distance of a displayed object. It may be desirable to employ the minimum amount of visual information needed for the task. This is frequently not possible because we lack clear knowledge of what is essential. Consequently, designers may opt for as realistic a scene as possible – they create a virtual reality. But, even virtual reality displays may lack perceptually important attributes of scenes they mimic. These missing attributes provide information used by the perceptual system, even though observers are not aware of using it. The ancient *moon illusion* – the fact that the horizon moon appears to be much larger than the elevated moon, even though its angular size is the same – provides a superb example of how perceptual information may be misconstrued by observers, including designers. The illusion is interesting for other reasons. For example, the history of this illusion illustrates how display technology can help to clarify our understanding of the perceptual system. Furthermore, it is an example of how enhanced understanding of perception reveals previously unnoticed problems for display technology.

The third edition of Helmholtz's *Physiological Optics,* contains comments by von Kries, one of its editors. He wrote:

> "In my opinion, any attempt to explain (the moon illusion) should start, first of all, with the fact that in gazing at the heavenly bodies we get a very definite impression of a certain *absolute* size... This apparent absolute size is, indeed, plainly out of all proportion to the angular diameter and the distance at which these objects are seen. For example, in my own case (which I believe is likewise true of many other persons), I get a very positive impression from the disc of the full moon that can readily be expressed in terms of an absolute magnitude, notwithstanding the fact that I may be perfectly aware of the absurdity of any such estimate. When the moon is high in the sky, I can fancy that it is about 20 cm in diameter, whereas, when it comes up above the horizon, I may estimate the diameter to be between 30 and 35 cm. An object that was really this size would have to be about 25 metres away in order to subtend the same visual angle as the moon." (1, p. 602)

The moon is far beyond the distance of any terrestrial object. Perceiving its size is an instance of perception at its limit. However, as von Kries implies, the perceptual system may treat the moon as if it is an ordinary terrestrial object. What is the evidence that this is true? Clearly, the perceptual system did not evolve to permit us to deal with objects at astronomical distances. The perceptual system must be adapted to identify objects, regardless of their distances from us. How could such a perceptual system deal with an object like the moon?

2. ON SIZE CONSTANCY

It is not our intention to review the elementary facts of size perception. However, a brief overview will help to set the stage for what follows. The angular size of an object in one dimension is inversely proportional to its distance. Thus, according to Euclid's law,

$$\text{Tan } \alpha = H/D$$

Where α = the angle subtended at the observer's eye by the object, H = the height of the object, and D = the distance between the eye and the object. The object's visual angle directly reflects the size of its retinal image. Clearly, if observers correctly perceive the linear size (H) of an object, then they must make use of both angular image size, and information reflecting the distance to the object.

In their classic study of *size constancy,* Holway and Boring (2) employed a disk of variable diameter at a distance of about 3 m. Subjects adjusted the diameter of this *variable* disk until it matched that of a *standard* disk at each of several greater distances (ranging up to ~30 m). This standard disk had a constant angular size of 1 deg. Consequently; it had a larger linear diameter at longer distances. With the environment illuminated, the diameter of the variable disk selected by the subjects tended to be proportional to the linear diameter of the standard disk at all distances. (For purposes of this paper we ignore a small tendency to overestimate the size of the standard disk.) Thus, the selected diameter was very close to the actual linear diameter of the more distant disk at each of its positions. However, with the disks in a darkened environment and viewed by one eye through a small aperture, subjects tended to make the size of the nearby disk more nearly the same size, regardless of the distance to the standard disk. These settings appeared to be a compromise between a match of the linear (distal) size of the disk and its angular size. The ability to estimate the linear size of an object at different distances is referred to as *size constancy,* and it clearly depends in large part upon the availability of cues to distance, which are reduced with monocular viewing in the darkened corridor.

The so-called distance cues derive from diverse sources that play complementary roles in space perception. For example, at distances up to 2 or 3 meters, the convergence angle of the eyes is related to perceiving differences in distance. Convergence acts to reduce diplopia (double vision), which occurs when an object is imaged at disparate retinal places. Another stimulus to convergence is the change in curvature of the eye's lens (accommodation). Accommodation creates a sharper image of an object. But it also initiates a corresponding change in convergence as an object's distance changes, even when one eye is occluded. The nervous system uses the blur circles and color fringes associated with spherical and chromatic aberrations of the eye in determining the required direction and magnitude of change in accommodation. The perceptual system automatically employs these subtle differences to compute relative distance, even though the human subject may be totally unaware of the diplopia, blur, and color fringes.

At distances greater than a few meters, accommodation and convergence are no longer effective. Then the system must rely on other kinds of information, such as perspective. Euclid's law plays a role here too. The classic image of parallel railroad tracks that appear to converge with distance in the picture plane is predictable from the law. Hence, we refer to *Euclidean cues*, e.g., linear perspective and texture perspective (3). Also, head movements result in differential angular velocities of images of objects and textural elements at different distances. This *motion parallax* (or *motion perspective*) and the related *kinetic depth effect* undoubtedly play important roles in depth perception.

There are many other cues to distance and depth. However, we briefly mention one more cue, *binocular disparity.* Since the two eyes view the world from two different positions (about 6.4 cm apart), they capture two slightly different images. Wheatstone (4) was the first to fully realize that this binocular parallax results in disparity of the images. This relative disparity makes it possible to perceive the solidity of objects, and also that they lie at different distances.

The binocular system is exquisitely sensitive to relative disparity, e.g., the disparity threshold may be as small as 1 arcsec (5). The moon lies beyond the limit of normal distance perception. Yet the binocular disparity between the moon and objects at 1 km is approximately 12 arcsec. Depth between the moon and objects more than 1 km away is likely to be perceptible, and is detectible well beyond the 2 or 3 m range of convergence. Of course, there is no way that the depth signified by this disparity can reflect the actual distance between the moon and the relatively disparate object.

Researchers are just now coming to grips with the problem of how and to what degree the different visual cues interact with each other to promote distance perception. Most of this literature involves the interactions among cues to distances within ~2 m of the observer, and very little of it deals with how cues to near distances may help scale cues to far distances. However, for now it suffices to note that many cues are known, some appear to be redundant, but others probably complement each other. Large disparities at close distances promote convergence and may inform the perceptual system about distance, but small disparities at great distances inform the perceptual system about differences in distance to objects, without activating convergence.

3. THE MOON'S PERCEIVED SIZE AND DISTANCE

Some of the earliest writers suggested that the moon's perceived distance varies with its elevation in the sky. Alhazen in the 11th century and Robert Smith in the 18th were among the most prominent of those who described versions of the so-called *apparent distance* theory of the moon illusion (6). Figure 1 is adapted from Smith (7). The actual path of the moon is a circle, with the observer at its center. Therefore, the angular size of the moon is constant, regardless of its elevation. (We ignore the extremely small eccentricity attributable to the radius of the earth.) However, if the sky is perceived as flattened, and if the moon is perceived as being on the "surface" of the sky, then the elevated moon must be perceived as smaller than the horizon moon. Thus, according to this theory, the illusion is due to differences in the apparent distance to the moon.

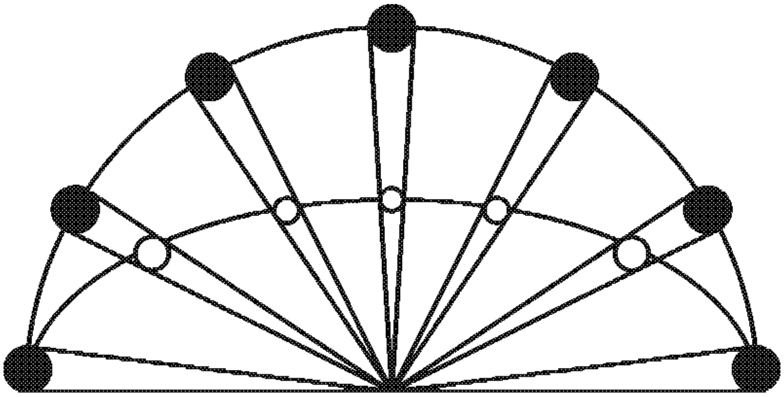


Figure 1. Regardless of its elevation, the distance between an observer (at the center of the horizontal line) and the moon remains constant (filled circles). However, a moon perceived as growing closer as its elevation increases (unfilled circles), must appear as growing smaller.

Von Kries implied that the horizon moon, a 0.5 deg diameter object, is perceived as being 30 – 35 m away. The zenith moon has the same angular size, but, for him, its distance is only about 20 m. We recall from Euclid's law that:

$$H = \alpha D$$

where α = visual angle in radians. However, nobody holds that observers always accurately perceive the distance to an object. This is certainly obvious when considering the moon or any other object at a vast distance, or even an object more than a few meters away in an empty space. As a general statement Euclid's law cannot be applicable to perceived size. Hence, to predict perceived size, investigators replaced physical distance with perceived distance. A strong form of this theory involves the so-called *size-distance invariance hypothesis* (SDIH) (8). SDIH holds that *perceived* size is proportional to *perceived* distance. Thus,

$$\mathit{H} = \alpha \mathit{D}$$

where H = perceived size and D = perceived distance. Perfect size constancy occurs only when perceived distance matches physical distance.

It may be of some interest to note that *Emmert's law*, which holds that the perceived size of an after image is proportional to its perceived distance, is a special instance of the SDIH. An after image has a constant retinal (angular) size. Thus, with α constant, an increase in D must be accompanied by a proportionate increase in H. Fig. 1 exemplifies Emmert's law. When the elevated moon is perceived as close, it is perceived as proportionately smaller. This is the essence of the classic *apparent distance theory* of the moon illusion.

There are several major problems that need to be resolved before SDIH can be tested. First, there are methodological problems related to defining *perceived size* and *perceived distance*. Second, there is a dearth of psychophysical data relating perceived distance to physical distance, especially over long distances.

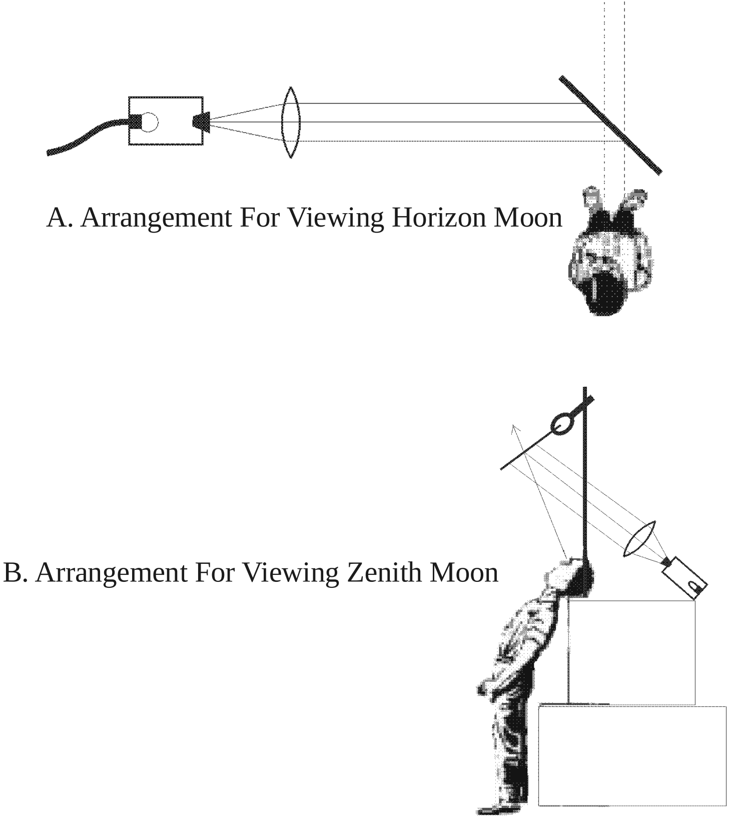
In the face of this lack of knowledge, Robert Smith's diagram (Fig. 1) is misleading. It forces us to assume that the moon's perceived size must be consistent with the SDIH, where D is the perceived distance to the sky. Let us entertain a more modest proposal. That is, the perceived size of an object of constant angular size is not simply proportional to perceived distance, but is some unknown monotonic function of perceived distance. In fact, the function relating perceived distance to physical distance undoubtedly depends upon many different conditions. We suggest that the Size Distance Invariance Hypothesis is only an approximation to this undefined function. Thus, if an object is viewed in an empty visual field, its perceived distance may be less than that of an object at the same physical distance but in a fully textured visual field. It is even possible that different non-linear functions relate perceived size to perceived distance, depending upon the available cues to distance. Thus, by this weak form of an apparent distance theory, the elevated moon appears as smaller because it is perceived as closer, even though the function relating perceived size to perceived distance is unknown.

4. METHODOLOGY REARS ITS UGLY HEAD

In the standard size constancy paradigm, a distant object is the standard, and the size of a nearby (variable) object is adjusted until it appears to match that of the standard. When applied to terrestrial objects, this method seems sensible. However, when attempting to measure the "size" of the moon, some important problems are manifested. For example, Holway and Boring (9,10) had three subjects (including themselves) attempt to match the size of the moon with one of 50 circular disks projected onto a screen 3.5 m away and off to one side. The basic data generated in these experiments were ratios, e.g., the diameter of the disk that matched the horizon moon was divided by the diameter of the disk that matched the elevated moon. Boring (11) complained about the difficulty of making these matches, comparing it to heterochromatic photometry where one tries to ignore differences in chromaticity while matching the intensities of two patches of different colors. The disks selected by subjects to match the moon were several degrees in diameter. But the moon subtends an angle of 0.5 deg. Curiously, after a subject made a selection, the experimenter reported that the match did not "look" right to him. However, by moving to a position near the subject, the experimenter found the match to be more acceptable. This suggests that the normal size constancy paradigm was not used in this experiment. If size constancy applied to the nearby disk, the match made by the subject should have seemed correct at any distance from the nearby disk. It suggests instead that subjects were using a method of magnitude estimation related to those in which subjects indicate their impression of an objects size by selecting some arbitrary number or exerting a force on a dynamometer. It is conceivable that a nearby disk, which, in this case is not commensurable with the moon, is also an arbitrary indicator of perceived magnitude.

Holway and Boring found that when the moon was viewed with eyes elevated in the head it was matched by a smaller disk than it was when the same moon was viewed with eyes level in the head. This was as true for supine as for erect subjects. Therefore, they attributed the effect to eye elevation and not to head elevation, nor to an inherent anisotropy of visual space (12, 13). However, Holway and Boring reported that their subjects were unaware that elevating their eyes resulted in a reduction in the size of the moon! This is consistent with observations described by Kaufman and Rock (1962). Therefore, Holway and Boring's illusion manifested itself as a difference in the size of the selected nearby disk. It was not possible for their subjects to employ a matching method in which they adjusted the size of one moon to match that of another, thereby obviating the need for the nearby variable disk.

Psychophysical measurements are subject to a variety of constant errors. Such errors have been attributed to the order of presentation of stimuli, the range of comparison stimuli, the step-sizes among them, and other factors as well. In fact, Kaufman and Rock (14) discovered that subjects often overestimate the size of the standard moon, whether it is elevated or at the horizon. This so-called *error of the standard* may be quite appreciable in magnitude, and there is some reason to believe that it increases with the distance separating the standard and variable stimuli. If it had been possible to control the angular size of the moon in the sky, then Holway and Boring might have measured the contribution that the error of the standard may have made to their illusion.



A. Arrangement For Viewing Horizon Moon

B. Arrangement For Viewing Zenith Moon

Figure 2. Schematic of first moon illusion apparatus used by Kaufman and Rock (15). A plate containing a small aperture was mounted on a light source and placed at the focus of a positive converging lens. This collimated the light emerging from the aperture. A virtual image of the aperture was reflected by a thin sheet of glass and was located at optical infinity. The diameter of the lens permitted both eyes to see the aperture superimposed either (A) on the horizon, or (B) at a high angle of elevation. Two units were employed so that subjects could look back and forth and instruct the experimenter to change the aperture of one of the unity until a satisfactory match was achieved.

To directly test the eye elevation hypothesis it is necessary that the observer be able to adjust the size of one moon to match that of the other. To accomplish this, Kaufman and Rock (15) made use of an optical apparatus based on the same principle as that of the heads-up display. A small disk of variable size was placed at the focus of a large diameter lens. The subject viewed the reflected image of this virtual moon in a beam splitter. As illustrated in Fig. 2, one virtual moon was viewed over the horizon, and the other at a high elevation in the sky. In point of fact, subjects tended to make the elevated moon physically larger than the horizon moon to achieve an acceptable match. Conversely, when adjusting the horizon moon to match the elevated moon, subjects had to make it smaller. The size of this moon illusion depended upon the viewing circumstances. For example, when viewed across a well-illuminated seascape, the size of the horizon moon was perceived as equal to that of the elevated moon when the latter was nearly twice the angular size of the horizon moon. The illusion was significantly larger when the sky overcast by clouds than when the sky was clear. When the actual horizon was occluded by a distant stand of trees the illusion was significantly smaller than when the horizon moon was viewed above a horizon that was over 5 km away. However, elevating the eyes had no effect on the illusion. With two moons at the same elevation, one of them viewed with eyes level and the other with eyes elevated, there was virtually no measurable illusion. Rock and Kaufman (16) concluded that the moon illusion was dependent upon the presence of a terrain. In fact, even when subjects looked up into a large right angle prism to view the terrain and horizon, the horizon moon was still perceived as larger than the moon seen in the empty sky, but with eyes level and the head erect.

Rock and Kaufman (16) concluded that the horizon moon does not look larger than the zenith moon because of either eye or head elevation. Rather, it depends upon the presence of terrain in one direction, and the magnitude of the illusion depends upon the distance to the horizon. When a vertical textured board occluded the terrain up to the horizon, the moon just above the horizon shrank dramatically in size. So it isn't just the texture of the terrain filling the lower half of the visual field -- the illusion is very much dependent upon the presence of distance cues provided by the terrain.

5. A SIZE-DISTANCE PARADOX

Kaufman and Rock considered their results to be consistent with the apparent distance theory (15, 16), although they expressed reservations about the precise applicability of SDIH. Even so, they concluded that the larger perceived size of the horizon moon was due to its being perceived at a greater distance than that of the elevated moon. These results were simply unattainable when the standard size constancy paradigm was employed. The nearby disk and the distant moon were incommensurable objects (which is why we suggest that subjects may have made magnitude estimations), and subject response bias may well play an enormous role in such experiments. However, one major objection was raised to Kaufman and Rock's interpretation. That is, when most people are asked which of the two moons appears as closer, they claim that the horizon moon is closer than the elevated moon. This flies in the face of any claim that the perceived distance to the moon determines its perceived size. Kaufman and Rock noted that their subjects spontaneously reported that the horizon moon appears as closer "because it is larger." This suggested to them that cues indicating that the horizon moon is at a greater distance determine its greater size. However, once that size is determined, subjects use it in making a conscious inference (judgment) that the moon is closer. Thus, the distance judgments reflect a bias based on experience, and do not reflect how the perceptual system responds directly to cues to distance. To check on this hypothesis they presented an elevated virtual moon with a much larger angular size than that of a virtual horizon moon. This size difference was obvious to the subjects. Nine of ten subjects reported that the apparently larger elevated moon was closer than the horizon moon. But when the horizon moon appeared to be larger than the elevated moon, subjects declared it to be closer.

It is widely accepted that relative size is a cue to distance. This is implicit in linear perspective where, according to Euclid's law, the nearer of two similar objects subtends a larger angle at the eye than the farther object. Gregory (17) argued that cues to distance operate to give the horizon moon a size appropriate to its perceived distance. However, since there are no cues to the distance of the elevated moon, it has a "default size", which happens to be smaller than that of the horizon moon. According to Gregory, the larger apparent size of the horizon moon operates in a "top-down" fashion to make it appear close when compared to the elevated moon, which, because of its smaller apparent size, appears as far. Of course, there is no obvious reason why the default size happens to be small. It makes at least equally good sense to assume that the elevated moon is perceived at a default distance, and data do suggest that this happens to be smaller than the distance to the horizon moon (18). Thus there may be a paradox: The perceived size of an object is determined by its perceived distance and its angular size, but its distance is determined by its size. If the size is angular size, then there is indeed a paradox. But if it is perceived size, then there may not be a paradox. More explicitly, if perceived size is not the same as angular size, then variations in perceived size need not determine perceived distance, and the paradox vanishes. Thus far, none of the commentators on this issue have made this distinction. In any event, many authors reject apparent distance theories on the grounds that the larger horizon moon is judged to be closer than the small-appearing elevated moon. How may this ostensible size-distance paradox be resolved?

6. ON THE MEANING OF 'SIZE'

As we have indicated, the traditional view is that the perceptual system computes size by taking information about distance into account. Recent theories propose an alternative view. They postulate that the elevated moon's small apparent size is not due to its perceived distance. Instead these theories assume that observers perceive the angular size of an object, and that the moon illusion is an illusion of angular and not of linear size. Of course, if the observer's perceived linear size rather than angular size, then the theories we are discussing would be entrapped by the size-distance paradox described above.

If a subject matches the length in meters of a distant rod to that of a nearby rod, as in a size constancy experiment, we refer to perceived linear size. Alternatively, if a subject adjusts the length of a nearby rod so that it subtends the same angle at the eye as a distant rod, we refer to perceived angular size. McCready (19, 20) considers perceiving angular size to be equivalent to the perception of the difference in visual directions to the two ends of the rod. Of course, an infinite number of distal rods of different lengths and orientations may subtend the same visual angle, and therefore visual angle *per se* cannot represent any specific object. Similarly, Roscoe (21) proposed that retinal size is perceptible. Perceiving a retinal image is tantamount to perceiving the distribution of activity across the mosaic of retinal receptors. Again, an infinite number of objects are capable of producing an identical retinal image.

Perception is of objects and not of neural activity, so strictly speaking, image size *per se* may not be perceptible. Moreover, at any given time a perception of an individual object is unique, even if the perception is invalid or illusory. Obviously, perceiving linear size is possible if and only if information regarding distance to the object is available and processed. If this information is incorrect (e.g., when convergence is increased by means of a prism), linear size is misperceived, but it still corresponds to a physical object at a particular distance. Hypothetically, if two objects were presented in the absence of all cues to distance, perceivers may determine if they have the same or different retinal sizes. However, this determination may equally well be based on the assumption that both distal objects are at the same distance.

Considerations such as these underlie the controversy concerning the perceptibility of angular size *per se* (22, 23, 24), and this controversy remains unsettled (8). It should be borne in mind that in actuality perceived size is of neither linear size nor of angular size. Since the visual system evolved to permit discrimination of size at a distance, perceived size is probably an imperfect *reflection* of linear size. Burbeck (25) provides convincing evidence that this is the case, as do McKee and Welch (26) whose evidence indicates that at short distances subjects do not have direct access to the angle subtended at the retina. Nevertheless, several theories attribute the moon illusion to a difference in the perceived angular size of the moon (20, 21, 27, 28). The moon's perceived angular size is ostensibly the predominant cue to its distance.

For the sake of the present discussion let us grant that one does perceive the angular size of the moon. How does a difference in its perceived angular size come about, when its physical angular size is the same at all elevations? Some investigators hold that the perceived angular size of the moon is directly affected by the accommodation of the eyes. Thus, according to Roscoe (21), accommodation increases while viewing the elevated moon and this leads directly to a reduction in its perceived angular size. Roscoe explicitly rejects the idea that accommodation acts in this case as a cue to the distance of the moon, as it is in the case of nearby objects. He asserts instead that accommodation has a direct effect on perceived angular size, and this is what determines perceived distance. It is known that in an empty field the eyes tend toward a nearby resting focus (29). Roscoe suggests that the elevated moon in the empty sky is not a sufficient stimulus for accommodating to infinity, so the eyes tend to accommodate for near. A reduction in perceived size due to increased accommodation is called *accommodative micropsia*.

Enright (27) suggested that rolling the eyes upward to view the elevated moon results in a transient increase in convergence, which in turn leads to a reduction in the perceived angular size of the moon. He also notes that viewing the moon across the terrain, as opposed to viewing the moon in an empty sky, leads to a lessening of convergence. Convergence to a distance less than the actual distance of an object, as when looking through base-out prisms, does results in an diminution of size --- a phenomenon known as *convergence micropsia* (30). However, sustained eye elevation may not produce anything more than a very small reduction in the moon's size. Therefore, it cannot account for the moon illusion. Enright's suggestion that a transient effect is implicated has never been directly tested, but it does not seem plausible. In summary, Roscoe, Enright and McCready all propose that oculomotor micropsia causes the perceived angular size of the elevated moon to be smaller than that of the horizon moon. The difference in perceived angular size then acts as a cue to distance, so that the elevated moon appears to be farther than the horizon moon.

Wagner et al. (28) do not attribute the illusion to oculomotor micropsia. Nevertheless, they consider the illusion to be one of a difference perceived angular size. In their theory the perceived angular size of the moon is determined by the ratio of its angular extent to that of its surrounding context. This causes the elevated moon to appear as smaller and therefore as farther.

7. TOWARDS A DECISION

By now it is obvious that apparent distance theories are diametrically opposed to the idea that perceived angular size of the moon determines its perceived distance. Recognizing this, Kaufman and Kaufman (31) tested two mutually exclusive hypotheses. The hypotheses are drawn directly from the fact that disparity alone does not determine the perceived distance to an object, nor the amount of depth between objects. Disparity must be calibrated by cues to distance, as will be made clear below. To test these hypotheses we designed a new stereoscopic heads-up display. This apparatus is described after we specify our two mutually exclusive hypotheses. The hypotheses are:

> 1. The binocular disparity of a point perceived as being halfway along the line of sight to the horizon moon is substantially smaller than the disparity of a point similarly perceived as being half the distance to the elevated moon seen against an empty sky.
>
> 2. The binocular disparity of a point perceived as being halfway along the line of sight to the elevated moon is substantially smaller than the disparity of a point similarly perceived as being half the distance to the moon seen above the terrain at the horizon. This is precisely the opposite of Hypothesis 1.

As we indicated above, the rationale for these hypotheses is based on the fact that binocular disparity alone can signify neither distance to objects nor the *amount* of depth between them. Disparity must be calibrated by information regarding distance. This follows from the geometry. In general, the relative disparity

$$\varrho = [a(E_1 - E_2)] \div [E_1 \times E_2].$$

Where,

a = distance between the eyes

E_1 = distance in meters to one object

E_2 = distance in meters to a more distant object.

If we let $(E_1 - E_2) = \delta$, then the depth

$$\delta = [\varrho\,(E_1 \times E_2)] / a..$$

That is, the depth associated with a particular binocular disparity ϱ is proportional to the product of the distances to the two objects. Where the depth, δ , is small relative to the viewing distance, the magnitude of the depth increases approximately as the square of the distance. This suggests that the perceived depth associated with a given disparity is powerfully affected by viewing distance. For example, Wallach and Zuckerman (32) demonstrated that the perceived depth associated with a relatively small fixed disparity increases approximately with the square of distance. By contrast, geometrically the linear size of an object of fixed angular size increases only as the first power of its distance. As with frontal size, the effect of distance on depth due to disparity probably diminishes as distance cues are made less effective. Unfortunately, most systematic studies of the effect of distance on disparity were limited to distances of about 2 m. However, Cormack (33) demonstrated that depth perceived in the after image of a stereogram increases in accord with the geometry of stereopsis up to a distance as great as 20 m. Further, he showed that depth continues to increase, but at a slower rate at greater distances. Therefore over long distances, for a constant binocular disparity, perceived depth is a monotonic function of perceived distance.

Oculomotor cues and pictorial cues (e.g., perspective) calibrate disparity (32). The weights given to these cues by the perceptual system may differ with distance. Thus, oculomotor cues become less effective at distances greater than 2 or 3 m, beyond which the perceptual system becomes increasingly dependent upon pictorial cues. As pointed out earlier, depth due to disparity is detectable at very great distances, well beyond the range of convergence and accommodation. Beyond the range of oculomotor cues, pictorial cues to distance must play an important role in determining the perceived depth represented by relative disparity.

8. HALF-DISTANCE EXPERIMENT

To test their alternative hypotheses, Kaufman and Kaufman (31) produced a stereoscopic heads up display. Subjects were presented with two virtual moons adjacent to each other on the sky. The "moons" were 0.6 deg diameter luminous disks produced on an IBM Thinkpad flat panel display mounted in a stereoscopic optical apparatus depicted in Fig. 3. One of them (the variable moon) was initially perceived as closer or at the same distance as the other (the reference moon). When near the horizon the reference moon had zero disparity relative to objects at the horizon. The active matrix screen of the computer was mounted horizontally in a frame 38.3 cm beneath a black screen containing two apertures, 6.4 cm apart. Each aperture contained a 38.3 cm focal length lens. A partially silvered front-surface 40 x 40 cm mirror was mounted at a controllable angle above the lenses so that subjects looking through the mirror viewed virtual images of 0.62 deg diameter "moons". Two moons were located under each lens to produce four virtual images at optical infinity. Subjects looking through the mirror fused the two sets of moons, and saw one pair of moons on the sky.

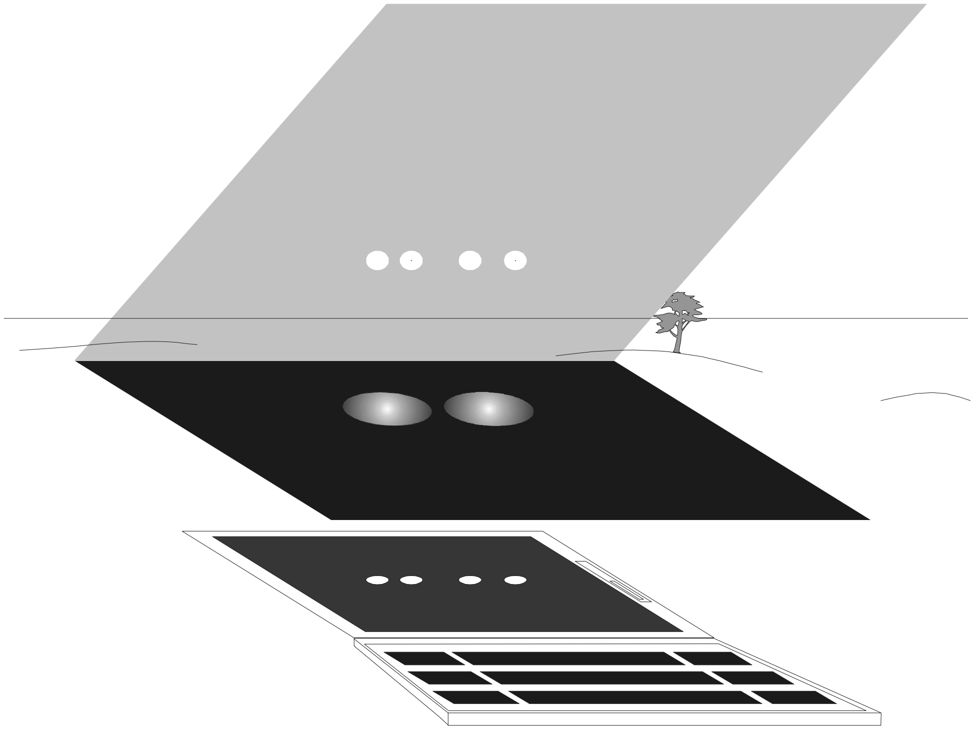


Figure 3. Schematic of stereoscopic heads-up display.

The image containing the stereogram of two moons was augmented by a landscape viewed from a hilltop on the C.W. Post campus of Long Island University in Brookville, NY. The moon that appeared closer also “seemed” smaller (although the image size of the two moons was always identical). The initial depth between the moons in the experiment was produced by a small, randomly selected, binocular disparity. Pressing a key increased or decreased the initial disparity. Subjects were asked to change the variable moon's distance so that it bisected the space between themselves and the distant reference moon. The angular sizes of both moons remained constant throughout the experiment. Another key press recorded this setting, and a new trial was begun. The elevation of the fused images was determined by the angle of the mirror that placed the reference moon either about 1.5 deg above the horizon, or at 45 deg to produce an elevated moon viewed through empty space against the sky. In both conditions the center of the half-image of the left-hand variable moon was always 1.35 deg from the reference moon. The corresponding distance in the right half-field was adjusted by pressing a key.

As stated earlier, the apparent distance theory predicts that a smaller disparity would be needed to bisect the distance between the subject and the horizon reference moon than with the elevated reference moon. Theories based on the proposition that the perceived distance to the moon is determined by its relative perceived size predict the opposite result. This follows from the fact that the reference moon above the horizon is perceived as larger than the elevated reference moon.

The experiment involved five male subjects and was conducted on the Long Island hilltop. Subjects looking through the combining glass and fusing the virtual moons saw them side-by-side and slightly above a very distant horizon or in an empty sky. The experiments were all performed midmorning on nearly cloudless days. The horizon was composed of hazy hills many kilometers away, across Long Island Sound (which was itself hidden by several kilometers of intervening terrain and treetops). When the moons were elevated there were no nearby features or clouds. Although all of the moons were identical in angular size (0.6 deg), it was evident that the horizon moon was much larger than the elevated moon.

The mean relative disparities at which the variable moon bisected the distance between the subject and the reference moon are shown for each subject in Table 1A. Corresponding estimates of the standard errors (SE) are included. The basic data show that for all subjects the angular disparity of the variable moon relative to the reference moon was greater when the moons were elevated than when the moons were viewed above the ground plane On average the relative disparity was 3.4 times greater for the elevated moons. This means that the elevated reference moon was perceived as much closer than the horizon reference moon, and that the elevated variable moon was much closer to the subject when it was perceived as half the distance to the reference moon.

TABLE 1

A: Mean disparities of elevated (M_E) and horizon (M_H) variable and moons relative to reference moons in arcmin at half-distance for each of five subjects, and estimates of standard errors (SE).

B: Mean disparities of moon when adjudged to be one half its original size.

A: HALF-DISTANCE DATA

Subject	M_H	M_E	SHE	SE_E
WH	6.293	21.976	0.259	1.189
BB	16.877	26.649	1.564	1.242
FD	5.409	23.158	0.334	1.027
WY	10.071	48.817	0.534	1.948
LK	3.132	20.292	0.203	0.242

B: HALF-SIZE DATA

Subject	M_H	M_E	SHE	SE_E
WH	52.382	215.308	1.385	8.703
BB	69.599	73.209	1.611	2.012
FD	51.983	92.365	1.230	4.463
MS	33.931	39.964	2.953	1.501
LK	23.777	34.618	0.482	0.797

To estimate the perceived distances to the variable moons when subjects had bisected the space between themselves and the reference moon, the average interocular distance of 0.064 m was divided by the mean disparities in radians and averages computed across subjects. Despite a wide range of mean half-distance settings, all five subjects placed the horizon variable farther away than they did the elevated variable, with ratios ranging from 1.58 to 6.48. On average, subjects placed the variable horizon moon at the same distance as an object 36.17 m away from them. In the case of the variable elevated moon the corresponding distance was 8.62 m. Thus, the horizon moon was placed approximately 4.2 times farther away than the elevated variable moon. These impressive results confirm Hypothesis 1, which is based on apparent distance theory, and are inconsistent with Hypothesis 2, which is associated with theories holding that the moon's perceived angular size determines its perceived distance.

We know that distance information scales disparity among objects at different distances. Based on the data of Kaufman and Kaufman, it is evident that differences in *apparent* size (as opposed to physical size) do not scale differences in depth between the variable and reference moons. Hence, illusory differences in size do not act as distance cues. They scale neither distance nor depth. Although differences in angular size do convey the impression of differences in depth, the relative size cue may itself require scaling through interaction with other cues.

9. THE MOON'S SIZE AND ITS PERCEIVED DISTANCE

Kaufman and Kaufman (31) noted that increasing the absolute disparity of a moon results in two concomitant changes. First, the moon appears to grow smaller. Second, it also appears to come closer. This is easily understood in the case of the horizon moon, because as the disparity increased observers were able to see the moon at the same distances as objects that were closer to them. However, without benefit of objects in the intervening space, the elevated moon exhibited the same behavior. As it grew smaller, it seemed to come closer. These observations were made in connection with another experiment in which they sought to test the applicability of SDIH (Emmert's law) to the perceived size of the moon.

According to Emmert's law, the perceived size of the moon is proportional to its perceived distance. However, a simple proportional relationship seems unlikely. This experiment tested the hypothesis that a moon at the half-distance should be perceived as one-half the size of a reference moon of the same angular size.

One moon was presented to each eye. These were fused to form a single moon that initially was at the distance of the reference moon (zero disparity). Five subjects decreased the distance to the moon (by increasing the absolute disparity) until it appeared to be one half its original size. This was done for the moon over the horizon, and for the elevated moon viewed in an empty sky.

All five subjects described the moon as drawing closer as its size diminished. The results are shown in Table 1B which contains the mean absolute disparities at which each subject judged the moon to be one-half its original size. The disparity at which the moon was judged to be one-half its original size was far greater than the disparity of the variable moon at the half-distance of the first experiment. Overall, the average distance to the horizon moon when it was judged to be one half its original size was 5.47 m. The corresponding distance of the elevated moon was 3.65 m. Therefore, in this experiment Emmert's law did not determine the magnitude of the moon illusion. Hence, the classic form of the apparent distance theory is not confirmed.

These results are consistent with those reported by Enright (34) who found only an 8% reduction in the size of a moon that was moved from a distance of about 3 Km to 60 m. Our results demonstrate that this effect becomes progressively larger so that the moon's size is reduced by half at distances within about 10 m. Consequently, the size of the moon varied as some positive function of distance. By contrast, if the moon's size had been the predominant cue to its distance, then if it had been seen as growing smaller (although its angular size was constant) it should have appeared to move away, or if it had been perceived as moving closer, its perceived size should have increased.

While this experiment showed that perceived size is not proportional to perceived distance, the horizon moon was still 1.5 times farther away than the elevated moon when it was judged to be one-half its original size. If the elevated moon had initially been perceived as more distant than the horizon moon, then it is plausible to consider that it should have reached its half-size when it was farther away than the horizon moon. Therefore, the experiment confirmed the proposition that the perceptual system responds to the horizon moon as though it is more distant than the elevated moon.

The perceived half-distance to the elevated variable moon was approximately 8.6 m. This is more than twice the distance at which the same moon is judged to be half its original size. Similarly, the half-distance in the horizontal direction is about 36 m. This is about 6 times the distance at which the horizon moon is judged to be one half its original size. It is possible that the moon had to be close enough to begin to engage the convergence system before the linear relationship defined by Emmert's law becomes a reasonable approximation to the data. This is an important issue for future display research. Clearly, the presence of terrain well beyond the effective distance of convergence led to a more rapid reduction in perceived size of the horizon moon as compared to the elevated moon.

10. THEORETICAL CONSIDERATIONS

Humans perceive distance, depth and size, not the cues that are used by the organism to enable those perceptions. The term *cue* represents properties of retinal images. They include such features as blur circles and color fringes, disparity, and both static and time-varying geometrical features. These cues had to be discovered by scientists and artists, and are not directly perceptible.

Cues to distance have effects on channels within the visual system where they interact at different levels to determine both perceived size and perceived relative distance. However, when subjects are asked if a moon that appears to be large is farther or closer than a moon that appears to be small, their judgments are based on the perceived size difference. They use different criteria when attention is called to different attributes of the scene. Hence, when the moon is absent (along with its larger or smaller size) subjects have no difficulty at all in reporting that the horizon is more distant than the zenith sky. This is an observation that can be made in daylight by any person who cares to look. The effect is even more pronounced when the sky is covered or partly covered with textured clouds. But the flattened appearance of the sky does not cause the moon illusion. Rather, whatever causes the moon illusion also causes the sky to appear as flattened, which is why the flattening is not visible at night.

Observers attend differently to different aspects of their perceptions. For example, when viewing the famous Ponzo illusion (Fig. 4), subjects can perceive that the converging lines are printed on flat paper, but they are also capable of perceiving them as parallel lines receding into the distance. The alternatives are given by the sensory data. This is not a conscious inference, but since the horizontal bars are perceived as differing in size, we must accept the possibility that the depth perception conveyed by the convergence of the lines has a direct impact on the perceptual system. Even so, both possible perceptions are present in the display.

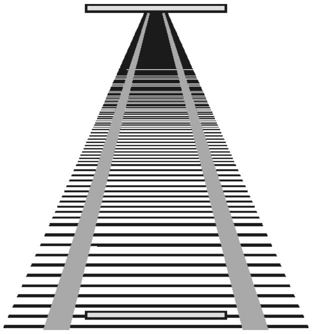


Figure 4. The Ponzo Illusion (see text).

At our latitude in New York the natural moon is never at an elevation higher than about 60 deg, and is most often much lower in the sky. Therefore, when viewing clouds or the elevated moon, on most occasions the terrain may be barely visible in peripheral vision. So some information regarding distance to the sky may be gleaned from peripheral vision and is certainly picked up via ordinary eye and head movements. Further, in natural circumstances the elevated moon may be only a few degrees above relatively nearby trees, tall buildings, and hills. In addition, the elevated moon could certainly be placed somewhat beyond the effective distance of convergence. This distance is approximately the same as the half-distance of our first experiment where the elevated moon was viewed in an empty sky. Strictly speaking, it isn't true that there are no cues to the distance of the elevated moon, although these cues are less salient than those associated with the terrain.

It has never been stressed that he the moon illusion is strongly dependent on cues related to the distance between the observer and external objects. We suggest that this is why the illusion is much weaker in simulations. For example, Rock and Kaufman (16) projected a stereogram of a simulated terrain, with a "moon" above its horizon, onto a wide screen relatively close to the observer. The horizon and moon were relatively distant, and the foreground was in the plane of the screen.

Subjects matched this moon to a slightly larger moon presented in on an empty screen at the same distance. The illusion was quite small. In this case disparity must have been scaled by the distance to the screen. Similarly, the illusion is also weak in ordinary pictures containing depth cues signifying that the moon is more distant than objects in the foreground (35). It seems obvious that the depth cues are scaled by the distance between the observer and the page. In the natural world the observer is part of the scene and cues are present to points covering a range of egocentric distances. This permits scaling of disparity and other depth cues extending out to much greater distances than is possible in a picture presented on a plane surface. This point should be emphasized for those interested in simulating visual scenes. To achieve veridical representations of depth, one must employ knowledge of how depth is scaled. Unfortunately, this knowledge is incomplete. So let us propose some fresh ideas.

The terrain leading up to the horizon moon presents a gradient of features at different distances. It is reasonable to propose that disparity among these features conveys information regarding the depth among them. This is also true of head movement parallax, where the relative angular velocities of elements of terrain vary inversely with their distance. Parallax provides information similar to that derived from binocular disparity. Depth given either by binocular disparity or by motion parallax must be calibrated by egocentric distance information. This information could be provided by convergence and, perhaps, by changes in angular size associated with locomotion (33, 34).

There is no terrain leading up to the elevated moon. Tree branches and rooftops may lie between the observer and the moon, but there is no continuum of depth intervals, as provided by the terrain. Hence, depth intervals separating points in space between the observer and the moon are ambiguous at best. Such considerations lead to the following hypotheses:

The threshold for discriminating differences in distance between points along the terrain is lower than that between points in the empty space surrounding the elevated moon (or of any isolated distant object, regardless of its elevation). Second, the threshold for discriminating objective (linear) depth between any pair of points on the terrain is lower than that between points in an empty field, even when stereoscopic cues are available. Third, assuming that size perception depends upon discrimination of differences in distance, the threshold for discriminating the size of the moon will be lower for the horizon moon than for the elevated moon. McKee and Welch (27) suggest how such experiments may be conducted, and we will take this up in detail below.

Given a more advanced version of the apparatus used by Kaufman and Kaufman, these hypotheses are quite easy to test out-of-doors in natural situations. Further, the hypotheses may be tested in different environments. For example, it is known that the moon illusion is weaker as the observer is moved to a high elevation. Is there an elevation of threshold for depth discrimination along the terrain under such circumstances? What is the effect of cloudiness on depth discrimination between the observer and a moon? Experiments based on these hypotheses may well change the nature of discourse about the moon illusion.

Rather than treat the problem as one in which differences in reported perceived distance determine the different perceived sizes of the moon, the problem is framed in terms of classical psychophysics. For example, as a spot of light moves toward a line, its apparent velocity will increase. This is predictable from Weber’s law. The apparent speed of the spot is increased by proximity to a visible feature. The distance between the spot and the feature becomes more evident as the space between them narrows. Similarly, Wallach and Lindauer (38) presented horizontal lines of slightly different length to each eye. However, when subjects viewed the lines in a stereoscope, most of them had great difficulty in seeing the line tilted in depth, despite the presence of the disparity. However, when two horizontal lines were presented to each eye, with the upper line shorter in the left eye and longer in the right, and the lower line longer for the left eye and shorter in the right, subjects had no difficulty at all in discerning two lines tilted in opposite directions in depth. The ability to discriminate differences in distance was powerfully enhanced by the presence of contrasting information. This information reduces uncertainty and lowers discrimination thresholds.

In summary, various investigators have employed the standard size constancy paradigm in which the moon is “matched” by a nearby disk. It is at least arguable that this is tantamount to using the method of magnitude estimation, as the nearby disk is not truly commensurate with the perceived size of the moon. Others have used optical displays in which the moon at one place on the sky is matched by a moon at another place on the sky. We refer to this as the *matching* method. The two methods have yielded disparate results. Now let us describe a different method in which we deal explicitly with the matter of discriminating distance and size, and not concern ourselves at all with subjective impressions of relative distance.

Using the apparatus described in the next section of this paper, we propose to employ the *single trial* method (26, 27). A single moon of constant angular size will be placed at one of seven or nine closely spaced distances (defined by their absolute binocular disparities or binocular parallaxes) from the observer. After several practice trials the observer will be asked if a particular moon is farther or closer than the moon placed at the average distance of the set of distances (disparities) presented in the block of trials. It is known that this task can be performed, and that it is possible to derive a psychometric function decribing how well depth is discriminated relative to the average distance of the set of disparities. It furnishes the difference threshold, which is the difference in distance between the median absolute disparity, and the point at which the distance has a 75% chance of being judged to be greater than the average distance. This distance threshold (the traditional JND) is related to the Weber ratio, and is an index of the uncertainty in the measurement. The larger the JND, the greater the variability or noise in discriminating depth in the vicinity of the average distance. This is to be done at each of several different absolute disparities, corresponding to distances ranging from a few meters to 100 meters or more. If, for example, Fechner's law holds (we do not expect it necessarily to hold), over long distances, the just noticeable increments in depth (disparity difference thresholds) will become larger as distance is increased. Perceived distance (deduced from discrimination data) could increase as the logarithm of physical distance. This implies that at far distances judgments become increasingly uncertain, and the ability to discriminate differences in distance less precise. If we imagine that this experiment is actually conducted, then it seems likely that at some point along the terrain subjects should become relatively insensitive to differences in distance. So, at some asymptote, the distance to the sky at the horizon will be indistinguishable from distance to some closer point along the terrain.

Now let us consider distance to an infinitely distant elevated moon in an empty sky. In our thought experiment it is almost obvious that at some relatively close distance subjects will no longer be able to discriminate the difference between the distance to a nearby "moon" and the infinitely distant moon. Uncertainty will increase much more quickly than when features of the terrain are visible. Therefore, points in space close to the observer will not be discriminable from points at the actual distance of the moon. This is entirely consistent with the finding of Kaufman and Kaufman that the elevated moon must get much closer to the observer to achieve the same degree of size loss as the horizon moon. It is also consistent with their half-distance experiment. The main reason for doing such an experiment is to establish that discrimination of differences in depth increases over a longer distance over the terrain than it does in empty space. Of course, the introduction of features such as clouds, texture that fills space but does not present distance cues, and other factors as well, have predictable results.

We should note that those who support the theory that perceived angular size determines perceived distance have actually framed an important question. We suggest an alternative, namely, size is meaningless without a prior determination of distance. These statements lend themselves to the formulation of alternative models that are amenable to test using discrimination methods (27).

If the computation of perceived angular size occurs prior to the computation of perceived distance, then the psychometric functions obtained in a size discrimination experiment will display less noise (their slopes will be steeper) than will the psychometric functions obtained from the depth discrimination data. Alternatively, if perceived distance is computed prior to perceived size, the reverse relation will hold.

The proposed experiment will be similar to the depth discrimination experiment, except that subjects will be asked to attend to the size of the moon. In actuality the moon's actual angular size will be kept constant. However, as we have already shown, the moon appears as larger at long distances than it does at relatively shorter distances. In this experiment distance will be controlled by disparity. A single moon will be presented at each of several different disparities. Subjects will be asked to indicate if a given moon is larger or smaller than the average moon in the set of stimuli they saw in a particular block of trials. After many trials a psychometric function will be computed to indicate the difference threshold (expressed in terms of angular disparity as well as distance) for differences in moon size. On other blocks of trials the set of disparities will be centered about mean values that are at different distances, ranging out to at least 100 m. A psychometric function will be computed for each of these sets. The data of this experiment together with that of the distance discrimination experiments will permit us to decide if size is computed first, or if distance is computed first.

Finally, let us revisit SDIH. If SDIH has validity, the ability to discriminate differences in the size of the moon presented at various distances should mirror the observed discriminability of differences in depth along the terrain, and also along a line projected into empty space to an elevated moon. The extent to which SDIH is applicable will also depend upon showing that the function relating perceived size to distance (as measured using the single trial discrimination method) varies with the same conditions that cause depth discrimination to vary. But to do such experiments we require a new apparatus.

11. AN ADVANCED STEREO DISPLAY

In order to properly measure the psychometric functions that describe the perceived depth response (and sensitivity) as a function of distance from an observer, we are now designing a new version of the augmented reality display used to study the moon illusion. The original 'Kaufman and Kaufman' apparatus was optimized for viewing artificial moons (at optical infinity). In order to study depth sensitivity as a function of distance it is essential to control several visual stimuli simultaneously. The new system is also a stereoscopic heads up display. It will include several new features to provide direct experimental control over accommodation, parallax, motion parallax (from head motion), image elevation, image size, and disparity.

11.1 Accommodation

The original moon illusion display contained a telescopic optical system with the plane of a tft-flat panel display source at the focus of the optical system. The resultant images were all at optical infinity. The new display will employ 5cm diameter 50cm FL lenses mounted on a translation stage controlled by a rotary stepper motor.

With a step size of 0.1mm, the stepper motor will be able to adjust the image distance at a rate of 1.0mm/second over a range of >5.0cm. This translates into a controllable image distance from 5 meters to infinity with a resolution of 1cm at 5 m (0.2%). Since the experiment is not image rate sensitive, it is a simple matter to avoid latency artifacts by presenting an image to a subject after the stepper motor has completed any translation of the optical stage. The subject will then be presented with consistent disparity and accommodative stimuli.

11.2 Binocular Parallax

As a stereoscopic heads up display, the subject can be presented with artificial stimuli superimposed optically on real scenes containing varying amounts of texture, and various cues to distance and depth. The disparity of elements on the terrain must be matched by that of virtual elements chosen as markers of points in the distance.

11.3 Motion Parallax

The original moon illusion experiment placed artificial moons at optical infinity. For artificial stimuli located closer to a subject, we plan to include the affect of motion parallax by tracking the head position and adjusting the pair of stereo images on the display accordingly. Obviously, any real objects viewed through the heads up augmented reality display will of themselves also introduce the affect of motion parallax. The challenge to presenting an accurate motion parallax effect for artificial stimuli is latency and disparity resolution. We discuss disparity resolution below. Latency will be limited by the frame rate of the monitor chosen. The Sony FD Trinitron CRT supports a resolution of 2048x1536 at a frequency of 75Hz.

11.4 Image elevation

The entire heads up display will rotate (manually) from a horizontal view to an inclination of >45 degrees. In the original apparatus the mirror was rotated to adjust the elevation of the moon.

11.5 Image Size

To create a stimulus 0.5 degrees in angular size will require an image size of 0.44 cm on the CRT or approximately 22 pixels in diameter. This image size will allow us to present the subjects with fairly complex stimuli.

11.6 Disparity

Using only the display Aperture Grille Pitch of 0.22mm, at 50cm focal distance our smallest change in disparity would be 2.52×10^{-3}degrees or 1.5 minutes of arc. In order to improve upon this resolution, we can select image stimuli to take advantage of sub-pixel motion techniques. By adjusting the relative intensity of adjacent pixels to visualize translations of less than one pixel in size, we can render translations of smaller than 1.5 minutes of arc. Using 5 levels of grayscale, a minimum disparity of 0.3 minutes of arc will allow us to study stimuli at a distance ½ Km (relative to stimuli at optical infinity). Ten levels of grayscale would allow us to compare stimuli at 11 Km distance with stimuli at optical infinity.

12. CONCLUSION

Just a few final words for those who are primarily interested in visual displays. We realize that the complexities and controversies alluded to in this paper may be confusing. However, there is a bottom line. Ultimately it will be shown that *some* information regarding egocentric distance to elements in visual displays is required to give users realistic impressions of size and distance. Disparity alone is insufficient. We suspect that this information is implicit in the fact that in real life observers are inside the scene, and that elements within the scene are at different optical distances from the observers. If we devise a virtual reality display containing all of the nominal cues to depth, it will not be anything like "real" so long as all of the information is in a single optical plane normal to the user's line-of-sight. Think of a display in which nearby objects in the foreground are actually nearby. This alone may permit the perceptual system to scale the cues that the designer so painstakingly provided for objects closer to the horizon of his virtual world.

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