Robust Optimization on E-Commerce Closed-Loop Supply Chain with Uncertain Environment

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Abstract

There are many uncertainty facts exists in e-commerce closed-loop supply chain. Linear optimization method is difficult to solve this complex network optimization model with uncertainty. This paper proposes a robust optimization model for handling the uncertainty of the demands,returns and transportation costs in a E-commerce closed-loop supply chain network design problem, which determined the best location of the logistics center and obtained the customer area distribution results by using example simulation; and the computational results show the robustness of the robust model in dealing with the disturbances of uncertainty parameters, which verified the feasibility and effectiveness of the model and method.

Keywords

E-commerce, closed-loopsupply chain, reverse logistics, uncertainty, robust optimization

1. Introduction

The closed-loop supply chain is delivered through the forward and reverse product recycling,which convert the open-loop process of "resources,production consumption and waste" into the closed-loop feedback cycle network of "resources,production,consumption and renewable resources",its essence is based on mesh chain in the process of integrating the forward/reverse supply chain^[1].

Xuet al.^[2]proposed a supply chain operating model which is constructed by using the robust linear programming method based on scenario analysis;Wang [3]developed a robust optimization model and algorithm for logistics center location and allocation under uncertain environment; Aryanezhad.et al.^[4]proposed a multi-objective nonlinear robust optimization model for multi-product multi-site aggregate production planning in a supply chain under uncertainty of cost parameters and demand. Dong et al.^[5]introduced the robust optimization modeland the differential honey badger algorithm solving the closed loop of fresh food in supply chainnetwork design problems efficiently; Zhang et al.^[6]proposed that the problem of emergency materialallocation under uncertain demand, comprehensively considering the matching degree of demandand demand time, an interval robust optimization model with the goal of maximizing the mean valueof comprehensive matching degree is constructed, and an improved adaptive genetic algorithmbased on random sampling is designed.

This paper proposes a robust optimization model for handling the uncertainty of the demands,returns and transportation costs, which determined the best location of the logistics center and obtained the customer area distribution results by using example simulation; and the computational results show the robustness of the robust model in dealing with the disturbances of uncertainty parameters, which verified the feasibility and effectiveness of the model and method.

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2. Robust Model with Uncertainties

2.1. Assumptions

To specify the study scope,four assumptions and simplification are postulated as follows.

- a. Customer demand for a single type of product.
b. The first level logistics center without inventor
- b. The first level logistics center without inventory capacity constraints.

c. The first level logistics center has adequate product inventory.

d. The first level logistics center can repair all the returns.
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- d. The first level logistics center can repair all the returns.

2.2. Model formulation

Sets and indices: R set of customer zones, $r \in R$; H set of the second level logistics centers, $h \in H$; M set of the first level logistics centers, $m \in M$; T set of products suppliers, $t \in T$. **Table 1**

Table 2

2.3. Robust model

E-commerce products are usually returned by the city station which is responsible for collecting and then back to second level logistics centers(H),and then shipped to first level logistics centers(M) for returns processing.At the same time,the demand caused by the returns is typically satisfied through a forward channel, the returned products are classified by the H and then shipped to the M $^{[7]}$.

The amount of returns of customer zones in the city stations(R) depend on the product return rates,the capacity of each H represent the overall capacity which can stacking new products and returns.We utilize product-specific coefficients as description for new product unit inventory capacity and returns unit inventory capacity.

E-commerce closed-loop supply chain network is a two levels' inventory system which hold appropriate product inventory for the M and the H.Firstly,it should be satisfied by the H when customers have a new product demand and returns,which can transverse scheduling the other same level centers when the H out of stock,and the demand should be transported urgently to the customers by M when all H is out of stock.

Figure1:A two levels' inventory system

 \rightarrow \rightarrow Note:forwardflowreverseflowtransversescheduling

The city station should back the returns to H,the returned products are classified by H and then shipped to M for final processing.

Based on Ben-Tal et al. [8], the uncertain robust linear optimization theory as follows:

In the Constraints(1) of theobjective function,the first two terms represent the fixed costs for *m* and *h* ,the following three terms represent the total cost of transportation and distribution between facilities point,the sixth term represents the total collection processing cost of returns at location *h* ,and the following one represents the total transportation from location *h* to location *m* and the total repairing cost of returns at location *m* ,and the remaining two terms represent the transverse scheduling transportation cost between location *h* and emergency transportation cost at location *m* .

$$
\sum_{m \in M} k_m F_m / V_m + \sum_{h \in H} b_n F_h / V_h + \sum_{t \in T} \sum_{m \in M} (\overline{G}_{tm} d_{tm} q_{tm} + \eta_{tm}^a) \n+ \sum_{m \in M} \sum_{h \in H} [(\overline{G}_{mh} d_{mh} q_{mh} + \eta_{mh}^b) + S_m q_{mh}] g_{mh} \n+ \sum_{h \in H} \sum_{r \in R} [(\overline{G}_{hr} d_{hr} q_{hr} + \eta_{hr}^c) + S_h q_{hr}] u_{hr} \n+ \sum_{r \in R} \sum_{h \in H} [(\overline{G}_{rh} d_{rh} q_{rh} + \eta_{rh}^c) + t_h q_{rh}] u_{rh} \n+ \sum_{h \in H} \sum_{m \in M} [(\overline{G}_{hm} d_{hm} q_{hm} + \eta_{hm}^c) + x_m q_{hm}] g_{hm} \n+ (\sum_{r \in R} (D_r u_{hr} + p_r u_{rh}) - \theta_h) f_h d_{hh} \tau_h \n+ (\sum_{r \in R} (D_r u_{hr} + p_r u_{rh}) - \sum_{h \in H} \theta_h) g_m d_{mr} I_{mr} g_{mh} \leq z
$$

Constraints(2) involves uncertainty related to transportation cost;

$$
-\eta_{tm}^a \leq \rho_a G_{tm}^a q_{tm} \leq \eta_{tm}^a, \forall t, m.
$$

\n
$$
-\eta_{mh}^b \leq \rho_b G_{mh}^b q_{mh} \leq \eta_{mh}^b, \forall m, h.
$$

\n
$$
-\eta_{hr}^c \leq \rho_c G_{hr}^c q_{hr} \leq \eta_{hr}^c, \forall h, r.
$$

\n
$$
-\eta_{rh}^d \leq \rho_d G_{rh}^d q_{rh} \leq \eta_{rh}^d, \forall r, h.
$$

\n
$$
-\eta_{tm}^e \leq \rho_e G_{mh}^e q_{mh} \leq \eta_{hm}^e, \forall h, m.
$$

Constraint set(3) assure that the location *m* has adequate product inventory;

$$
\sum_{t \in T} q_{tm} - \sum_{h \in H} q_{mh} g_{mh} \ge 0, \quad \forall m \in M \qquad (3)
$$

Constraints (4)-(7) involve material flow between facilities.

$$
\sum_{m \in M} q_{mh} g_{mh} - \sum_{r \in R} q_{hr} u_{hr} = 0, \quad \forall h \in H(4)
$$

$$
\sum_{h\in H} q_{hr} \ge \overline{D}_r + \rho_\nu G_r^v, \quad \forall r \in R,\tag{5}
$$

$$
\sum_{h \in H} q_{rh} \le \overline{p}_r + \rho_w G_r^w, \quad \forall r \in R,
$$
\n
$$
\sum_{r \in R} q_{rh} u_{rh} - \sum_{m \in M} q_{hm} g_{hm} = 0, \quad \forall h \in H \quad (7)
$$

Constraints(8)and (9) enforce the binary on corresponding decision variables.

$$
\tau_h = \begin{cases} 1, & \theta_h < \sum_{r \in R} (D_r u_{hr} + p_r u_{rh}), \quad \forall h \in H \\ 0, & \text{else.} \end{cases} \tag{8}
$$

$$
l_{mr} = \begin{cases} 1, & \sum_{h \in \mathcal{H}} \theta_h < \sum_{r \in \mathcal{R}} (D_r u_{hr} + p_r u_{rh}), & \forall r \in \mathcal{R} \\ 0, & \text{else.} \end{cases} \tag{9}
$$

Constraints(10) and(11) ensure that a customer zone r is assigned to exactly one h for service and a second level logistics center *h* is assigned to exactly one *m* for service respectively.

$$
\sum_{h \in \mathcal{H}} u_{hr} = \sum_{h \in \mathcal{H}} u_{rh} = 1, \quad \forall r \in \mathcal{R} (10)
$$

$$
g_{mh} = \sum g_{hm} = 1, \quad \forall h \in \mathcal{H} (11)
$$

 $\sum_{m\in M} g_{_{mh}} = \sum_{m\in M} g_{_{hm}} = 1, \ \ \ \forall h \in$ $\sum_{m \in M} g_{m h} = \sum_{m \in M} g_{h m} = 1, \quad \forall h \in H$ (11) Constraint(12) represents that it will have relevant product flow only when the facility is selected,

 $r \in R$ $m \in$

w is a very large number.

$$
q_{t_m} \leq \mathbf{w} k_m, q_{m h} \leq \mathbf{w} k_m, q_{h r} \leq \theta_h b_h, q_{r h} \leq \theta_h b_h, (12)
$$

$$
q_{h_m} \leq \theta_h b_h, \ \forall m \in M, \forall h \in H
$$

Constraints (13) and (14) represent the maximum capacity limit.

$$
\sum_{m\in M} q_{mh} g_{mh} - \theta_h b_h \le 0, \quad \forall h \in H \qquad (13) \qquad \sum_{r\in R} \gamma D_r u_{hr} + \sum_{r\in R} \beta p_r u_{rh} \le Q_h b_h, \quad \forall h \in H \qquad (14)
$$

Constraint(15) and (16) limit the range of the variables.

riables.
\n
$$
g_{mh}, g_{hm}, k_m, b_h, u_h, u_m \in \{0,1\}, \quad \forall r \in R,
$$
\n
$$
h \in H, m \in M.
$$
\n(15)

$$
\eta_{lm}^a, \eta_{mh}^b, \eta_{hr}^c, \eta_{rh}^d, \eta_{hm}^e \ge 0, \quad \forall t, m, h, r. \tag{16}
$$

Then,the cost minimization model as follows:

min z
s.t.
$$
equ. (1) - (16)
$$
 (17)

3. Computational Experiments

The basic data of example come from vehicle routing problem in the database^[8], both the deterministic and robust models are solved by Lingo optimization software and only considers the single-cycle closed-loop supply chain problem with single-product. Assuming that the supply chain network consist of two product suppliers, two alternative first level logistics centers, six alternative second level logistics centers and 14 city stations customer zone. The coordinate of product suppliers shown as in Table 3;the basic data of first level logistics centers as shown in Table 4;Table5 and 6 report the basic data of second level logistics centers and city stations customer zone; the nominal value of unit

transportation between facilities are $G_{tm} = G_{mh} = G_{r} = G_{hm} = 0.051$.

Table 3

Coordinate Of Product Suppliers

Table 4

Basic Data Of First Level Logistics Centers

Table 5

Basic Data Of Second Level Logistics Centers s_h = 0.8, t_h = 1, s_h = 0.8, θ_h = 1000, θ_h = 1600, γ= 1, β= 2

h	Coordinate	F_h
1	(15, 19)	80000
\mathcal{P}	(19,75)	100000
3	(31, 87)	130000
4	(71, 41)	100000
5	(61, 83)	110000
հ	(59,51)	90000

In addition, $\delta_r = 0.3$,unit transverse scheduling transportation cost $f_h = 0.4$,unit emergency transportation $\cos t_{g_m} = 0.8$, the above experiment is solved by Lingo optimization software.

To assess the performance of robust optimization model,the experiments are performed under three different uncertainty levels (i.e., $\rho = 0.3, 0.6, 1$),the uncertainty levels of the model are assumed to be equal to (i.e., $\rho_a = \rho_b = \rho_c = \rho_d = \rho_e = \rho_v = \rho_w$) to analyze the impact of the objective function value, and five random experiments are generated in the uncertainty set (i.e., $[$ nominal value – ρG^* , nominal value + ρG^*) on the each corresponding uncertainty level to analyze the performance of robust optimization model,and we use the circumstance of facilities construction and the standard deviation of objective function values of the robust optimization model to assess the performance of robust optimization model.

The results under the uncertainty of transportation costs,demand and returns as shown in Table 7; the standard deviation contrast of objective function values in robust optimization model as shown in Table 8.

Table 7

Experiments results

Table 8

Standard deviation of objective function values in robust optimization model

Uncertainty level ρ	The standard deviation of objective value in robust optimization model
0.3	137
0.6	518
	631

The results showed that the robust model opened the second $I\!I\!I$ facility spot and the second, fourth, sixth h facility spot under different uncertainty level, it can be seen that robust optimization model has good stability in dealing with uncertainty circumstances, and the fluctuation of objective function value is small, and the objective function value in robust model has a small standard deviation, which can handle the disturbances of uncertainty better and more conductive to the overall stability of the system.

Figure 2 illustrated the wave phenomena of objective function value under the uncertainty level $\rho = 0.6$, it can be seen that robust optimization model has better stability in dealing with uncertainty circumstances,and the fluctuation of objective function value is more small than deterministic model,and the objective function value in robust model has a lower standard deviation,which can handle the disturbances of uncertainty better and more conductive to the overall stability of the system.

Figure 2:Fluctuations of the objective function value

4. Conclusions

Since there are many uncertainty in e-commerce supply chain network design problems, this paper based on the recent robust optimization theory proposed an e-commerce closed-loop supply chain robust optimization model under the uncertainty of transportation costs, demand and returns. Robust optimization model was designed based on the deterministic model in order to handle the disturbances of uncertainty parameters of the system, five random experiments are generated under each different uncertainty levels, and computational results show the superiority of the robust model in dealing with the disturbances of uncertainty parameters, which also has better robustness. This paper's work mainly proposed a e-commerce closed-loop supply chain robust optimization model based on the recent robust optimization theory and applied to the uncertainty supply chain network. However, this paper only considers a single-product, single-objective problem, and the form of uncertainty set has limitations, thus the direction of future research work focused on multi-product set, multi-objective and other forms of uncertainty set robust optimization problems.

5. References

- [1] Jr V Guide, R Daniel,V Jayaraman,J D Linton, Building contingency planning for closed-loop supply chains with product recovery, Journal of Operations Management, $2(2)$, (2003) : 35-37, ,.
- [2] J W Xu, X Y Huang, H F Guo, Robust optimizations for operating of closed-loop supply chain with uncertain demands, Systems Engineering and Electronics,30(2),(2008):283-287.
- [3] B H Wang, S W He, Robust Optimization Model and Algorithm for Logistics Center Location and Allocation under Uncertain Environment, Journal of Transportation Systems Engineering and Information Technology,9(2),(2009):69-74,.
- [4] S.M.J. Mirzapour Al-e-hashem, H. Malekly, M.B. Aryanezhad, A Multi-objective Robust Optimization Model for Multi-product Multi-site Aggregate Production Planning In A Supply Chain Under Uncertainty, Int.J.Production Economics, 34(1),(2011):28-42,.
- [5] Dong H, Lin G D,Robust optimization design of fresh closed-loop supply chain network based on, Application Research of Computers, Vol.39, (2022). doi:10.19734/j.issn.1001-3695.2022.03.0097
- [6] zhang X, Zhang H Y, Yuan X M, interval robust optimization of emergency materials allocation based on matching degree under uncertain demand,Systems Engineering,(2022).doi:https://kns.cnki.net/kcms/detail/43.1115.n.20220505.1336.002.html
- [7] Gopalakrishan Easwaran, Halit Uster, A closed_loop supply chain network design problem with integrated forward and reverse channel decisions, IEEE Transactions,5(42)(2010):779-792.
- [8] Augerat.CVRP Instances [DB/OL].(2009).doi[:http://osiris.tuwien.ac.at/~wgarn/vehicler](http://osiris.tuwien.ac.at/~wgarn/vehicle)outing/neo/problem%20Instances/ CVRPinstances.html.