On the Throughput of a Relay-Assisted Cognitive Radio MIMO Channel with Space Alignment

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***Abstract*—We study the achievable rate of a multiple antenna relay-assisted cognitive radio system where a secondary user (SU) aims to communicate instantaneously with the primary user (PU). A special linear precoding scheme is proposed to enable the SU to take advantage of the primary eigenmodes. The used eigenmodes are subject to an interference constraint fixed beforehand by the primary transmitter. Due to the absence of a direct link, both users exploit an amplify-and-forward relay to accomplish their transmissions to a common receiver. After decoding the PU signal, the receiver employs a successive interference cancellation (SIC) to estimate the secondary message. We derive the optimal power allocation that maximizes the achievable rate of the SU respecting interference, peak and relay power constraints. Furthermore, we analyze the SIC detection accuracy on the PU throughput. Numerical results highlight the cognitive rate gain achieved by our proposed scheme without harming the primary rate. In addition, we show that the relay has an important role in increasing or decreasing PU and SU rates especially when varying its power and/or its amplifying gain.**

***Index Terms*—MIMO space alignment , underlay cognitive radio, successive interference cancellation, amplify-and-forward relay.**

I. INTRODUCTION

The concept of cognitive radio (CR) was presented as a solution to overcome the spectrum inefficient allocation [1]. In this concept, cognitive/secondary users (SU) share the spectrum of licensed/primary users (PU) without affecting the primary communication [2]. During the last years, many sophisticated techniques have been presented to enhance point-to-point as well as cognitive communications, e.g., relaying and multiple-input multiple-output (MIMO).

Relaying consists of inserting additional nodes in the network that retransmit the received signal to the destination in order to enhance reliability and reduce the communication cost in terms of power [3]–[6]. Relaying is very efficient in cell edge cases in which the source transmission requires high power that may lead to reduced rate due to the remoteness from the destination. In particular, in CR communications, when the destination is far, a high power transmission from the cognitive user may affect the primary communication. In some cases, there is no direct link between the source and the destination. Thus, a relay is necessary to guarantee reliable communications. There are mainly three relaying techniques: i) Amplify-and-Forward (AF) [7] in which the relay amplifies the received signal before broadcasting it to the destination. ii) Decode-and-Forward (DF) [8] where the relay decodes the signal and then re-encodes it before retransmission. iii) Compress-and-Forward (CF) [3] in which the relay compresses the received signal and forwards an estimation of it. On the other hand, MIMO communications are based on adopting multiple antennas at the receiver and at the transmitter in order to increase the throughput/reliability by exploiting the spatial multiplexing/diversity [9], [10]. The fact of spearheading the power over multiple antennas enhances remarkably the spectral efficiency even with only two antennas [11]. The MIMO relaying capacity was deeply studied in [5], [6]. Adopting MIMO power allocation within a CR framework has been studied previously in e.g., [12]–[15]. In [12], MIMO space alignment was adopted but without relaying.

In this paper, we investigate the combination of both relaying and MIMO techniques in the CR network communication. Our objective is to examine the maximum achievable rate of the cognitive user as well as the effect of relay parameters on both primary and cognitive rates. This study is motivated by the fact that the cognitive user may share the relay of the primary user in addition to the spectrum. Hence, the corresponding secondary gain and the effect on the primary user need to be analyzed. The SU maximizes its rate, by allocating optimally its power among its antennas depending on the communication environment including the primary communication activity. In our setting, after a special precoding at the PU transmitter, some free eigenmodes are unused and thus can be freely exploited by the SU. Nevertheless, the SU transmits also

The work of M.-S. Alouini was made possible by NPRP grant #5-250-2-087 from the Qatar National Research Fund (a member of Qatar Foundation). The statements made herein are solely the responsibility of the authors.

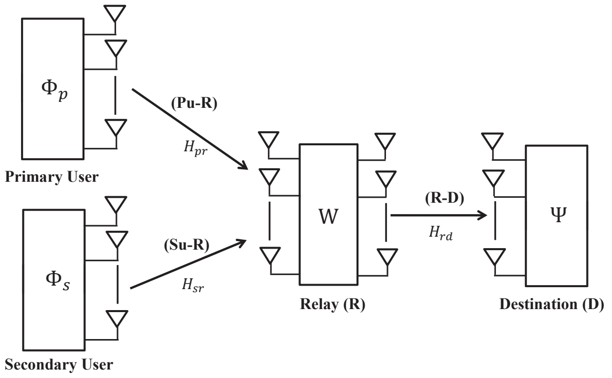


Figure 1: An uplink spectrum sharing communication in presence of a relay.

through the used eigenmodes but respecting an interference constraint tolerated by the PU. That is, the secondary signal is sent on both the free and the non-free eigenmodes. Then, the whole signal is amplified and retransmitted to the destination where the primary signal is decoded first as it is expected to be the strongest one since the SU signal is always limited by the interference threshold forced by the PU. We adopt a Successive Interference Cancellation (SIC) decoder [16] in order to decode the PU and the SU signals. We also study the accuracy of the SIC decoder on the cognitive power allocation.

The rest of this paper is organized as follows. In Section II, the system model is presented. Section III describes the precoding and decoding strategies. SU achievable rate expressions are derived for various SIC accuracies in Section IV. Numerical results are presented in Section V. Finally, the paper is concluded in Section VI.

II. SYSTEM MODEL

We consider the uplink communication scenario depicted by Fig.1 where "PU" and "SU" are interested in transmitting their signals simultaneously to a common destination "D". We assume that there is no direct link between the transmitters and the common receiver. A relay "R" is introduced to ensure the communication between the terminals by amplifying the received signal and forwarding it to the destination D. PU, as a licensed user, exploits the channel while the SU, as an unlicensed node, is allowed to share opportunistically the spectrum and to access the channel under some constraints to maintain a certain Quality of Service (QoS) of the primary communication. Each node is equipped with N antennas, and the channel gain matrices representing the links between the PU and R (PU-R), between SU and R (SU-R), and between R and D (R-D) are denoted by $\boldsymbol{H_{pr}}$, $\boldsymbol{H_{sr}}$, and $\boldsymbol{H_{rd}}$, respectively. The transmission between the transmitters and the common receiver takes place during two time slots. In the first time slot, PU and SU terminals transmit simultaneously their signal to the relay where the complex received vector is given by:

$$\boldsymbol{y_R} = \boldsymbol{H_{pr}}\boldsymbol{\Phi_p}\boldsymbol{s_p} + \boldsymbol{H_{sr}}\boldsymbol{\Phi_s}\boldsymbol{s_s} + \boldsymbol{z_R}, \tag{1}$$

where $\boldsymbol{H_{pr}}$ and $\boldsymbol{H_{sr}}$ are assumed to be independent, $\boldsymbol{\Phi_p}$ and $\boldsymbol{\Phi_s}$ are the linear precoding matrices applied at the PU and SU, and $\boldsymbol{s_p}$ and $\boldsymbol{s_s}$ are independent and identically distributed (i.i.d) complex Gaussian signals transmitted by PU and SU, respectively. For $i \in \{p, s\}$, we consider $\boldsymbol{P_i} = \mathbb{E}[\boldsymbol{s_i}\boldsymbol{s_i}^h]$ to be the covariance matrix of the vector $\boldsymbol{s_i}$, where $\mathbb{E}[\cdot]$ is the conditional expectation over all channel realizations and $.^h$ designates the transpose conjugate operator. This covariance matrix is subject to a power constraint $Tr\ (\boldsymbol{\Phi_i}\boldsymbol{P_i}\boldsymbol{\Phi_i}^h) \leq P_{tot}$ where $Tr\ (\boldsymbol{A}) = \sum_j A(j,j)$ is the trace of the matrix $\boldsymbol{A}$, and P_{tot} is the total power budget considered, without loss of generality, to be the same for both users. Finally, $\boldsymbol{z_R}$ indicates a zero mean additive white Gaussian noise (AWGN) vector at the relay with an identity covariance matrix, $\boldsymbol{I_N}$. During the second time slot, the relay amplifies the signal $\boldsymbol{y_R}$ through a gain matrix denoted $\boldsymbol{W}$. Then, it retransmits the signal to the common destination D. The received signal $\boldsymbol{y_D}$ at the receiver D, is expressed as follows

$$\boldsymbol{y_D} = \boldsymbol{H_{pd}}\boldsymbol{\Phi_p}\boldsymbol{s_p} + \boldsymbol{H_{sd}}\boldsymbol{\Phi_s}\boldsymbol{s_s} + \boldsymbol{z}, \tag{2}$$

where $\boldsymbol{H_{pd}} = \boldsymbol{H_{rd}}\boldsymbol{W}\boldsymbol{H_{pr}}$, $\boldsymbol{H_{sd}} = \boldsymbol{H_{rd}}\boldsymbol{W}\boldsymbol{H_{sr}}$ and $\boldsymbol{z} = \boldsymbol{H_{rd}}\boldsymbol{W}\boldsymbol{z_R} + \boldsymbol{z_D}$, where $\boldsymbol{z_D}$ is a AWGN vector at the destination D with an identity covariance matrix, $\boldsymbol{I_N}$. Note that the covariance matrix of the equivalent noise $\boldsymbol{z}$, $\boldsymbol{Q_z}$, can be written as follows:

$$\boldsymbol{Q_z} = \mathbb{E}[\boldsymbol{z}\boldsymbol{z}^h] = \boldsymbol{I_N} + \boldsymbol{H_{rd}}\boldsymbol{W}\boldsymbol{W}^h\boldsymbol{H_{rd}}^h. \tag{3}$$

In our framework, we assume that full Channel State Information (CSI) is available at the receiver and at the transmitters (i.e., PU-R, SU-R and R-D channel gains). Since the receiver at destination is common to both transmitters, PU and SU signals are subject to a mutual interference that may cause a significant deterioration to both primary and secondary performances. Therefore, in order to protect the licensed PU, we adopt an interference constraint [17] imposed by PU to force the SU transmission to be below a certain threshold per receive antenna. Let us denote such an interference threshold I_{th}.

III. SPACE ALIGNMENT PRECODING WITH INTERFERENCE TEMPERATURE THRESHOLD

This section introduces the proposed linear precoding and decoding matrices used to maximize the SU rate while respecting the PU's QoS. At the same time, the proposed scheme is also employed to exploit the space alignment technique, presented in [13], which permits to the SU to transmit through the unused primary eigenmodes. In fact, by having a perfect

CSI of the PU-R and R-D links at the PU transmitter in addition to the knowledge of the fixed relay amplification matrix gain $\boldsymbol{W}$, the PU can optimally allocate its power in order to maximize its achievable rate. By applying a Singular Value Decomposition (SVD) to $\boldsymbol{H_{pd}}$, the PU transmits through parallel channels characterized by their associated eigenmodes. The SVD of the matrix is denoted $\boldsymbol{H_{pd}} = \boldsymbol{U\Lambda V}^h$ where $\boldsymbol{U}$ and $\boldsymbol{V}$ are two unitary matrices and $\boldsymbol{\Lambda}$ is a diagonal matrix that contains the ordered singular values of $\boldsymbol{H_{pd}}$ denoted as $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_N$. To transform the PU MIMO relay channel to N parallel channels, we employ the linear precoding $\boldsymbol{\Phi_p}$ at the PU node and the decoding $\boldsymbol{\Psi}$ at the destination, respectively, as follows:

$$\boldsymbol{\Phi_p} = \boldsymbol{V} \text{ and } \boldsymbol{\Psi} = \boldsymbol{U}. \tag{4}$$

Thus, the output received signal after decoding becomes:

$$\boldsymbol{r} = \boldsymbol{\Psi}^h \boldsymbol{y_D} = \boldsymbol{\Lambda s_p} + \boldsymbol{U}^h \boldsymbol{H_{sd}} \boldsymbol{\Phi_s s_s} + \tilde{\boldsymbol{z}}, \tag{5}$$

where $\tilde{\boldsymbol{z}} = \boldsymbol{U}^h \boldsymbol{z}$ remains a zero mean AWGN with a covariance matrix $\boldsymbol{Q_{\tilde{z}}}$ given as follows:

$$\boldsymbol{Q_{\tilde{z}}} = \boldsymbol{I_N} + \boldsymbol{U}^h \boldsymbol{H_{rd}} \boldsymbol{W} \boldsymbol{W}^h \boldsymbol{H_{rd}}^h \boldsymbol{U}. \tag{6}$$

In order to maximize its rate, the licensed user PU forces the interference per receive antenna, $j = 1, \ldots, N$, caused by the vector $\boldsymbol{s} = \boldsymbol{U}^h \boldsymbol{H_{sd}} \boldsymbol{\Phi_s s_s}$ to not exceed a fixed I_{th}, i.e., the covariance matrix of $\boldsymbol{s}$ noted $\boldsymbol{Q_s}$ satisfies the condition: $Q_s(j,j) \leq I_{th}$ for the j^{th} antenna. Therefore, the PU considers this eventual interference as a noise and aims to maximize its achievable rate R_p. Note that this rate is considered to be the worst case scenario or a lower bound of the primary rate as the interference threshold, I_{th}, may not be reached by the SU. Consequently, the PU "real" achieved rate is greater or equal to this lower bound and is mainly derived by considering the actual SU interference instead of I_{th}. Hence, the optimal PU power and the rate lower bound are derived by solving the following optimization problem:

$$\underset{\boldsymbol{P_p}}{\text{maximize}} \;\; R_p = \sum_{j=1}^{N} \log_2 \left(1 + \frac{P_p(j,j)\lambda_j^2}{I_{th} + Q_{\tilde{z}}(j,j)} \right) \tag{7}$$

$$\text{s.t. } Tr\ (\boldsymbol{P_p}) \leq P_{tot}, \tag{8}$$

$$Tr\left(\boldsymbol{H_p P_p H_p}^h + I_{th} \boldsymbol{H_s H_s}^h + \boldsymbol{W W}^h \right) \leq P_R, \tag{9}$$

where $\boldsymbol{H_p} = \boldsymbol{W H_{pr} \Phi_p}$ and $\boldsymbol{H_s} = \boldsymbol{W H_{sr} \Phi_s}$. The constraint (9) indicates that the amplified signal power at the relay $\boldsymbol{W y_R}$ has to respect the total relay's power budget P_R. This optimization problem is convex as the objective function (7) is convex and the constraints are linear [18]. Hence, we apply the Lagrangian method to solve this problem. We first compute the Lagrangian function and then find its derivative with regards to each $P_p(j,j)$. The optimal power is given such as the derivative is equal to zero and is given,$\forall j = 1, \ldots, N$, by:

$$P_p^*(j,j) = \left[\frac{1}{\mu_p + \eta_p \sum_{i=1}^{N} |H_p(j,i)|^2} - \frac{I_{th} + Q_{\tilde{z}}(j,j)}{{\lambda_j}^2} \right]^+, \tag{10}$$

where $[.]^+ = \max(0,.)$. μ_p and η_p are the Lagrangian multipliers corresponding to the primary total power constraint and the relay total power constraint expressed in (8) and (9), respectively. From (10), when the channel gain is poor, i.e, λ_j's have small values, we note that the number of used eigenmodes by PU can be less than the total number of antennas N. This case occurs when the optimal power allocated to the j^{th} antenna is zero (i.e., $P_p^*(j,j) = 0$). Consequently, the SU can freely exploit the unused eigenmodes. We denote by n ($0 \leq n < N$) the number of unused eigenmodes. Then, we distinguish two sets of eigenmodes: $N-n$ eigenmodes used by the PU and n unused eigenmodes that can be freely exploited by the SU. In order to allow the SU to transmit in all the eigenmodes by respecting a certain interference temperature threshold I_{th} when sharing the used eigenmodes, we choose $\boldsymbol{\Phi_s}$ as follows:

$$\boldsymbol{\Phi_s} = (\boldsymbol{H_{sd}})^{-1} \boldsymbol{U}. \tag{11}$$

without loss of generality we assume that $\boldsymbol{H_{sd}}$ is invertible otherwise $(\boldsymbol{H_{sd}})^{-1}$ can be taken as the pseudo-inverse of $\boldsymbol{H_{sd}}$. Note that, since the SU knows the PU CSI, (i.e., $\boldsymbol{H_{pr}}$ and $\boldsymbol{H_{rd}}$), the unitary matrix $\boldsymbol{U}$ can be computed at the SU transmitter. We assume here that there is a feedback through which the receiver can broadcast this information to the cognitive user. This is not a very benign assumption as feedback CSI is adopted in most wireless communication protocols. Consequently, the received signal is expressed as

$$\begin{aligned} r_{Dj} &= \lambda_j s_{pj} + s_{sj} + \tilde{z}_j,\ \forall j = 1, \ldots, N-n, \\ r_{Dj} &= s_{sj} + \tilde{z}_j,\ \forall j = N-n+1, \ldots, N. \end{aligned} \tag{12}$$

Typically, the SU signal is always constrained by the interference threshold forced by the PU. Thus, in order to decode the SU signal, we propose to employ a SIC in order to remove the effect of the (strongest) signal, $\boldsymbol{s_p}$ from the received signal. Note that the SU signal, transmitted over the n free eigenmodes, is only constrained by the total power constraints at the SU terminal and the relay. Note that (10) is similar to a waterfilling power allocation solution [19]. Note that the actual PU rate is given by

$$R_p^{\text{real}} = \sum_{j=1}^{N} \log_2 \left(1 + \frac{P_p^*(j,j)\lambda_j^2}{P_s^*(j,j) + Q_{\tilde{z}}(j,j)} \right), \tag{13}$$

where $\boldsymbol{P_s^*}$ is SU the optimal power.

IV. SECONDARY USER ACHIEVABLE RATE

In this section, we investigate the achievable rate of SU using the proposed strategy described in Section III depending on the SIC performance. First, we derive the SU optimal power allocation assuming a perfect SIC (a sort of genie SIC). Then, we investigate the gain in performance with an imperfect SIC (i.e., totally erroneous SIC). We introduce a parameter α ($0 \leq \alpha \leq 1$) that corresponds to the probability of detecting the PU signal $\boldsymbol{s_p}$ correctly before applying the SIC.

A. Perfect SIC

In this case, we assume that the PU signal is always decoded perfectly, i.e., $\hat{s}_{p_j} = s_{p_j}, \forall j = 1, \ldots, N-n$, where $\hat{s}_{p_j}$ is the estimated PU signal at the jth receive antenna. Hence, the PU effect cancellation is performed correctly ($\alpha = 1$) and, in this case, the output received signal after the SIC decoding, $\tilde{\boldsymbol{r}}$, is written as

$$\tilde{\boldsymbol{r}} = \boldsymbol{r} - \boldsymbol{\Lambda}\hat{\boldsymbol{s}}_{\boldsymbol{p}} = \boldsymbol{s_s} + \tilde{\boldsymbol{z}}. \tag{14}$$

Notice that the SU throughput is independent of the secondary channel gain. Indeed, the proposed precoding scheme described in (11) converts the secondary channel to a unitary channel. Consequently, the maximum achievable rate ${R_s}^{(\alpha=1)}$ is obtained after solving the following optimization problem:

$$\max_{\boldsymbol{P_s}} {R_s}^{(1)} = \sum_{j=1}^{N} \log_2\left(1 + \frac{P_s(j,j)}{Q_{\tilde{z}}(j,j)}\right) \tag{15}$$

$$\text{s.t. } Tr(\boldsymbol{\Phi_s P_s {\Phi_s}}^h) \leq P_{tot}, \tag{16}$$

$$Tr\left(\boldsymbol{H_p P_p^* {H_p}}^h + \boldsymbol{H_s P_s {H_s}}^h + \boldsymbol{W W}^h\right) \leq P_R, \tag{17}$$

$$P_s(j,j) \leq I_{th}, \forall j = 1, \ldots, N-n, \tag{18}$$

where $\boldsymbol{P_p^*}$ is the optimal PU power obtained after solving the optimization problem given in (7)-(9). This problem is also convex as the objective function is convex and the three constraints are linear. Similarly to (9), when allocating its power, SU has to satisfy the relay power constraint (17) while considering the PU power obtained in (10). By using the invariance of the *Trace* operator under the cyclic permutation, the constraint (16) can be written as $Tr(\boldsymbol{{\Phi_s}^h \Phi_s P_s}) \leq P_{tot}$. By defining the matrix $\boldsymbol{A_s} = \boldsymbol{{\Phi_s}^h \Phi_s}$, (16) becomes $Tr(\boldsymbol{A_s P_s}) \leq P_{tot}$. Since the constraint (18) is a peak constraint we divide the problem into two subproblems with the same objective function but with constraints (16),(17) for the first subproblem and with the constraint (18) in the second. Then we take the minimum between the two solutions. For the first subproblem, we, again, use the Lagrange method [18] to find the optimal solution. For the second subproblem, it is clear that I_{th} is the optimal solution $\forall j = 1, \ldots, N-n$. Finally, the resulting power profile is given as follows:

$$P_s^*(j,j) = \begin{cases} \min\left\{\left[\frac{1}{\mu A_s(j,j) + \eta\sum_{i=1}^{N}|H_s(j,i)|^2} - Q_{\tilde{z}}(j,j)\right]^+, I_{th}\right\}, \\ \qquad \forall j = 1, \ldots, N-n, \\ \left[\frac{1}{\mu A_s(j,j) + \eta\sum_{i=1}^{N}|H_s(j,i)|^2} - Q_{\tilde{z}}(j,j)\right]^+, \\ \qquad \forall j = N-n+1, \ldots, N, \end{cases} \tag{19}$$

where μ and η are the Lagrange multipliers associated to the peak and the relay power constraint, respectively. Note that, the optimal power does not depend on the primary transmission. In addition, when the PU does not tolerate any interference, i.e. $I_{th} = 0$, the SU is still able to transmit using the free eigenmodes and the corresponding rate is note as the "free eigenmodes rate".

B. Imperfect SIC

In Section IV-A, we considered the ideal case when capacity achieving codes are employed by the PU transmitter. Since the PU rate is smaller than the PU mutual information, arbitrary low decoding error probability is achievable. In this subsection, we assume that instead of using capacity achieving codes, PU employs more practical coding schemes and thus decoding errors are unavoidable no matter how small the PU rate is. To capture this setting, we have introduced the parameter α. In this case, we investigate the extreme scenario ($\alpha = 0$) when the receiver decodes the cognitive message after employing an imperfect SIC where the interference power at each antenna is equal to $\mathbb{E}\left[\left|\tilde{\lambda}_j\left(s_{p_j} - \hat{s}_{p_j}\right)\right|^2\right] = 2P_p^*(j,j){\lambda_j}^2$. Then, the SU achievable rate is obtained by solving the following optimization problem:

$$\max_{\boldsymbol{P_s}} {R_s}^{(0)} = \sum_{j=1}^{N-n} \log_2\left(1 + \frac{P_s(j,j)}{Q_{\tilde{z}}(j,j) + 2P_p^*(j,j){\lambda_j}^2}\right) + \sum_{j=N-n+1}^{N} \log_2\left(1 + \frac{P_s(j,j)}{Q_{\tilde{z}}(j,j)}\right) \tag{20}$$

$$\text{s.t. } Tr(\boldsymbol{A_s P_s}) \leq P_{tot}, \tag{21}$$

$$Tr\left(\boldsymbol{H_p P_p^* {H_p}}^h + \boldsymbol{H_s P_s {H_s}}^h + \boldsymbol{W W}^h\right) \leq P_R, \tag{22}$$

$$P_s(j,j) \leq I_{th}, \forall j = 1, \ldots, N-n. \tag{23}$$

This problem is also convex and the optimal power is computed similarly to the perfect SIC case by using the Lagrange method, the optimal power is given by

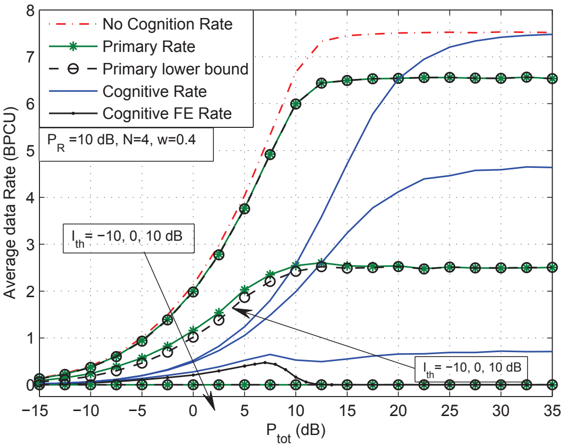
$$P_s^*(j,j) = \begin{cases} \min\Big\{ \Big[\frac{1}{\mu A_s(j,j)+\eta\sum_{i=1}^{N}|H_s(j,i)|^2} \\ \qquad - \big(Q_{\tilde{z}}(j,j) + 2P_p^*(j,j){\lambda_j}^2\big)\Big]^+, I_{th}\Big\}, \\ \qquad \forall j = 1,\dots,N-n, \\ \Big[\frac{1}{\mu A_s(j,j)+\eta\sum_{i=1}^{N}|H_s(j,i)|^2} - Q_{\tilde{z}}(j,j)\Big]^+, \\ \qquad \forall j = N-n+1,\dots,N, \end{cases} \tag{24}$$

where μ and η are the Lagrange multipliers associated to constraints (21) and (22), respectively. We notice, here, that the optimal power depends on the primary power and eigenmodes which means that the secondary is adapting its power continuously with the variation of the primary channel state.

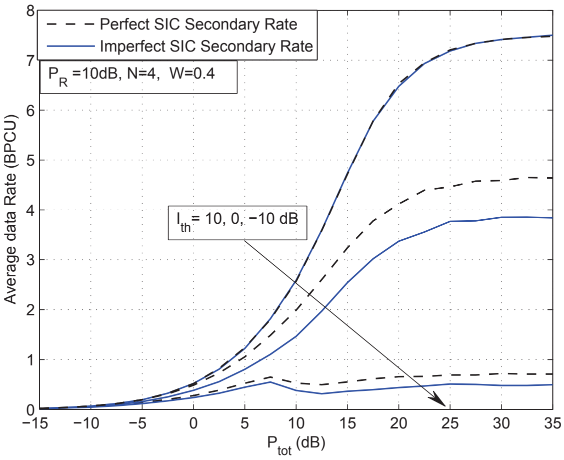
V. NUMERICAL RESULTS

In our numerical results, we consider a Rayleigh fading channel in which the channel gains are complex Gaussian random variables with zero mean and unit variance, and we choose $N = 4$ antennas, and the rates expressed in bits per channel use (BPCU). For simplicity, we assume that the relay's amplification matrix is diagonal and is given by: $\boldsymbol{W} = w \times \boldsymbol{I_N}$ where w is a positive scalar and $\boldsymbol{I_N}$ is the N-dimension identity matrix. Note that the proposed scheme can be applied with any fixed amplification gain matrix. The optimization of $\boldsymbol{W}$ is left to a future extension of this work.

In Figure 2.a, we plot the PU lower bound, PU and SU achievable rates as a function of P_{tot} for $P_R = 10$ dB and $w = 0.4$ with perfect SIC ($\alpha = 0$). Recall that the lower bound of the PU corresponds to the achievable rate found after solving (7) while the actual PU rate is computed using (13) .To measure the performance of the proposed system, we plot the rate limits when $I_{th} = 0$ which gives the upper bound of the PU rate: "no cognition rate" and the lower bound of the SU rate: "free eigenmodes (FE) rate". We show that the space alignment technique allows the SU to achieve a rate up to 0.5 BPCU when using only the FE, i.e. there is no tolerated interference from the PU. However, this rate becomes zero when P_{tot} exceeds 12 dB since, in this case, the PU is using all the eigenmodes. Then, depending on the tolerated interference threshold I_{th} the SU rate is considerably enhanced especially when P_{tot} is high since the power constraints (16) become relaxed. Note that the PU remains satisfied as its rate is always above the lower bound (dashed black curves). This can be explained by the fact that the secondary power is limited by the peak power constraint and not by the interference constraint. At high values of P_{tot}, the rates saturate at a certain value which depends mainly on the relay's power. That is, even if the PU and SU have high power, the received signal at the destination is limited by the relay's power. We also note that if I_{th} is high enough (i.e., more than 0 dB), the cognitive rate may exceed the primary rate starting from a certain P_{tot} value, in this setting $P_{tot} = 10$ dB. In Figure 2.b, the SU rate with perfect and imperfect SIC is presented for $P_R = 10$ dB to quantify the rate loss when $\alpha = 1$. We notice that the loss is small for high values of I_{th} For instance, for $I_{th} = 10$ dB, there is a match between the perfect and imperfect SIC rate.



(a) Perfect SIC.



(b) Imperfect SIC.

Figure 2: PU and SU Rates versus P_{tot}.

Figure 3 shows the effect of the relay's power, P_R, on the PU and SU rates with different values of P_{tot}. First, we notice that even without cognition, the rates stagnate at high values of P_R since the power budget P_{tot} is exceeded by the relay's power. In the case of cognition in Fig. 3.a, for fixed I_{th}, when P_R is low, the cognitive rate departures from the no cognition upper bound. Meanwhile, the PU is only achieving the lower bound rate since its power is limited, in (9), by the low relay's power and the terms involving $I_{th}\boldsymbol{H_s}{\boldsymbol{H_s}}^h$ which

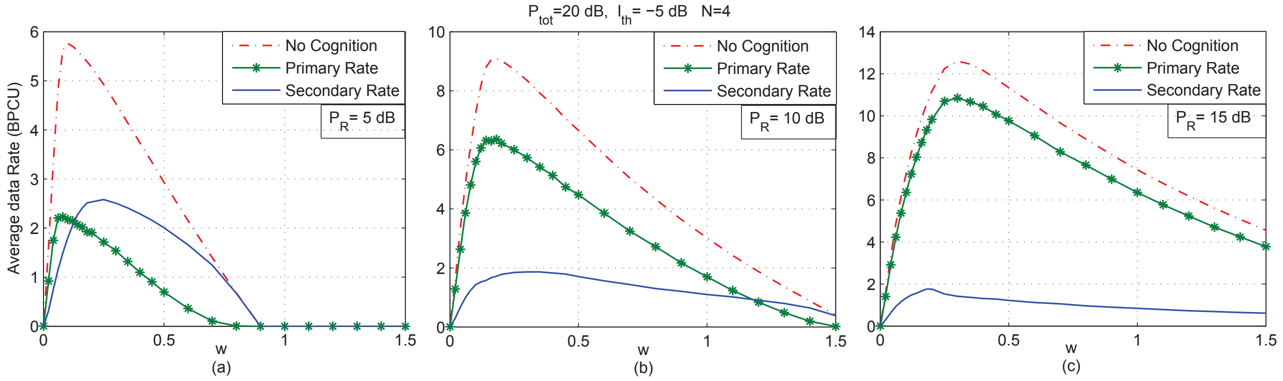
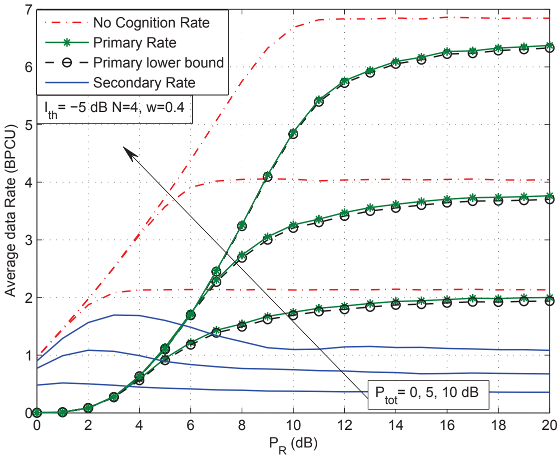
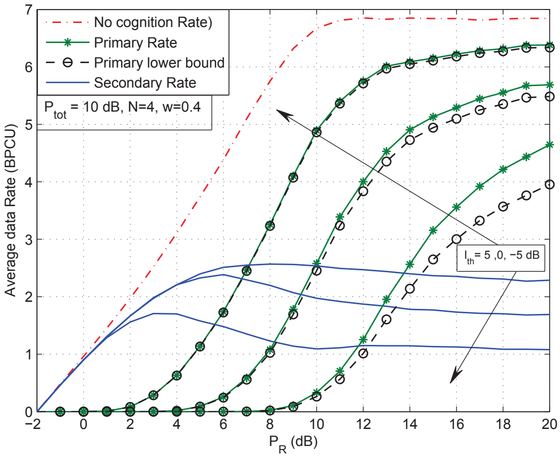


Figure 4: PU and SU rates with perfect SIC versus w.



(a) $I_{th} = -5$ dB and $P_{tot} = 0, 5, 10$ dB.



(b) $I_{th} = 5, 0, -5$ dB and $P_{tot} = 10$ dB.

Figure 3: PU and SU Rates with perfect SIC versus P_R.

is independent of P_R. Hence the optimal PU power, P_p^*, is limited and close to zero. Meanwhile, the SU power in (17) is limited by the relay's power P_R in addition $\boldsymbol{H_p P_p^* H_p}^h$ which is already very low, consequently the rate of the relay is, in this regime, fully dedicated to the SU. However, when P_R becomes greater, the cognitive rate stagnates or decreases while the primary rate increases remarkably to the no cognition upper bound. Hence, the choice of P_R is important to the PU since the SU rate is almost the same. Figure 3.b shows that as I_{th} increases, the PU rate is shifted far from the upper bound due to the tolerated interference. Meanwhile, the SU rate is relatively high at low P_R but stagnates to a certain value at high P_R. We also note in Fig. 3.b, that the SU rate reaches a maximum and decreases again as P_R increases. This is due to the fact that at low P_R, the SU power is only limited by the relay power constraint and as P_R increases the power increases. However, at high P_R values, the effect of the interference constraint appears and the SU becomes almost constant. The relay's power is considered as an envelope of the SU and PU rates at low and high values, respectively.

In Figure 4, we highlight the effect of the relay amplification matrix gain $\boldsymbol{W}$ on PU and SU rates for different values of P_R. Recall that, in our numerical results, we chose $\boldsymbol{W} = w \times \boldsymbol{I_N}$, which is not necessarily the optimal choice but is a simple one to quantify the effect of this matrix on the system performance. We notice that even with no cognition the rate reaches its maximum for a certain value of w before decreasing to zero as w increases. The reason behind this rate shape is that increasing w enhances the power as the relay power constraint is not reached. When reached, i.e., the values of w are large, the terminal power should be small in order to respect the constraint and as w increases further, the power should be near zero. In the CR framework, the shape of the rate is similar but lower than the no cognition rate. The optimal w giving the maximum rate is slightly different for PU and SU and can favor one over the other as shown in Figure 4.a. This fact is noted for low values of P_R, i.e., the interference constraint is not limiting the SU power. However, at high P_R, the PU rate is higher and converges to the no cognition rate.

VI. CONCLUSION

In this paper, we investigated the achievable rate of a cognitive radio system consisting of primary and secondary users assisted by a common amplify-and-forward relay. We proposed a particular linear precoding scheme based on the space alignment strategy. By adopting this strategy, we computed the optimal power allocation for the cognitive user under power, interference and relay's power constraints. We have also derived the optimal power in different settings (perfect and imperfect successive interference cancellation) in order to give upper and lower bounds of the cognitive rate. In our numerical results, we showed that our scheme insures a non-zero cognitive rate up to a certain budget power and this rate is considerably enhanced as the interference threshold is higher. In addition, we found that the relay affects both primary and secondary rates through the relay's power and the structure of the amplification gain.

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