Nearness of Objects.
Approximation Space Model Revisited

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Dedicated to Zdzisław Pawlak

Abstract. This paper is devoted to a nearness relation in an Efremovič-proximity space. The basic approach is to consider the nearness of the upper and lower approximation of a set introduced by Z. Pawlak during the early 1980s as a foundation for rough sets. Two forms of nearness relations are considered, namely, a spatial EF- and a descriptive EF-relation. This leads to a study of the nearness of objects either spatially or descriptively in the approximation of a set. The 2007 nearness approximation space model is refined and extended in this paper, leading two new forms of nearness approximation spaces. There is a natural transition from the two forms of approximation introduced in this article to nearness of information granules. This leads to the study of methods of inducing approximations of nearness relations for information granules and the benefits of this approach for approximate reasoning over granular computations.

Key words: Approximation space, EF-proximity space, information granules, nearness relation, rough sets

1 Introduction

This paper introduces an extension of the Pawlak approximation space model with a nearness relation in an EF-proximity space [1]. The basic approach is to consider the nearness of the upper and lower approximations of a set, introduced by Z. Pawlak during the early 1980s as cornerstones in the foundations of rough sets [2–5]. Two forms of nearness relations are considered, namely, a spatial nearness relation defined in a traditional Efremovič (EF) proximity space [6] and a relation defined on a descriptive EF-proximity space [7–9]. This leads to a study of the nearness of objects either spatially or descriptively in the approximation of a set. The 2007 nearness approximation space model introduced in [10, §3] is refined and extended in this paper, leading to two new forms of nearness approximation spaces. There is a natural transition from the two forms of approximation introduced in this article to nearness in a generalized approximation space (GAS). In this article, approximation spaces are also considered in the more general context of information granules (recently, this has led to what is known as a rough granule calculus [11]). In keeping with the original nearness approximation space model, Mitchell analogy-making [12] is revisited and viewed in a more general setting in reasoning about concepts [10, §5].

2 Spatial and Descriptive Nearness

Let δ on a nonempty set X be a nearness (proximity) relation. For subsets B, C in X, we write $B\ \delta\ C$ (meaning B is *spatially near* C), provided $B \cap C$ is nonempty. If B is not near (far from) C (denoted by $A\ \underline{\delta}\ C$), then $B \cap C$ is empty. Sets that are far from each other are called remote sets. Such nearness (proximity) relation is also called a discrete proximity [1].

Example 1. **Sample Remote Sets**.
Let the nonempty set X with subsets A, B, C be represented by the picture in Fig. 1.

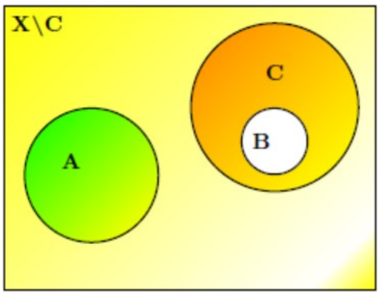

Fig. 1. $\mathbf{B}\ \delta\ \mathbf{C}, \mathbf{A}\ \underline{\delta}\ \mathbf{C}$ and $\mathbf{B}\ \underline{\delta}\ \mathbf{X}\backslash\mathbf{C}$

In this picture, $B\ \delta\ C$, since $B \cap C = B \neq \emptyset$. Also, $A\ \underline{\delta}\ C$ in Fig. 1. The sets A, C are examples of remote sets, since they are far from each other.

■

The relation $\delta \subseteq \mathcal{P}(X) \times \mathcal{P}(X)$ is an **Efremovič proximity** (also called an **EF-proximity**), if and only if, for $A, B, C \in \mathcal{P}(X)$, the following axioms hold [6, 1].

(**EF**.1) $A\ \delta\ B$ implies A and B are not empty.
(**EF**.2) $A \cap B \neq \emptyset$ implies $A\ \delta\ B$.
(**EF**.3) $A\ \delta\ B$ implies $B\ \delta\ A$ (symmetry).
(**EF**.4) $A\ \delta\ (B \cup C)$, if and only if, $A\ \delta\ B$ or $A\ \delta\ C$.
(**EF**.5) Efremovič axiom:

$$A\ \underline{\delta}\ B \text{ implies } A\ \underline{\delta}\ C\ \&\ B\ \underline{\delta}\ X \backslash C \text{ for some } C \subseteq X.$$

The pair (X, δ) is called an **EF-proximity space**. An EF-proximity is **separated**, provided it satisfies (EF.6).

(**EF**.6) $\{x\}\ \delta\ \{y\}$ implies $x = y$.

Example 2. **Illustration for the Efremovič axiom**.
Assume that (X, δ) is an EF-space, where the set X with subsets A, B are represented by the picture in Fig. 1. $A\ \underline{\delta}\ B$ (A and B are remote sets) and we can find C so that $B\ \underline{\delta}\ X \backslash C$, *i.e.*, B is far from the complement of C (denoted by C^c).

■

2.1 Pawlak Approximation Space

Let X be a nonempty set of objects, Φ a set of functions that represent object features. For simplicity of reasoning, we assume that these are real valued functions. We define an equivalence relation $\underset{\Phi}{\sim}$ by

$$\underset{\Phi}{\sim} = \{(x, y) \in X \times X : \text{for all } \phi \in \Phi, \phi(x) = \phi(y)\},$$

where $\phi(x) \in \Re^k$ for some k, where $\Re$ is the set of reals. $\Phi(x)$ is called the *feature value vector of* x.

The relation $\underset{\Phi}{\sim}$ is usually called an Φ-indiscernibility relation [3]. The pair $(X, \underset{\Phi}{\sim})$ is an approximation space, introduced by Z. Pawlak [2]. Let us assume that $x \in X$. Then $[x]_\Phi$ is an equivalence class in the partition $X_{/\sim_\Phi}$. Unions of equivalence classes from $X_{/\sim_\Phi}$ are called Φ-definable subsets of X.

The lower and upper approximations of $E \subseteq X$ (denoted by $\Phi_* E$, $\Phi^* E$, respectively) are defined by

$$\Phi_* E = \bigcup_{[x]_\Phi \subseteq E} [x]_\Phi, \ \textit{lower approximation of } E,$$
$$\Phi^* E = \bigcup_{[x]_\Phi \cap E \neq \emptyset} [x]_\Phi, \ \textit{upper approximation of } E.$$

Let us observe that the lower approximation and the upper approximation of E can be equivalently defined by

$$\Phi_* E = \bigcup_{x \in X:\ \Phi(x) \in\ \Phi(E)^+} [x]_\Phi,$$

$$\Phi^* E = \bigcup_{x \in X:\ \Phi(x) \in\ \Phi(E)} [x]_\Phi,$$

where $\Phi(E) = \{\Phi(x) : x \in E\}$, $\Phi(E)^+ = \{v \in \Phi(E) : \forall x(\Phi(x) = v \to x \in E)\}$.

The boundary region $bd_\Phi(E)$ of E relative to Φ is defined by

$$bd_\Phi(E) = \Phi^* E \setminus \Phi_* E.$$

Let $(X, \underset{\Phi}{\sim}, \delta_\Phi)$ denote a *descriptive nearness approximation* space, which is a Pawlak approximation space endowed with the descriptive EF-proximity relation δ_Φ.

2.2 Descriptive EF-Proximity Space

A descriptive EF-proximity is briefly presented in this section (see, *e.g.*, [8, 7]). Let X be a nonempty set, x a member of X, $\Phi = \{\phi_1, \ldots, \phi_n\}$ a set of functions that represent features of each x. Let $\Phi(x)$ denote a feature vector for the object x, *i.e.*, a vector of feature values that describe x. A feature vector provides a description of an object. Let A, E be subsets of X. Let $\Phi(A)$, $\Phi(E)$ denote sets of descriptions of members of A, E, respectively. Then we have $\Phi(x) \in \Phi(A)$ iff $x \in \Phi^*(A)$ for any $x \in X$.

The expression $A\ \delta_\Phi\ E$ reads A *is descriptively near* E. Similarly, $A\ \underline{\delta}_\Phi\ E$ denotes that A is descriptively far (remote) from E. The descriptive proximity of A and E is defined by

$$A\ \delta_\Phi\ E \Leftrightarrow \Phi(A) \cap \Phi(E) \neq \emptyset.$$

The descriptive remoteness of A and E (denoted by $A\ \underline{\delta}_\Phi\ E$) is defined by

$$A\ \underline{\delta}_\Phi\ E \Leftrightarrow \Phi(A) \cap \Phi(E) = \emptyset.$$

From the above definition one can obtain the following proposition:

Proposition 1. *Let $(X, \underset{\Phi}{\sim}, \delta_\Phi)$ be a descriptive nearness approximation space and let $A, E \subseteq X$. Then, for descriptively near sets, the following statements are equivalent.*
(a) $A\ \delta_\Phi\ E$,
(b) $\exists x \in X : [x]_\Phi \cap A \neq \emptyset$ *and* $[x]_\Phi \cap E \neq \emptyset$,
(c) $\Phi^* A \cap \Phi^* E \neq \emptyset$.

This result is illustrated in Figure 2.

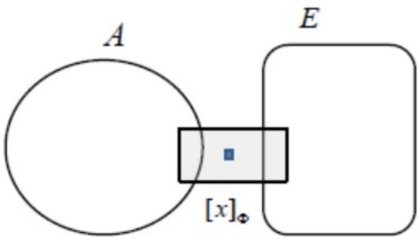

Fig. 2. Disjoint sets A and E are near because there exists an indiscernibility class with nonempty intersection with both sets A, E.

Example 3. **Sample Descriptively Remote Sets**.
Let the nonempty set X with subsets A, B, C, E represented by the picture in Fig. 3. Also, let Φ contain functions $\phi_g, \phi_o, \phi_y, \phi_w$ used to measure the intensity of the colours green (g), orange (o), yellow (y), and greylevel intensity (w) of points x in X. In this picture, $A \ \delta_\Phi \ E$, since the description of A matches the description of E. In addition, $A \ \underline{\delta}_\Phi \ C$ and $A \ \underline{\delta}_\Phi \ B$ in Fig. 1, since the description of A does not match the descriptions of B and C. The sets A, B, C are examples of pairwise descriptively remote sets, since their descriptions are far from each other.

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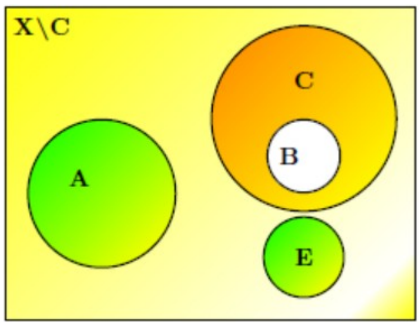

Fig. 3. $A \ \delta_\Phi \ E$, $\mathbf{A} \ \underline{\delta} \ \mathbf{C}$ and $\mathbf{B} \ \underline{\delta} \ \mathbf{X \backslash C}$

Define the descriptive intersection $\underset{\Phi}{\cap}$ of subsets A and B of X by

$$A \underset{\Phi}{\cap} B = \{x \in X : \Phi(x) \in \Phi(A) \cap \Phi(B)\}.$$

We have $A \underset{\Phi}{\cap} B = \Phi^*(A) \cap \Phi^*(B)$. Hence, from Proposition 1, we also have $A \delta_\Phi B$ iff $A \underset{\Phi}{\cap} B \neq \emptyset$.

Example 4. **Sample Descriptive Intersection**.
For the nonempty set X with subsets A, B, C, E be represented by the picture in Fig. 3, observe that the sets A and E are spatially remote, but descriptively near. In fact,

$$A \underset{\Phi}{\cap} E = \Phi^* A \neq \emptyset \text{ since } \Phi(A) = \Phi(E) \neq \emptyset.$$

By contrast, the sets B and C are spatially near. However, they are descriptively remote, since

$$B \underset{\Phi}{\cap} C = \emptyset. \qquad \blacksquare$$

For any two nonempty sets A and B, descriptive union is defined by

$$A \underset{\Phi}{\cup} B = \{x \in X : \Phi(x) \in \Phi(A) \cup \Phi(B)\}.$$

We have $A \underset{\Phi}{\cup} B = \Phi^*(A \cup B)$.

The definition of the descriptive proximity relative to Φ can be generalized as follows. Let us assume that $\Phi(x) \in \Re^k$, where $\Re$ is the set of reals and $r \subseteq \mathcal{P}(\Re^k) \times \mathcal{P}(\Re^k)$, where $\mathcal{P}(\Re^k)$ is the powerset of $\Re^k$. Then one can define a binary relation $\delta_{\Phi,r}$ by $A\ \delta_{\Phi,r}\ E$, if and only if, $r(\Phi(A), \Phi(E))$.

The binary relation $\delta_{\Phi,r}$ is a **descriptive Efremovič-proximity** (EF-proximity), provided the following axioms are satisfied for subsets A, B, C of X.

($\boldsymbol{EF_{\Phi,r}}$.1) $A\ \delta_{\Phi,r}\ B$ implies $A \neq \emptyset, B \neq \emptyset$.
($\boldsymbol{EF_{\Phi,r}}$.2) $A \underset{\Phi}{\cap} B \neq \emptyset \Rightarrow A\ \delta_{\Phi,r}\ B$.
($\boldsymbol{EF_{\Phi,r}}$.3) $A\ \delta_{\Phi,r}\ B \Rightarrow B\ \delta_{\Phi,r}\ A$
($\boldsymbol{EF_{\Phi,r}}$.4) $A\ \delta_{\Phi,r}\ (B \cup C) \Leftrightarrow A\ \delta_{\Phi,r}\ B$ or $A\ \delta_{\Phi,r}\ C$.
($\boldsymbol{EF_{\Phi,r}}$.5) $A\ \underline{\delta}_{\Phi,r}\ B \Rightarrow A\ \underline{\delta}_{\Phi,r}\ C$ and $B\ \underline{\delta}_{\Phi,r}\ C^c$ for some $C \subseteq X$.

The pair $(X, \delta_{\Phi,r})$ is called a **descriptive EF-proximity space**. Points are descriptively distinct if they have different descriptions. For distinct points $x, y \in X$, a descriptive EF-proximity is separated, if and only if, it satisfies

($\boldsymbol{EF_{\Phi,r}}$.6) $\{x\}\ \delta_{\Phi,r}\ \{y\} \Leftrightarrow \Phi(x) = \Phi(y)$ (EF-Proximity Separation Axiom).

Example 5. **EF-Proximity Based on Colour**.
Let the subsets $A, B, C \subseteq X$ be represented by the coloured circular regions in Fig. 1. Let Φ contain probe functions representing various colours of the picture elements in Fig. 1. The assumption made here is that a picture element is the smallest visible part of the picture (a pixel) and each picture elements has discernible features such as green, orange, yellow, white. The labels $X \backslash C$ (complement of C, also written C^c), A, B, C identify the parts of the picture. Axiom (EF.5) is satisfied in this depiction of the subsets of X, since the colour of A is far from the colour B and B is descriptively far from the C^c. It is easy to verify that the remaining EF axioms are satisfied. Hence, (X, δ_Φ) is an example of a descriptive EF-proximity space.

$\blacksquare$

By considering different functions Φ, one can obtain different proximities and approximations of sets. Let us consider some illustrative examples.

Example 6. **Descriptive Classes**. In Fig. 4(a), the partition of the set X has

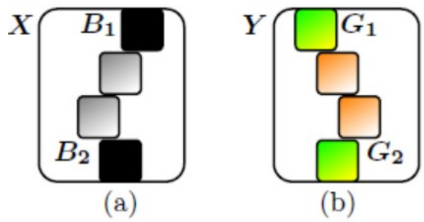

Fig. 4. Spatially remote and descriptively near sets

three equivalence classes, namely, the class containing white picture elements represented along the border of the box □, the class containing black picture elements in the pair of boxes ■ (sets labelled B_1, B_2) and the class containing grey picture elements in the pair of grey boxes ■. Let $x \in B_1$. Notice, for example, that the pair of sets B_1, B_2 are spatially remote sets but descriptively near sets and they belong to the equivalence class $[x]_{\underset{\Phi}{\sim}}$, *i.e.*,

$$
\begin{aligned}
&B_1 \ \underline{\delta} \ B_2 \text{ (spatially remote sets)},\\
&B_1 \ \delta_\Phi \ B_2 \text{ (descriptively near sets)},\\
&[x]_{\underset{\Phi}{\sim}} = B_1 \underset{\Phi}{\cup} B_2 \text{ (class = descriptive union)}.
\end{aligned}
$$

Again, in Fig. 4(b), the partition of the set Y has three equivalence classes, namely, the class containing white picture elements represented along the border of the box □, the class containing green picture elements in the pair of boxes ■ (sets labelled G_1, G_2) and the class containing orange picture elements in the pair of boxes ■. Let $y \in G_1$. Again, for example, the pair of sets G_1, G_2 are spatially remote sets but descriptively near sets and belong to the equivalence class $[y]_{\underset{\Phi}{\sim}}$, *i.e.*,

$$
\begin{aligned}
&G_1 \ \underline{\delta} \ G_2 \text{ (spatially remote sets)},\\
&G_1 \ \delta_\Phi \ G_2 \text{ (descriptively near sets)},\\
&[y]_{\underset{\Phi}{\sim}} = G_1 \underset{\Phi}{\cup} G_2 \text{ (class = descriptive union)}. \qquad \blacksquare
\end{aligned}
$$

Example 7. **Descriptive Approximation Spaces**.
Let X be a nonempty set of picture elements in Fig. 5(a), Φ a set of functions used to extract colours from members of X, $g \in G_1, b \in B_1$, w a member of a set W of white picture elements, and $E_X \subseteq X, E_Y \subseteq Y$ (represented by dotted

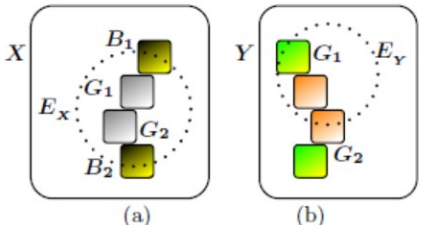

Fig. 5. Sample descriptive approximation spaces

circles in Fig. 5(a) and Fig. 5(b), respectively). The descriptive approximation space $(\Phi(X), \sim_{\Phi})$ is represented in Fig. 5(a), where

$$
\begin{aligned}
[g]_{\Phi} &= G_1 \underset{\Phi}{\cup} G_2,\\
[b]_{\Phi} &= B_1 \underset{\Phi}{\cup} B_2,\\
\Phi_* E_X &= [g]_{\Phi} \cup \{[w]_{\Phi} :\ w \in W \ \&\ [w]_{\Phi} \subseteq E_X\},\\
\Phi^* E_X &= [g]_{\underset{\sim}{\Phi}} \cup [b]_{\Phi} \cup \{[w]_{\Phi} :\ w \in W \ \&\ [w]_{\Phi} \cap E_X \neq \emptyset\}.
\end{aligned}
$$

Since, for example, the sets B_1, B_2 are partly in and partly outside E_X, the set E_X is a rough set. A similar line of reasoning leads to the conclusion that E_Y in Fig. 5(b) is a rough set.

3 Nearness of Information Granules

Granular Computing (GC) becomes a hot topic in many application areas where it is necessary to search for or discover complex structural objects called information granules used, e.g., for inducing classifiers for vague concepts (see, e.g., [13]). This approach may be necessary for approximate reasoning about dynamic complex objects, *e.g.*, for expressing that complex dynamic granule representing a flock of birds is now near another complex granule representing a forest or that two complex dynamic granules representing cells, recorded by using electron microscopy, are now near. In particular, this seems to be necessary for realization of the *computing with word* paradigm proposed by Lotfi A. Zadeh [14, 15].

Information granules [14–16] have often complex structures and can be represented as objects of information systems or decision tables on different levels of hierarchical modeling (see,*e.g.*, [17]). Then attributes are defined over such complex granules used as objects in such information systems. One can again use the presented above approach for descriptive nearness using attribute value vectors over the granules. However, there is also an issue of nearness in a particular context. One of the possible approaches is to consider granules in the framework of mereology or rough-mereology (see,*e.g.*, [18–21]), which makes it possible to consider nearness of granules that are parts of some more complex granules.

Note that in real-life applications, it is difficult to obtain an analytical form for a nearness relation. Approximation of such a relation, as one of the relations in an ontology of granules, should be learned from incomplete data. This process usually will require interaction with domain experts for acquiring relevant features (attributes) for approximation of that relation. Discovery of these relevant attributes can be achieved using the ontology approximation methodology developed in a number of papers and summarized in [17]. In inducing approximations of nearness relations from data, one can use the approximate Boolean reasoning approach (see, *e.g.*, [22, 23]), assuming that relevant features for approximation have already been discovered.

Another problem with nearness relations in real-life applications is that they lead to vague concepts. The temporary approximations of such relations can be obtained using the rough set approach in combination with other soft computing methods and hierarchical modeling. Note that it is necessary to change adaptively these approximations due to interactions with the dynamically changing environment and other information sources.

Hence, in real-life applications, the nearness can be considered as a process of dynamically changing information granules [13]. In different process stages, these information granules represent the current semantical meanings of complex vague concept of nearness. This requires to use advanced methods of learning supported, *e.g.*, by domain knowledge expressing the context in which the nearness concept is considered, methods for new relevant feature extraction, *e.g.*, domain ontology approximation and adaptation strategies. This approach is quite different from the traditional approach based on axiomatic definition of nearness introduced, *e.g.*, in proximity spaces. Note that this remark is also true for many other complex vague concepts, *e.g.*, related to risk analysis.

4 Conclusions and Future Research

In the paper, we discussed two approaches to nearness. The first approach has the roots in the approach developed for proximity spaces. The second approach arises from real-life projects, where nearness becomes a complex vague concept dependent on the context. The latter approach requires advanced methods for approximation of complex vague concepts, *e.g.*, based on approximation of domain ontology.

We would like to address two research issues linking our considerations on nearness of granules with our previous considerations on proximity relations. The first one is related to methods of inducing approximations of nearness relations for information granules satisfying crisp constraints assumed for nearness relations. Assuming that such approximations can be obtained, one can consider the second issue based on characterization of benefits on approximate reasoning over granular computations with approximations of nearness relations satisfying such constraints.

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