# Imagining Contexts*

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**Abstract.** The aim of this paper is to present a formal semantics inspired by the notion of Mental Imagery, largely researched in Cognitive Science and Experimental Psychology, that grasps the full significance of the concept of context. The outcomes presented here are considered important for both the Knowledge Representation and Philosophy of Language communities for two reasons. Firstly, the semantics that will be introduced allows to overcome some unjustified constraints imposed by previous quantificational languages of context, like flatness or the use of constant domains among others, and increases notably their expressive power. Secondly, it attempts to throw some light on the debate about the relation between meaning and truth by formally separating the conditions for a sentence to be meaningful from those that turn it true within a context.

## 1 Introduction

In human communication every sentence is uttered in a context and interpreted in a context. These contexts are regarded as the set of facts that hold true at the time of utterance and interpretation respectively. If a sentence refers unambiguously to a fact that is universally accepted, it will be considered true regardless of the differences between the context of the agent who uttered it and the agent who interprets it. This is the case of mathematics, which is based on an unambiguous formal language that expresses facts derived from a set of universally accepted axioms, such as the Zermelo-Fraenkel set theory. Quine [1] referred to these sentences as "eternal" and looked for a language whose sentences were all of this kind. However, in contrast with the language of mathematics, human language, and consequently that of any form of artificial intelligence, is highly dependent on context. And as a consequence of this, Quine's project turned to be a difficult enterprise, that could result even impossible, if the notion of context is not included in the characterization of the truth function.

Although the interest in a formal theory of context within the AI community had already been present years before, there was no official research programme in this direction until in 1993 McCarthy [2] presented it as a candidate solution

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for the problem of generality [3]. Since then, many logics [4–7] have emerged with the aim of capturing all of the common-sense intuitions about contextual reasoning that were introduced in [2]. However, most of these languages only deal with the propositional case and are therefore unable to treat contexts as first-class citizens included in the domain of discourse, what is one of the main desiderata behind the formalization of contexts. Only the quantificational logic of context presented in [6] is capable of formulating statements that predicate on contexts. Nevertheless, its semantics is too restrictive and imposes counter-intuitive constraints like flatness [5] or the use of constant domains [8] among others. Due to the lack of an adequate solution to the challenges posed in [2], Guha and McCarthy [9] restated the initial motivations by providing a classification on the different kinds of contexts that a satisfactory logic of context should be able to represent.

In parallel with the research in contextual reasoning developed in AI, the theory of a mental representation of ideas in the form of mental images [1] has been largely researched by cognitive scientists, experimental psychologists and philosophers [10]. Although there exist a number of controversies on how these images are formed or if after all they are images or not, the common thesis is that mind can recreate quasi-perceptual experiences similar to those that are presumed to be caused by external factors. According to the *analog* or *quasi-pictorial* theory of imagery [11], the human ability for the interpretation of symbols is equivalent to the recreation of quasi-perceptual experiences by mind. The memory of past perceptual experiences and their possible recombination are the basis of the imagery that an agent uses when interpreting a sentence.

We endorse the quasi-pictorial theory of imagery and argue that by taking it as an inspiration we can develop a logic of context that meets the challenges introduced by [2]. This inspiration is mainly realized in two features of our semantics. Firstly, in contrast with the truth-conditional theory of meaning, in our logic the meaning of a sentence will be regarded as a set of quasi-perceptual experiences instead of as a set of worlds. Secondly, a sentence will be considered to be supported by a context if its meaning is part of the image produced by the interpretation of that context. We claim that this separation between meaning and truth is necessary to grasp the concept of context in its full extent.

In this paper we present a semantics that formalizes a conceptualization of a quasi-pictorial theory of Mental Imagery by which it notably increases the expressiveness of previous logics of context and overcomes some difficulties posed by them. The paper is structured as follows. First, we introduce informally the main features of the logic and compare it with previous logics of context and other formalisms. Second, the language of our logic and its formation rules are described. Third, we define a model of interpretation inspired by a conceptualization of a theory of imagery and subsequently explain the associated theory of

[1] It must be noted that all along this document we do not use the term "image" with a static connotation but we refer to both instantaneous and durative quasi-perceptual recreations. Besides, it is not limited to visual experiences but to all the kinds of experiences an agent can perceive through its senses.

meaning, together with the characterization of the truth function and the way we resolve traditional problems like existence and denotation. We end the paper by extracting the conclusions and envisaging some future work that we plan to undertake in this line of research.

## 2 The Logic

Our logic cannot simply be defined as an extension to predicate calculus, because there are some fundamental aspects in the semantics that turn it very different from the classical model theoretic semantics of first order logic. However, we can compare the expressive power of both logics and say that the logic presented here increases the expressiveness of predicate calculus with identity by adding the following capabilities:

1. Like in previous logics of context [4–7], formulas can be stated in a context. Therefore, there is no contradiction in asserting a formula and its negation while they are in different contexts. However, in contrast with those logics of context based on the *ist* predicate [4–6], we do not force every sentence to be preceded by a context. Instead, if a sentence is not preceded by a context, it will be assumed to be a description of actuality. The reason for this is that, unlike [2], we do not judge intuitively correct to state that actuality can be transcended and therefore we consider it to be the outermost context. Nevertheless, this does not make our logic differ on the transcendence capabilities described in [4–6], because what it is claimed as unlimited transcendence by these approaches is actually limited in each context tree by its respective initial context $k_0$.
2. Formulas in our language can refer to contexts and quantify over them like in [4, 6]. In our logic it is not allowed, however, to predicate on any context, but only on those that are accessible from the context under which the formula in question is being asserted. References to non-accessible contexts will therefore fail to denote.
3. A given contextualized formula can be quoted or not depending on the context in terms of which that formula is being expressed. In order to express a formula in terms of the context in which is being contextualized it will need to be quoted. Otherwise, it will be assumed that the meaning of the terms used on that formula correspond to the outer context. The use of quotation marks in a formula will therefore allow for the abstraction from the meaning of its terms and the disambiguation of the indexicals it may contain.
4. Like suggested in [12], we differentiate between internal and external negations. While the external negation of a formula can be satisfied even if its terms fail to denote or the formula is meaningless, the internal negation of a formula requires that it is meaningful and its terms succeed to denote in the context in which it is being asserted.
5. Formulas can express a parthood relation between the references of two terms. This relation will result particularly useful when formalizing normalcy assumptions between contexts [9]. If a context is said to be part of other

context and the latter supports a set of normalcy conditions expressed in the form of universal quantifications, all these conditions will consequently become normalcy assumptions in the former context.

In addition to the mentioned expressive capabilities, the semantics we present overcomes some counter-intuitive restrictions imposed by [6] and adds some novel ways of dealing with meaningless sentences, existence and designation. Below are roughly introduced the fundamental aspects that characterize this semantics:

1. An image is a mereologically structured object.
2. The meaning of the non-logical symbols of our logic ranges over a set that contains the imagery an agent possesses. This set is partially ordered according to two mereological parthood relations that will be introduced in the next section. In terms of possible worlds semantics, the imagery set is equivalent to a kind of *possibilia*. The meaning function assigns to each constant symbol a subset of the imagery containing all the possible counterparts [13] that it can denote.
3. Like [6] we differentiate between individuals of the *discourse* sort and the *context* sort. Like the rest of individuals contexts are interpreted as images mereologically structured. The domain of discourse of a context consists of its set of grounded parts. Therefore, each context defines in a natural and flexible way its own domain of discourse over which the denotation of terms of the discourse sort ranges. This domain is equivalent to *actuality*. In a given context the denotation of terms of the context sort ranges over the set of contexts that are accessible from it, which is equivalent to its set of figured parts.
4. In contrast with Intensional Logic [14–16], the denotation of constant symbols is not a function from contexts or states to members of the domain of discourse. Instead, the object denoted by a constant is the unique member of the intersection of the meaning of that constant and the grounded part expansion of the context in question, if it is of the discourse sort, or the set of contexts that are accessible from it, if it is of the context sort. This will help to determine whether a symbol succeeds to denote under a certain context.
5. Unlike in [6], there is no flatness restriction among contexts. In other words, the set of axioms holding at a particular context depends on the context from which it is accessed.

We will make use of classical Extensional Mereology [17] for the elaboration of the semantics. The binary relation *"is part of"* will be represented by the symbol $\preccurlyeq$ in our model. Therefore, if an object $x$ is said to be part of an object $y$, we will write $x \preccurlyeq y$. In addition to the classical operators of mereology we will make use of the part-expansion of an object. This operation is defined below.

**Definition 1.** *Given an object $\Gamma$, its part-expansion $\downarrow \Gamma$ is the set containing every part of $\Gamma$.*

$$\downarrow \Gamma = \{x : x \preccurlyeq \Gamma\} \tag{1}$$

## 3 Formal System

### 3.1 Syntax

A language $\mathcal{L}$ of our logic is any language of classical two-sorted predicate calculus with identity and a infinite set of non-logical symbols, together with a parthood relation and a set of symbols to express the contextualization, the quotation, and the internal negation of a formula. For simplicity we will make no use of functions. Below is the list of logical symbols of our language and the notational convention we will use for the non-logical ones:

1. $n$-ary predicate symbols: $P^n$ , $P_1^n$ , $P_2^n$ , ...
2. Constants of the discourse sort: $a$ , $a_1$ , $a_2$ , ...
3. Constants of the context sort: $k$ , $k_1$ , $k_2$ , ...
4. Variables of both sorts: $x$ , $x_1$ , $x_2$ , ...
5. External and internal negation: $\neg$ , $^-$
6. Connectives: $\vee$ , $\wedge$ , $\supset$
7. Quantifiers: $\forall$ , $\exists$
8. Identity: $=$
9. Parthood: $\leq_g$ , $\leq_f$
10. Quotation marks: “ , ”
11. Auxiliary symbols: : , [ , ]

Given a language $\mathcal{L}$, we will use $\mathbb{C}$ to refer to the set of constants of the discourse sort, and $\mathbb{K}$ to refer to the set of constants of the context sort. The set of variables of both sorts will be given by $\mathbb{V}$, while $\mathbb{P}$ will denote the set of predicates.

**Definition 2.** *The set of terms $\mathbb{T}$ and well-formed formulas (wffs) $\mathbb{W}$ are inductively defined on their construction by using the following formation rules:*

1. *Each variable or constant of any sort is a term.*
2. *If $t_1, \ldots, t_n$ are terms and $P^n$ is an n-ary predicate, then $P^n(t_1, \ldots, t_n)$ and $\overline{P^n}(t_1, \ldots, t_n)$ are wffs.*
3. *If $t_1$ and $t_2$ are terms, then $t_1 = t_2$, $t_1 \leq_g t_2$ and $t_1 \leq_f t_2$ are wffs.*
4. *If $A$ is a wff, then $[\neg A]$ is a wff.*
5. *If $A$ and $B$ are wffs, then $[A \vee B]$ , $[A \wedge B]$ and $[A \supset B]$ are wffs.*
6. *If $A$ is a wff and $x$ is a variable of any sort, then $(\forall x)[A]$ and $(\exists x)[A]$ are wffs.*
7. *If $A$ is a wff and $k$ is a constant of the context sort, then $[k : A]$ and $[k : \text{“}A\text{”}]$ are wffs.*

It must be noted that the treatment of the parthood relation as a logical symbol of our logic entails that its axiomatization as a transitive, reflexive and antisymmetric relation will be included in the set of axioms of the logic itself. We will refer to the axioms of Extensional Mereology [17] for this.

### 3.2 A Model of Interpretation

In our attempt to elaborate a formal semantics inspired by a quasi-pictorial theory of mental imagery, we consider that an image is a mereologically structured object and therefore it is a whole composed of parts. An image will be said to model the set of facts that its parts support and consequently the truth value assigned to a sentence will be relativized to the context under which is being considered. However, we will differentiate between two kinds of parts of which images may consist, namely *grounded* and *figured* parts. It is easy to understand the intuition behind this differentiation if we consider an example in which an agent is situated in an augmented-reality scenario. In this situation the agent will perceive some objects as genuine parts of reality and others as artificial objects recreated by some kind of device. We will say that the former objects are part of the actuality constructed by this agent in a *grounded* sense while the latter objects are part of the actuality constructed by this agent in a *figured* sense. Our intuition is that contexts, like those artificially recreated objects of the example, exist and are part of reality in a figured sense.

In order to capture these two different senses of parthood, the model structure will include a grounded parthood relation and a figured parthood relation. While the former will determine the domain of those objects of the discourse sort, the latter will determine the domain of those objects of the context sort and their accessibility. A formal definition of the model structure is given below.

**Definition 3.** *In this system a model, $\mathfrak{M}$, is a structure $\mathfrak{M} = \langle \mathcal{I}, \preccurlyeq_g, \preccurlyeq_f, \Omega, \mathcal{M} \rangle$ whose components are defined as follows:*

1. *$\mathcal{I}$ is a non-empty set. It consists of all the imagery an agent can recreate at the moment she is performing the interpretation.*
2. *$\preccurlyeq_g$ is a partial ordering on $\mathcal{I}$. It is therefore a transitive, reflexive, and antisymmetric relation on $\mathcal{I}$. It denotes the mereological parthood relation on the members of $\mathcal{I}$ in a grounded sense.*
3. *$\preccurlyeq_f$ is a partial ordering on $\mathcal{I}$. It is therefore a transitive, reflexive, and antisymmetric relation on $\mathcal{I}$. It denotes the mereological parthood relation on the members of $\mathcal{I}$ in a figured sense.*
4. *$\Omega$ is a distinguished member of $\mathcal{I}$. It represents the image of actuality constructed by the agent performing the interpretation.*
5. *$\mathcal{M}$ is a function from the non-logical symbols of $\mathcal{L}$ to a mapping from members of $\mathcal{I}$ to subsets of $\mathcal{I}$. $\mathcal{M}$ denotes the meaning function that assigns each constant of $\mathcal{L}$ a mapping from contexts to its set of possible denotations and each predicate of $\mathcal{L}$ a mapping from contexts to its extension over $\mathcal{I}$. In the definition of $\mathcal{M}$ given below we use the standard mathematical notation $\mathcal{P}(X)$ to refer to the powerset of $X$.*

$$\mathcal{M} : \begin{cases} \mathbb{C} \to [\mathcal{I} \to \mathcal{P}(\mathcal{I})] \\ \mathbb{K} \to [\mathcal{I} \to \mathcal{P}(\mathcal{I})] \\ \mathbb{P} \to [\mathcal{I} \to \mathcal{P}(\mathcal{I}^n)] \end{cases} \tag{2}$$

### 3.3 Meaning

As introduced in the previous section, the meaning of a non-logical symbol does not depend on the context under which the truth of the statement in which it occurs is being considered. We consider that the meaning of a term is a subset of $\mathcal{I}$ that contains all the images that term can possibly denote. This is the set of possible counterparts [13] that term stands for. In the same way, the meaning of a predicate symbol is defined as a subset of $\mathcal{I}^n$. The meaning of those non-logical symbols that are not included in the vocabulary of an agent will be equivalent to the empty set. We will refer to these symbols as meaningless.

On the other hand, the meaning assigned to a constant or a predicate will vary depending on whether the sentence is being asserted using the terms of one context or the terms of other. This is the reason why the introduction of quotation marks is important. A context will have to quote the report of a sentence in order to dissociate itself from the meaning given to the non-logical symbols in that sentence. This resolves the problems with statements containing ambiguous references [9].

**Assignment.** We proceed to define the assignment function in our logic.

**Definition 4.** *If $x$ is a variable of any sort, an assignment into $\mathfrak{M}$ is a function $\varphi$ such that $\varphi(x)$ is a subset of $\mathcal{I}$.*

$$\varphi : \mathbb{V} \rightarrow [\mathcal{I} \rightarrow \mathcal{P}(\mathcal{I})] \tag{3}$$

It will be useful to introduce the concept of $x$-variant assignment for the characterization of the truth function that will be presented in the next section.

**Definition 5.** *An assignment $\psi$ is an $x$-variant of an assignment $\varphi$ if $\varphi$ and $\psi$ agree on all variables except possibly $x$.*

**Valuation.** As usual we will define the valuation of the non-logical symbols of our logic in terms of the assignment and meaning functions. However, as we have mentioned before, the valuation of the terms of the logic will not yet result in the denotation of these, because the latter is relativized to the context under which the truth of a particular formula is being considered. We will define this notion more formally in the next section.

**Definition 6.** *Given a model $\mathfrak{M} = \langle \mathcal{I}, \preccurlyeq_g, \preccurlyeq_f, \Omega, \mathcal{M} \rangle$, an assignment $\varphi$ and a context $\Delta$ member of $\mathcal{I}$, a valuation $\mathcal{V}^{\mathfrak{M}}_{\varphi,\Delta}$ of the non-logical symbols of our language into $\mathfrak{M}$ under $\varphi$ and in terms of $\Delta$ is defined as follows:*

1. $\mathcal{V}^{\mathfrak{M}}_{\varphi,\Delta}(t) = \varphi(t)(\Delta)$ *if $t$ is a variable.*
2. $\mathcal{V}^{\mathfrak{M}}_{\varphi,\Delta}(t) = \mathcal{M}(t)(\Delta)$ *if $t$ is a constant of any sort.*
3. $\mathcal{V}^{\mathfrak{M}}_{\varphi,\Delta}(P^n) = \mathcal{M}(P^n)(\Delta)$ *if $P^n$ is an $n$-ary predicate.*

**Meaningful Formula.** We do not need to check the truth of a formula with regard to a context in order to know whether it is meaningful or not. This will only depend on the valuation of the terms and predicates it contains. Informally, we will say that a sentence is meaningful with regard to a model constructed by an agent if this agent can recreate some image for each of the terms in the sentence and at least one of these images is included in the set to which she would attribute the predicate in question. For example, let us take the sentence "the smell of your jacket is red". If "the smell of your jacket" and "red" are interpreted as they are usually in English, an English speaking agent will not be able to recreate an image of the smell of someone's jacket that is included in the set of images to which she would attribute the red colour. Therefore we will say that this sentence is meaningless for that agent. Below is presented a more formal definition of meaningful formula:

**Definition 7.** *An atomic wff expressing the $P^n$-ness of a sequence of terms $t_1, \ldots, t_n$ in terms of a context $\Delta$ is said to be meaningful in a model $\mathfrak{M}$ if and only if there exists some assignment $\varphi$ such that the cardinality of the intersection of the cartesian product of the valuations of $t_1, \ldots, t_n$ under $\varphi$ in terms of $k$ and the valuation of $P^n$ under $\varphi$ in terms of $\Delta$ is equal or greater than one.*

$$\begin{aligned} &P^n\left(t_1, \ldots, t_n\right) \text{ is a meaningful formula iff} \\ &\quad \left|\left[\mathcal{V}_{\varphi,\Delta}^{\mathfrak{M}}\left(t_1\right) \times \cdots \times \mathcal{V}_{\varphi,\Delta}^{\mathfrak{M}}\left(t_n\right)\right] \cap \mathcal{V}_{\varphi,\Delta}^{\mathfrak{M}}\left(P^n\right)\right| \geq 1 \end{aligned} \tag{4}$$

Note that the condition that the valuation of each of the non-logical symbols included in the sentence must be different from the empty set is implicit in this definition. Therefore, if a sentence is to be meaningful in a model, all of its non-logical symbols must be meaningful in that model as well.

This definition can be extended to sentences expressing the parthood or identity relation between two terms. The set of meaningful formulas will be trivially defined by induction on their construction.

### 3.4 Truth

The meaning of the non-logical symbols of a formula cannot determine by itself its truth value. In our logic the truth value of a formula is relativized to the context in which it is asserted. As we have said in the definition of a model in our logic, the image of a context supports the set of facts that are supported by its parts. Therefore, the first requisite for a formula to be supported by a context is that at least one counterpart of each of the terms of that sentence is part of the image of that context. On the other hand, one and only one counterpart can be part of the image of the same context or otherwise the term will be an ambiguous designator in that context. In this section, we define in what conditions a term succeeds to denote when it is used in a particular context and how the truth function is characterized according to this definition of denotation.

**Denotation.** Below we define formally the conditions under which a term succeeds to denote when considered under a certain context.

**Definition 8.** *A term $t$ succeeds to denote into a model $\mathfrak{M}$ under an assignment $\varphi$ in terms of a context $\Delta$ when considered under a context $\Gamma$ if and only if the cardinality of the intersection of the valuation of $t$ under $\varphi$ in terms of $\Delta$ with the grounded part-expansion of $\Gamma$, if $t$ is of the discourse sort, or the figured part-expansion of $\Gamma$, if $t$ is of the context sort, is a singleton.*

$$\begin{cases} \left|\mathcal{V}^{\mathfrak{M}}_{\varphi,\Delta}(t) \cap \downarrow_g \Gamma\right| = 1 & \textit{if } t \textit{ is of the discourse sort} \\ \left|\mathcal{V}^{\mathfrak{M}}_{\varphi,\Delta}(t) \cap \downarrow_f \Gamma\right| = 1 & \textit{if } t \textit{ is of the context sort} \end{cases} \tag{5}$$

**Definition 9.** *A term $t$ fails to denote into a model $\mathfrak{M}$ under an assignment $\varphi$ in terms of a context $\Delta$ when considered under a context $\Gamma$ if and only if the cardinality of the intersection of the valuation of $t$ under $\varphi$ in terms of $\Delta$ with the grounded part-expansion of $\Gamma$, if $t$ is of the discourse sort, or the figured part-expansion of $\Gamma$, if $t$ is of the context sort, is zero.*

$$\begin{cases} \left|\mathcal{V}^{\mathfrak{M}}_{\varphi,\Delta}(t) \cap \downarrow_g \Gamma\right| = 0 & \textit{if } t \textit{ is of the discourse sort} \\ \left|\mathcal{V}^{\mathfrak{M}}_{\varphi,\Delta}(t) \cap \downarrow_f \Gamma\right| = 0 & \textit{if } t \textit{ is of the context sort} \end{cases} \tag{6}$$

**Definition 10.** *A term $t$ is an ambiguous designator into a model $\mathfrak{M}$ under an assignment $\varphi$ in terms of a context $\Delta$ when considered under a context $\Gamma$ if and only if the cardinality of the intersection of the valuation of $t$ under $\varphi$ in terms of $\Delta$ with the grounded part-expansion of $\Gamma$, if $t$ is of the discourse sort, or the figured part-expansion of $\Gamma$, if $t$ is of the context sort, is greater that one.*

$$\begin{cases} \left|\mathcal{V}^{\mathfrak{M}}_{\varphi,\Delta}(t) \cap \downarrow_g \Gamma\right| > 1 & \textit{if } t \textit{ is of the discourse sort} \\ \left|\mathcal{V}^{\mathfrak{M}}_{\varphi,\Delta}(t) \cap \downarrow_f \Gamma\right| > 1 & \textit{if } t \textit{ is of the context sort} \end{cases} \tag{7}$$

If a term succeeds to denote then we will say that it denotes that image that, at the same time, is in its meaning and is part of the image of the context in consideration. This is formally defined below.

**Definition 11.** *If a term $t$ succeeds to denote into a model $\mathfrak{M}$ under an assignment $\varphi$ in terms of a context $\Delta$ when considered under a context $\Gamma$, its denotation $\mathcal{V}^{\mathfrak{M},\Gamma}_{\varphi,k}(t)$ into $\mathfrak{M}$ under $\varphi$ in terms of $\Delta$ when considered under $\Gamma$ is that unique element that is member of the intersection of the valuation of $t$ under $\varphi$ in terms of $\Delta$ with the grounded part-expansion of $\Gamma$, if $t$ is of the discourse sort, or the figured part-expansion of $\Gamma$, if $t$ is of the context sort.*

$$\mathcal{V}^{\mathfrak{M},\Gamma}_{\varphi,\Delta}(t) =_{def} (\iota x) \begin{cases} x \in \left[\mathcal{V}^{\mathfrak{M}}_{\varphi,\Delta}(t) \cap \downarrow_g \Gamma\right] & \textit{if } t \textit{ is of the discourse sort,} \\ x \in \left[\mathcal{V}^{\mathfrak{M}}_{\varphi,\Delta}(t) \cap \downarrow_f \Gamma\right] & \textit{if } t \textit{ is of the context sort.} \end{cases} \tag{8}$$

**Truth Function.** Once the conditions under which a term succeeds to denote and the value that takes its denotation have been defined, we can proceed to characterize the truth function on a model $\mathfrak{M}$ by induction on the construction of the wffs of our logic.

**Definition 12.** *Truth ($\Vdash$), with respect to an assignment $\varphi$ into a model $\mathfrak{M} = \langle \mathcal{I}, \preccurlyeq_g, \preccurlyeq_f, \Omega, \mathcal{M} \rangle$, is characterized as follows:*

1. *A context $\Gamma$ included in the imagery $\mathcal{I}$ of a model $\mathfrak{M}$ supports the assertion [internal negation] of the $P^n$-ness of a sequence of terms $t_1, \ldots, t_n$ under an assignment $\varphi$ in terms of a context $\Delta$ included in $\mathcal{I}$ if and only if every term $t_1, \ldots, t_n$ succeeds to denote under $\varphi$ in terms of $\Delta$ when considered under $\Gamma$ and the tuple formed by the denotations of $t_1, \ldots, t_n$ under $\varphi$ in terms of $\Delta$ when considered under $\Gamma$ belongs to [the complement of] the valuation of $P^n$ under $\varphi$ in terms of $\Delta$.*

$$\begin{aligned} &\mathfrak{M}, \Gamma \Vdash_{\varphi,\Delta} P^n\left(t_1, \ldots, t_n\right) \text{ iff} \\ &\quad t_1, \ldots, t_n \text{ succeed to denote under } \varphi \text{ in terms of } \Delta \text{ in } \Gamma \\ &\quad \text{and } \langle \mathcal{V}^{\mathfrak{M},\Gamma}_{\varphi,\Delta}\left(t_1\right), \ \ldots \ , \mathcal{V}^{\mathfrak{M},\Gamma}_{\varphi,\Delta}\left(t_n\right) \rangle \in \mathcal{V}^{\mathfrak{M}}_{\varphi,\Delta}\left(P^n\right) \end{aligned} \tag{9}$$

$$\begin{aligned} &\mathfrak{M}, \Gamma \Vdash_{\varphi,\Delta} \overline{P^n}\left(t_1, \ldots, t_n\right) \text{ iff} \\ &\quad t_1, \ldots, t_n \text{ succeed to denote under } \varphi \text{ in terms of } \Delta \text{ in } \Gamma \\ &\quad \text{and } \langle \mathcal{V}^{\mathfrak{M},\Gamma}_{\varphi,\Delta}\left(t_1\right), \ \ldots \ , \mathcal{V}^{\mathfrak{M},\Gamma}_{\varphi,\Delta}\left(t_n\right) \rangle \in \left[\mathcal{V}^{\mathfrak{M}}_{\varphi,\Delta}\left(P^n\right)\right]^C \end{aligned} \tag{10}$$

2. *A context $\Gamma$ included in the imagery $\mathcal{I}$ of a model $\mathfrak{M}$ supports the assertion of the identity relation between two terms $t_1$ and $t_2$ under an assignment $\varphi$ in terms of a context $\Delta$ included in $\mathcal{I}$ if and only if $t_1$ and $t_2$ succeed to denote under $\varphi$ in terms of $\Delta$ when considered under $\Gamma$ and the denotations of $t_1$ and $t_2$ under $\varphi$ in terms of $\Delta$ when considered under $\Gamma$ are equal.*

$$\begin{aligned} &\mathfrak{M}, \Gamma \Vdash_{\varphi,\Delta} t_1 = t_2 \text{ iff} \\ &\quad t_1 \text{ and } t_2 \text{ are proper descriptions under } \varphi \text{ in terms of } \Delta \text{ in } \Gamma \\ &\quad \text{and } \mathcal{V}^{\mathfrak{M},\Gamma}_{\varphi,\Delta}\left(t_1\right) = \mathcal{V}^{\mathfrak{M},\Gamma}_{\varphi,\Delta}\left(t_2\right) \end{aligned} \tag{11}$$

3. *A context $\Gamma$ included in the imagery $\mathcal{I}$ of a model $\mathfrak{M}$ supports the assertion of the grounded[figured] parthood relation of a term $t_1$ into a term $t_2$ under an assignment $\varphi$ in terms of a context $\Delta$ included in $\mathcal{I}$ if and only if $t_1$ and $t_2$ succeed to denote under $\varphi$ in terms of $\Delta$ when considered under $\Gamma$ and the denotation of $t_1$ under $\varphi$ in terms of $\Delta$ when considered under $\Gamma$ is a grounded[figured] part of the denotation of $t_2$ under $\varphi$ in terms of $\Delta$ when*

*considered under* $\Gamma$.

$$\begin{aligned}&\mathfrak{M},\Gamma \Vdash_{\varphi,\Delta} t_1 \leq_g t_2 \text{ iff}\\ &\quad t_1 \text{ and } t_2 \text{ succeed to denote under } \varphi \text{ in terms of } \Delta \text{ in } \Gamma\\ &\quad \text{and } \mathcal{V}^{\mathfrak{M},\Gamma}_{\varphi,\Delta}(t_1) \preccurlyeq_g \mathcal{V}^{\mathfrak{M},\Gamma}_{\varphi,\Delta}(t_2)\end{aligned} \tag{12}$$

$$\begin{aligned}&\mathfrak{M},\Gamma \Vdash_{\varphi,\Delta} t_1 \leq_f t_2 \text{ iff}\\ &\quad t_1 \text{ and } t_2 \text{ succeed to denote under } \varphi \text{ in terms of } \Delta \text{ in } \Gamma\\ &\quad \text{and } \mathcal{V}^{\mathfrak{M},\Gamma}_{\varphi,\Delta}(t_1) \preccurlyeq_f \mathcal{V}^{\mathfrak{M},\Gamma}_{\varphi,\Delta}(t_2)\end{aligned} \tag{13}$$

4. *A context* $\Gamma$ *included in the imagery* $\mathcal{I}$ *of a model* $\mathfrak{M}$ *supports the external negation of a formula* $A$ *under an assignment* $\varphi$ *in terms of a context* $\Delta$ *included in* $\mathcal{I}$ *if and only if it does not support* $A$ *under* $\varphi$ *in terms of* $\Delta$.

$$\mathfrak{M},\Gamma \Vdash_{\varphi,\Delta} \neg A \text{ iff not } \mathfrak{M},\Gamma \Vdash_{\varphi,\Delta} A \tag{14}$$

5. *The following clauses are defined as usual.*

$$\mathfrak{M},\Gamma \Vdash_{\varphi,\Delta} A \wedge B \text{ iff } \mathfrak{M},\Gamma \Vdash_{\varphi,\Delta} A \text{ and } \mathfrak{M},\Gamma \Vdash_{\varphi,\Delta} B \tag{15}$$

$$\mathfrak{M},\Gamma \Vdash_{\varphi,\Delta} A \vee B \text{ iff } \mathfrak{M},\Gamma \Vdash_{\varphi,\Delta} A \text{ or } \mathfrak{M},\Gamma \Vdash_{\varphi,\Delta} B \tag{16}$$

$$\mathfrak{M},\Gamma \Vdash_{\varphi,\Delta} A \supset B \text{ iff not } \mathfrak{M},\Gamma \Vdash_{\varphi,\Delta} A \text{ or } \mathfrak{M},\Gamma \Vdash_{\varphi,\Delta} B \tag{17}$$

6. *A context* $\Gamma$ *included in the imagery* $\mathcal{I}$ *of a model* $\mathfrak{M}$ *supports the universal quantification of a variable* $x$ *in a formula* $A$ *under an assignment* $\varphi$ *in terms of a context* $\Delta$ *included in* $\mathcal{I}$ *if and only if* $\Gamma$ *supports* $A$ *for every* $x$*-variant assignment* $\psi$ *under which* $x$ *succeeds to denote in terms of* $\Delta$ *when considered under* $\Gamma$.

$$\begin{aligned}&\mathfrak{M},\Gamma \Vdash_{\varphi,\Delta} (\forall x)[A] \text{ iff}\\ &\quad \text{for every } x\text{-variant assignment } \psi, \text{ if } x \text{ succeeds to denote}\\ &\quad \text{under } \psi \text{ in terms of } \Delta \text{ in } \Gamma \text{ then } \mathfrak{M},\Gamma \Vdash_{\psi,\Delta} A\end{aligned} \tag{18}$$

7. *A context* $\Gamma$ *included in the imagery* $\mathcal{I}$ *of a model* $\mathfrak{M}$ *supports the existential quantification of a variable* $x$ *in a formula* $A$ *under an assignment* $\varphi$ *in terms of a context* $\Delta$ *included in* $\mathcal{I}$ *if and only if* $\Gamma$ *supports* $A$ *under some* $x$*-variant assignment* $\psi$ *under which* $x$ *succeeds to denote in terms of* $\Delta$ *when considered under* $\Gamma$.

$$\begin{aligned}&\mathfrak{M},\Gamma \Vdash_{\varphi,\Delta} (\exists x)[A] \text{ iff}\\ &\quad \text{for some } x\text{-variant assignment } \psi,\\ &\quad x \text{ succeeds to denote under } \psi \text{ in terms of } \Delta \text{ in } \Gamma\\ &\quad \text{and } \mathfrak{M},\Gamma \Vdash_{\psi,\Delta} A\end{aligned} \tag{19}$$

8. *A model* $\mathfrak{M}$ *supports a formula* $A$ *under an assignment* $\varphi$ *if and only if the image of actuality* $\Omega$ *in* $\mathfrak{M}$ *supports* $A$ *under an assignment* $\varphi$ *in terms of* $\Omega$.

$$\mathfrak{M} \Vdash_{\varphi} A \text{ iff } \mathfrak{M},\Omega \Vdash_{\varphi,\Omega} A \tag{20}$$

9. *A context $\Gamma$ included in the imagery $\mathcal{I}$ of a model $\mathfrak{M}$ supports a formula $A$ contextualized in a context $k$ under an assignment $\varphi$ in terms of a context $\Delta$ included in $\mathcal{I}$ if and only if $k$ succeeds to denote under $\varphi$ in terms of $\Delta$ when considered under $\Gamma$ and its denotation under $\varphi$ in terms of $\Delta$ when considered under $\Gamma$ supports $A$ under $\varphi$ in terms of $\Delta$.*

$$\begin{aligned} &\mathfrak{M}, \Gamma \Vdash_{\varphi,\Delta} k : A \textit{ iff} \\ &\quad k \textit{ succeeds to denote under } \psi \textit{ in terms of } \Delta \textit{ in } \Gamma \\ &\quad \textit{and } \mathfrak{M}, \mathcal{V}^{\mathfrak{M},\Gamma}_{\varphi,\Delta}(k) \Vdash_{\varphi,\Delta} A \end{aligned} \tag{21}$$

10. *A context $\Gamma$ included in the imagery $\mathcal{I}$ of a model $\mathfrak{M}$ supports the quotation a formula $A$ contextualized in a context $\Delta$ under an assignment $\varphi$ in terms of a context $\Delta$ included in $\mathcal{I}$ if and only if $\Delta$ succeeds to denote under $\varphi$ in terms of $\Delta$ when considered under $\Gamma$ and its denotation under $\varphi$ in terms of $\Delta$ when considered under $\Gamma$ supports $A$ under $\varphi$ in terms of its denotation under $\varphi$ in terms of $\Delta$ when considered under $\Gamma$.*

$$\begin{aligned} &\mathfrak{M}, \Gamma \Vdash_{\varphi,\Delta} k : \text{“}A\text{”} \textit{ iff} \\ &\quad k \textit{ succeeds to denote under } \psi \textit{ in terms of } \Delta \textit{ in } \Gamma \\ &\quad \textit{and } \mathfrak{M}, \mathcal{V}^{\mathfrak{M},\Gamma}_{\varphi,\Delta}(k) \Vdash_{\varphi,\mathcal{V}^{\mathfrak{M},\Gamma}_{\varphi,\Delta}(k)} A \end{aligned} \tag{22}$$

**Definition 13.** *A formula $A$ is said to be valid if and only if it is supported by every image $\Gamma$ of every model $\mathfrak{M}$ under every assignment $\varphi$ and in terms of every context $\Delta$.*

$$\Vdash A \textit{ iff } (\forall \mathfrak{M}) (\forall \Gamma) (\forall \varphi) (\forall \Delta) [\mathfrak{M}, \Gamma \Vdash_{\varphi,\Delta} A] \tag{23}$$

As can be appreciated from the definition of the truth function, the principle of bivalence holds with regard to both external and internal negations. However, the bivalence with respect to the internal negation of a formula under a certain context holds if and only if all its terms succeed to denote when considered under that context and the sentence is meaningful, while the bivalence with regard to the external negation of a formula holds regardless of these conditions. Therefore the validity of the principle of bivalence can only be determined locally in the case of internal negations. This differentiation resolves how to deal with foreign languages [7] in the quantificational case. Concretely, this means that if a sentence is expressed in a language different from the one an agent knows, then the actuality constructed by this agent will not support that sentence neither its internal negation. Below are formally expressed the principles of bivalence with regard to both kinds of negation.

$$\begin{aligned} &\mathfrak{M}, \Gamma \Vdash_{\varphi,\Delta} P^n(t_1, \ldots, t_n) \vee \overline{P^n}(t_1, \ldots, t_n) \text{ iff} \\ &\quad t_1, \ldots, t_n \text{ are proper descriptions under } \varphi \text{ in terms of } \Delta \text{ in } \Gamma \\ &\quad \text{and } P^n(t_1, \ldots, t_n) \text{ is a meaningful sentence.} \end{aligned} \tag{24}$$

$$\Vdash A \vee \neg A \quad \text{in any case.} \tag{25}$$

On the other hand, in line with Modal Realism [13], we treat the universal quantifier as implicitly ranging over actuality. Therefore, as can be seen in the equation (16), only those assignments under which $x$ succeeds to denote in the context in consideration are required to validate the quantified formula.

The equations (18) and (19) show how this semantics facilitates entering into an inner context from a relative actuality and reversely transcending back from it. As can be seen in the equation (19), when entering a context the use of quotation marks entails the change of the context in terms of which the non-logical terms are valuated by the one in which we are entering.

## 4 Conclusions

In this paper we have presented a formal semantics for a logic of context that is inspired by a quasi-pictorial theory of Mental Imagery [11], which is a very active research area in the disciplines of Cognitive Science and Experimental Psychology. The semantics we have elaborated not only addresses how to interpret the reasoning between contexts but also increases the expressivity of previous logics of context by adding some new constructors to the set of logical symbols. Among these are the quotation marks that, like in natural language, enable an agent to use the terms in which another agent expresses herself and the parthood relation, which results very useful when formalizing normalcy assumptions between contexts. On the other hand, we have shown a characterization of the truth function that allows the differentiation between external and internal negations [12], what is necessary in order to adjust the principle of bivalence to the case of meaningless sentences or foreign languages [7].

Besides, this semantics has proved to overcome some unjustified restrictions that were imposed by previous quantificational logics of context [6], like flatness or the use of constant domains among others. This makes our logic more intuitively appropriate for accommodating the concept of context that Guha and McCarthy restated in [9].

From the point of view of the Philosophy of Language, we have elaborated a theory of meaning that provides a novel solution to the classical problems of meaningless sentences, designation and existence. The separation between meaning and truth that we have formalized allows to identify these cases and to deal with them adequately when it comes to evaluate the truth value of the sentences of our language.

At the moment of writing this paper, we are looking into a complete and sound axiomatization that allows to give a definition of derivability adequate for our logic. We also plan to research into how to accommodate temporal concepts, like events or actions, in this formalism.

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