Distributed SINR Balancing for MISO Downlink Systems via the Alternating Direction Method of Multipliers

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***Abstract*—We provide a distributed algorithm for the radio resource allocation problem in multicell downlink multi-input single-output systems. Specifically, we consider the problem of signal-to-interference-plus-noise ratio (SINR) balancing subject to total transmit power constraints. We propose a consensus-based distributed algorithm, and a fast solution method via alternating direction method of multipliers. Numerical results show that the proposed distributed algorithm converges to the optimal solution.**

***Index Terms*—Distributed optimization, multicell networks, signal-to-interference-plus-noise ratio balancing, alternating direction method of multipliers (ADMM), second-order cone program (SOCP).**

I. Introduction

We provide a distributed algorithm for the signal-to-interference-plus-noise ratio (SINR) balancing problem subject to total transmit power constraints at the base stations (BS), for multicell downlink systems with linear precoding. The BSs are assumed to have multiple antennas while all the receivers are equipped with single antenna.

Centralized methods for the SINR balancing problem has been proposed in [1]–[5]. Unfortunately, the centralized method is not practical for the resource allocation due to high overhead required for collecting all channel state information at the central processing unit. Therefore, to share the workload of the central controller and to overcome impelling backhaul the distributed algorithm are more desirable in practice.

The considered problem is quasiconvex [6]. Thus centralized methods based on bisection search [7] are commonly used, e.g. [4], [6]. By combining the bisection search and the uplink-downlink SINR duality, a distributed algorithm for multi-input single-output (MISO) system is proposed in [8]. The algorithm in [8] is a hierarchical iterative method which consists of outer and inner iterations, where the bisection search is carried out in the outer iteration and the uplink-downlink SINR duality is used for the inner iteration.

The main contribution of our paper is to propose a consensus-based distributed algorithm, and a fast solution method via alternating direction method of multipliers (ADMM) [9]. The ADMM turns the original problem into a series of iterative steps, namely, *local variable* update, *global variable* update, and *dual variable* update [9]. The local variable and dual variable updates are carried out independently in parallel by all BSs, while the global variable update is carried out by BSs coordination.

The remainder of this paper is organized as follows. The considered MISO system model and problem formulation are described in Section II. The distributed algorithm is derived in Section III. The numerical results are presented in Section IV, and Section V concludes our paper.

Notations: All boldface lower case and upper case letters represent vectors and matrices respectively, and calligraphy letters represent sets. The notation $\mathbb{C}^T$ denotes the set of complex T-vectors, $|x|$ denotes the absolute value of the scalar x, $\|\mathbf{x}\|_2$ denote the Euclidean norm of the vector $\mathbf{x}$, $\mathbf{I}$ denotes the identity matrix, and $\mathcal{CN}(\mathbf{m}, \mathbf{C})$ denotes the complex circular symmetric Gaussian vector distribution with mean $\mathbf{m}$ and covariance matrix $\mathbf{C}$. The superscript $(\cdot)^{\mathrm{H}}$ and $(\cdot)^\star$ is used to denote a Hermitian transpose of a matrix and a solution of an optimization problem, respectively.

II. System Model and Problem Formulation

A multicell MISO downlink system, with N base stations each equipped with T transmit antennas is considered. The set of all BSs is denoted by $\mathcal{N}$, and we label them with the integer values $n = 1, \ldots, N$. The *transmission region* of each BS is modeled as a disc with radius R_{BS} centered at the location of the BS (see Figure 1). Single data stream is transmitted for each user. We denote the set of all data streams in the system by $\mathcal{L}$, and label them with the integer values $l = 1, \ldots, L$. The transmitter node (i.e., the BS) of lth data stream is denoted by $tran(l)$ and the receiver node of lth data stream is denoted by $rec(l)$. We have $\mathcal{L} = \cup_{n\in\mathcal{N}}\mathcal{L}(n)$, where $\mathcal{L}(n)$ denotes the set of data streams transmitted by nth BS.

The antenna signal vector transmitted by nth BS is given by

$$\mathbf{x}_n = \sum_{l\in\mathcal{L}(n)} d_l \mathbf{m}_l, \qquad (1)$$

where $d_l \in \mathbb{C}$ and $\mathbf{m}_l \in \mathbb{C}^T$ represent the information symbol and the transmit beamformer associated to lth data stream, respectively. We assume that d_l is normalized such

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that $\mathrm{E}|d_l|^2 = 1$. Moreover, we assume data streams are independent, i.e., $\mathrm{E}\{d_l d_j^*\} = 0$ for all $l \neq j$, where $l, j \in \mathcal{L}$.

The signal received at $rec(l)$ can be expressed as

$$
\begin{aligned}
y_l \;&=\; d_l \mathbf{h}_{ll}^{\mathrm{H}} \mathbf{m}_l + \sum_{j \,\in\, \mathcal{L}(tran(l)),\, j \,\neq\, l} d_j \mathbf{h}_{jl}^{\mathrm{H}} \mathbf{m}_j \\
&\quad + \sum_{n \in \mathcal{N} \setminus \{tran(l)\}} \; \sum_{j \,\in\, \mathcal{L}(n)} d_j \mathbf{h}_{jl}^{\mathrm{H}} \mathbf{m}_j + n_l,
\end{aligned}
\tag{2}
$$

where $\mathbf{h}_{jl}^{\mathrm{H}} \in \mathbb{C}^{1\times T}$ is the channel matrix between $tran(j)$ and $rec(l)$, and n_l is circular symmetric complex gaussian noise with variance σ_l^2. Note that the second right hand term in (2) represents the intra-cell interference and the third right hand term represents the out-of-cell interference. The received SINR of lth data stream is given by

$$
\Gamma_l = \frac{|\mathbf{h}_{ll}^{\mathrm{H}} \mathbf{m}_l|^2}{\sigma_l^2 + \sum\limits_{j \,\in\, \mathcal{L}(tran(l)),\, j \,\neq\, l} |\mathbf{h}_{jl}^{\mathrm{H}} \mathbf{m}_j|^2 + \sum\limits_{n \in \mathcal{N} \setminus \{tran(l)\}} z_{nl}^2}, \tag{3}
$$

where $z_{nl}^2 = \sum_{j\in\mathcal{L}(n)} |\mathbf{h}_{jl}^{\mathrm{H}} \mathbf{m}_j|^2$ represents the power of the out-of-cell interference from nth BS to $rec(l)$.

The out-of-cell interference term in (3) (i.e., $\sum_{n\in\mathcal{N}\setminus\{tranl(l)\}} z_{nl}^2$) prevents resource allocation on an intra-cell basis and demands BSs cooperation/coordination. To avoid unnecessary coordination between far apart located BSs, we make the following assumption: transmission from nth BS *interfere* the lth data stream (transmitted by BS $b \neq n$) only if the distance between nth BS and $rec(l)$ is smaller than a threshold R_{int}[1]. The disc with radius R_{int} centered at the location of any BS is referred to as the *interference region* of the BS (see Figure 1). Thus, if nth BS located at a distance larger than R_{int} to $rec(l)$, the associated z_{nl} components are set to zero[2]. Based on the assumption above, we can express Γ_l as

$$
\Gamma_l = \frac{|\mathbf{h}_{ll}^{\mathrm{H}} \mathbf{m}_l|^2}{\sigma_l^2 + \sum\limits_{j \,\in\, \mathcal{L}(tran(l)),\, j \,\neq\, l} |\mathbf{h}_{jl}^{\mathrm{H}} \mathbf{m}_j|^2 + \sum\limits_{n\in\mathcal{N}_{\mathrm{int}}(l)} z_{nl}^2},
$$

where $\mathcal{N}_{\mathrm{int}}(l) \subseteq \mathcal{N}\setminus\{tran(l)\}$ is the set of out-of-cell interfering BSs that are located at a distance less than R_{int} to $rec(l)$. For example, in Figure 1, we have $\mathcal{N}_{\mathrm{int}}(2) = \{2\}$, $\mathcal{N}_{\mathrm{int}}(8) = \{1\}$, and $\mathcal{N}_{\mathrm{int}}(l) = \emptyset$ for all $l \in \{1, 3, 4, 5, 6, 7\}$.

Providing fairness among the users with per BS power constraint (i.e., $\sum\limits_{j\in\mathcal{L}(n)} \|\mathbf{m}_l\|_2^2 \leq p_n^{\max}$) is an important resource

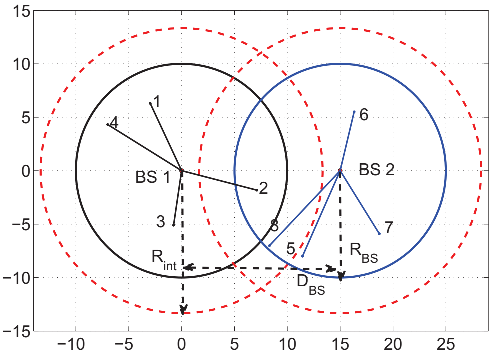


Fig. 1: Multicell network: $\mathcal{N} = \{1, 2\}$, $\mathcal{L}(1) = \{1, 2, 3, 4\}$, $\mathcal{L}(2) = \{5, 6, 7, 8\}$, $\mathcal{L}_{\mathrm{int}} = \{2, 8\}$. The area inside solid-lined circle around BSs represent the associated transmission region of each BS, and the area inside dash-lined circle around BSs represent the associated interference region of each BS.

allocation problem. One way[3] of providing fairness among the users is by maximizing the minimum SINR [6], which can be formulated as

$$
\begin{array}{ll}
\text{maximize} & \min\limits_{l\in\mathcal{L}} \left(\dfrac{|\mathbf{h}_{ll}^{\mathrm{H}} \mathbf{m}_l|^2}{\sigma_l^2 + \sum\limits_{\substack{j \,\in\, \mathcal{L}(tran(l)),\\ j \,\neq\, l}} |\mathbf{h}_{jl}^{\mathrm{H}} \mathbf{m}_j|^2 + \sum\limits_{n\in\mathcal{N}_{\mathrm{int}}(l)} z_{nl}^2} \right) \\
\text{subject to} & z_{nl}^2 = \sum\limits_{j\in\mathcal{L}(n)} |\mathbf{h}_{jl}^{\mathrm{H}} \mathbf{m}_j|^2, l \in \mathcal{L}_{\mathrm{int}}, n \in \mathcal{N}_{\mathrm{int}}(l) \\
& \sum\limits_{j\in\mathcal{L}(n)} \|\mathbf{m}_l\|_2^2 \leq p_n^{\max}, \quad n \in \mathcal{N},
\end{array}
\tag{4}
$$

with variables $\{\mathbf{m}_l\}_{l\in\mathcal{L}}$ and $\{z_{nl}\}_{l\in\mathcal{L}_{\mathrm{int}}, n\in\mathcal{N}_{\mathrm{int}}(l)}$, where $\mathcal{L}_{\mathrm{int}}$ denotes the set of all data streams that are subject to the out-of-cell interference, i.e., $\mathcal{L}_{\mathrm{int}} = \{l | l \in \mathcal{L}, \mathcal{N}_{\mathrm{int}}(l) \neq \emptyset\}$. Finally, to improve the readability of the paper we summarize a list of sets used in this paper in Table I.

Set	Description
$\mathcal{N}$	Set of all BSs
$\mathcal{L}$	Set of all data streams
$\mathcal{L}(n)$	Set of data streams transmitted by nth BS
$\mathcal{N}_{\mathrm{int}}(l)$	Set of out-of-cell BSs interfering to lth data stream
$\mathcal{L}_{\mathrm{int}}$	Set of all data streams that are subject to the out-of-cell interference
$\mathcal{I}_{\mathrm{int}}(n)$	Set of links for which BS n acts as the out-of-cell interferer
$\mathcal{L}_{\mathrm{int}}(n)$	Set of links in BS n that are affected by the out-of-cell interference

TABLE I: Summary of a list of sets.

[1]Similar assumptions are made, e.g., in [10] in the context of arbitrary wireless networks.

[2]The value of R_{int} is chosen such that the power of the interference term is below the noise level and this commonly used approximation is made to avoid unnecessary coordinations between distant BSs. The effect of nonzero z_{nl} terms can be accurately modeled by changing the statistical characteristics of noise n_l at $rec(l)$. However, those issues are extraneous to the main focus of the paper.

[3]A more general SINR balancing problem which can set priority of each users (keeping the SINR values of data streams to a fixed ratio) [11, Section IV-C] can be formulated. However to simplify the presentation, we consider maximization of the minimum SINR. Note that the proposed decentralized method can be easily generalized to the more general problem considered in [11, Section IV-C].

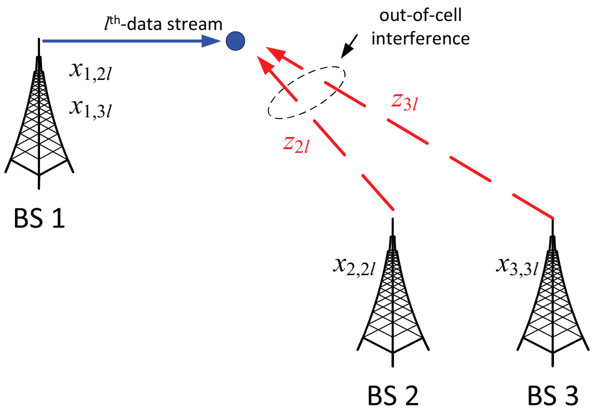


Fig. 2: BS 2 and BS 3 are coupled with BS 1 due to coupling variables z_{2l} and z_{3l}, respectively. To distribute the problem, local copy $x_{1,2l}$ of z_{2l} at BS 1 and local copy $x_{2,2l}$ of z_{2l} at BS 2 are introduced. Similarly, local copy $x_{1,3l}$ of z_{3l} at BS 1 and local copy $x_{3,3l}$ of z_{3l} at BS 3 are introduced.

III. SINR BALANCING

In this section we derive a distributed algorithm for problem (4). First, we equivalently reformulate the original problem (4) in a form of *global consensus* problem. Then, we derive the proposed distributed algorithm based on ADMM [9].

A. An equivalent reformulation

We start by reformulating SINR balancing problem (4) in the epigraph form as

$$\begin{array}{ll} \text{minimize} & -\gamma \\ \text{subject to} & \dfrac{|\mathbf{h}_{ll}^{\mathrm{H}}\mathbf{m}_l|^2}{\sigma_l^2 + \sum\limits_{j\,\in\,\mathcal{L}(t_l),\, j\neq l}|\mathbf{h}_{jl}^{\mathrm{H}}\mathbf{m}_j|^2 + \sum\limits_{n\in\mathcal{N}_{\text{int}}(l)} z_{nl}^2} \geq \gamma, \\ & \qquad\qquad\qquad\qquad l\in\mathcal{L} \\ & z_{nl}^2 \geq \sum\limits_{j\in\mathcal{L}(n)} |\mathbf{h}_{jl}^{\mathrm{H}}\mathbf{m}_j|^2, l\in\mathcal{L}_{\text{int}}, n\in\mathcal{N}_{\text{int}}(l) \\ & \sum\limits_{j\in\mathcal{L}(n)} \|\mathbf{m}_l\|_2^2 \leq p_n^{\max}, \quad n\in\mathcal{N}, \end{array} \tag{5}$$

where the variables are γ, $\{\mathbf{m}_l\}_{l\in\mathcal{L}}$, and $\{z_{nl}\}_{l\in\mathcal{L}_{\text{int}},n\in\mathcal{N}_{\text{int}}(l)}$. Problem (4) and (5) are equivalent as it can be easily shown (e.g., by contradiction) that the second inequality holds with equality at the optimal point.

Recall that z_{nl}^2 in problem (5) represents power of the out-of-cell interference caused by nth BS at $rec(l)$, and hence, variable z_{nl} couples exactly two BSs (i.e., BS n and BS $tran(l)$). Furthermore, the SINR variable γ coupes all BSs via SINR constraints. We use consensus technique to distribute problem (5) over the BSs. The method consist of introducing at each BS local copies of the coupling variables z_{nl} and γ.

Let us define $x_{k,nl}$ as the local copy of z_{nl} at BS k. Thus for each z_{nl}, we make two local copies, i.e., $x_{n,nl}$ at BS n and $x_{tran(l),nl}$ at BS $tran(l)$ (see Figure 2). Furthermore, let us define α_n as the local copy of γ at BS n. Then problem (5) can be written equivalently in a *global consensus* form as

$$\begin{array}{ll} \text{minimize} & -\gamma \\ \text{subject to} & \dfrac{|\mathbf{h}_{ll}^{\mathrm{H}}\mathbf{m}_l|^2}{\sigma_l^2 + \sum\limits_{j\,\in\,\mathcal{L}(n),\, j\neq l}|\mathbf{h}_{jl}^{\mathrm{H}}\mathbf{m}_j|^2 + \sum\limits_{b\in\mathcal{N}_{\text{int}}(l)} x_{n,bl}^2} \geq \alpha_n, \\ & \qquad\qquad\qquad\qquad n\in\mathcal{N}, l\in\mathcal{L}(n) \\ & x_{n,nl}^2 \geq \sum\limits_{j\in\mathcal{L}(n)} |\mathbf{h}_{jl}^{\mathrm{H}}\mathbf{m}_j|^2, l\in\mathcal{L}_{\text{int}}, n\in\mathcal{N}_{\text{int}}(l) \\ & \sum\limits_{j\in\mathcal{L}(n)} \|\mathbf{m}_l\|_2^2 \leq p_n^{\max}, \quad n\in\mathcal{N} \\ & x_{k,nl} = z_{nl}, \quad k\in\{n, tran(l)\}, l\in\mathcal{L}_{\text{int}}, \\ & \qquad\qquad\qquad n\in\mathcal{N}_{\text{int}}(l) \\ & \alpha_n = \gamma, \quad n\in\mathcal{N}, \end{array} \tag{6}$$

with variables γ, $\{\mathbf{m}_l\}_{l\in\mathcal{L}}$, $\{\alpha_n\}_{n\in\mathcal{N}}$, $\{x_{k,nl}\}_{k\in\{n,tran(l)\},l\in\mathcal{L}_{\text{int}},n\in\mathcal{N}_{\text{int}}(l)}$, and $\{z_{nl}\}_{l\in\mathcal{L}_{\text{int}},n\in\mathcal{N}_{\text{int}}(l)}$. Note that in the SINR constraints of problem (6), we replaced z_{bl} by the local copy $x_{n,bl}$, γ by the local copy α_n, and used $\mathcal{L} = \cup_{n\in\mathcal{N}}\mathcal{L}(n)$. In the second inequality constraints of (6) we replaced z_{nl} by the local copy $x_{n,nl}$. The set of equality constraints of (6) are called *consistency constraints*, and they enforce the local copies $\{x_{k,nl}\}_{k\in\{n,tran(l)\}}$ and $\{\alpha_n\}_{n\in\mathcal{N}}$ to be equal to the corresponding global variable z_{nl} and γ, respectively.

In problem (6) the first and the third set of inequality constraints are separable in $n\in\mathcal{N}$ (one for each BS). Also, it can be easily shown that the second set of inequality constraints of (6) are separable in $n\in\mathcal{N}$. To do this, let us denote $\mathcal{I}_{\text{int}}(n)$ the set of links for which BS n acts as an out-of-cell interferer, i.e., $\mathcal{I}_{\text{int}}(n) = \{l | l\in\mathcal{L}_{\text{int}}, n\in\mathcal{N}_{\text{int}}(l)\}$. Then, by noting that the sets $\{(n,l) | l\in\mathcal{L}_{\text{int}}, n\in\mathcal{N}_{\text{int}}(l)\}$ and $\{(n,l) | n\in\mathcal{N}, l\in\mathcal{I}_{\text{int}}(n)\}$ are identical, the second set of inequality constraints of (6) can be written as

$$x_{n,nl}^2 \geq \sum_{j\in\mathcal{L}(n)} |\mathbf{h}_{jl}^{\mathrm{H}}\mathbf{m}_j|^2, \quad n\in\mathcal{N}, l\in\mathcal{I}_{\text{int}}(n), \tag{7}$$

which is separable in $n\in\mathcal{N}$. Observe that without the consistency constraints, problem (6) can now be easily decoupled into N subproblems, one for each BS.

We next express problem (6) more compactly. To do this, we first express the consistency constraints of problem (6) more compactly by using vector notations, which denote the local and global variables associated with BS n. By using the equivalence between the sets $\{(n,l) | l\in\mathcal{L}_{\text{int}}, n\in\mathcal{N}_{\text{int}}(l)\}$ and $\{(n,l) | n\in\mathcal{N}, l\in\mathcal{I}_{\text{int}}(n)\}$, let us express the fourth constraints of problem (6) as

$$\begin{array}{lcll} x_{n,nl} & = & z_{nl}, & n\in\mathcal{N}, l\in\mathcal{I}_{\text{int}}(n) \\ x_{tran(l),nl} & = & z_{nl}, & l\in\mathcal{L}_{\text{int}}, n\in\mathcal{N}_{\text{int}}(l). \end{array} \tag{8}$$

In the first set of equalities of (8), $\{x_{n,nl}\}_{l\in\mathcal{I}_{\text{int}}(n)}$ is a set of local variables associated with BS n. Similarly, to find a set of local variables that are associated with BS n in the second set of equalities of (8), let us define $\mathcal{L}_{\text{int}}(n)$ the set of links in BS n that are affected by the out-of-cell interference, i.e., $\mathcal{L}_{\rm int}(n) = \{l | l \in \mathcal{L}_{\rm int} \cap \mathcal{L}(n)\}$. Then by noting $\mathcal{L}_{\rm int} = \cup_{n\in\mathcal{N}}\mathcal{L}_{\rm int}(n)$ we can rewrite (8) as

$$\begin{array}{rcll} x_{n,nl} & = & z_{nl}, & n \in \mathcal{N}, l \in \mathcal{I}_{\rm int}(n) \\ x_{tran(l),bl} & = & z_{bl}, & n \in \mathcal{N}, l \in \mathcal{L}_{\rm int}(n), b \in \mathcal{N}_{\rm int}(l). \end{array} \tag{9}$$

Clearly, in the second set of equalities of (9) [4], $\{x_{tran(l),bl}\}_{l\in\mathcal{L}_{\rm int}(n), b\in\mathcal{N}_{\rm int}(l)}$ is a set of local variables that is associated with BS n. We now denote (9) compactly using vector notation. Let us define vectors $\mathbf{x}_n$ and $\mathbf{z}_n$ as [5]

$$\begin{array}{rcl} \mathbf{x}_n & = & \{\{x_{n,nl}\}_{l\in\mathcal{I}_{\rm int}(n)}, \{x_{tran(l),bl}\}_{l\in\mathcal{L}_{\rm int}(n), b\in\mathcal{N}_{\rm int}(l)}\} \\ \mathbf{z}_n & = & \{\{z_{nl}\}_{l\in\mathcal{I}_{\rm int}(n)}, \{z_{bl}\}_{l\in\mathcal{L}_{\rm int}(n), b\in\mathcal{N}_{\rm int}(l)}\}. \end{array} \tag{10}$$

Then (9) can be compactly written as

$$\mathbf{x}_n = \mathbf{z}_n, \quad n \in \mathcal{N}. \tag{11}$$

Note that $\mathbf{x}_n$ is a collection of the local variables that are associated with BS n, and $\mathbf{z}_n$ is a collection of the global variables that are associate with the components of variable $\mathbf{x}_n$.

Furthermore, for the sake of brevity, let us define the matrix $\mathbf{M}_n = [\mathbf{m}_l]_{l\in\mathcal{L}(n)}$ obtained by concatenating the column vectors $\mathbf{m}_l$, the following set

$$\mathcal{C} = \left\{ \mathbf{M}_n, \mathbf{x}_n, \alpha_n \,\middle|\, \begin{array}{l} \dfrac{|\mathbf{h}_{ll}^{\rm H}\mathbf{m}_l|^2}{\sigma_l^2 + \sum\limits_{\substack{j\in\mathcal{L}(n)\\ j\neq l}} |\mathbf{h}_{jl}^{\rm H}\mathbf{m}_j|^2 + \sum\limits_{b\in\mathcal{N}_{\rm int}(l)} x_{n,bl}^2} \geq \alpha_n, \\ \hfill l \in \mathcal{L}(n) \\ x_{n,nl}^2 \geq \sum\limits_{j\in\mathcal{L}(n)} |\mathbf{h}_{jl}^{\rm H}\mathbf{m}_j|^2, l \in \mathcal{I}_{\rm int}(n) \\ \sum\limits_{j\in\mathcal{L}(n)} \|\mathbf{m}_l\|_2^2 \leq p_n^{\max} \end{array} \right\}, \tag{12}$$

and the following indicator function $I_n(\mathbf{M}_n, \mathbf{x}_n, \alpha_n)$

$$I_n(\mathbf{M}_n, \mathbf{x}_n, \alpha_n) = \begin{cases} 0 & (\mathbf{M}_n, \mathbf{x}_n, \alpha_n) \in \mathcal{C} \\ \infty & \text{otherwise.} \end{cases} \tag{13}$$

Then by using notations (11), (12) and (13) consensus problem (6) can be rewritten compactly as

$$\begin{array}{ll} \text{minimize} & -\gamma + \sum\limits_{n\in\mathcal{N}} I_n(\mathbf{M}_n, \mathbf{x}_n, \alpha_n) \\ \text{subject to} & \mathbf{x}_n = \mathbf{z}_n, \quad n \in \mathcal{N} \\ & \alpha_n = \gamma, \quad n \in \mathcal{N}, \end{array} \tag{14}$$

with variables γ and $\{\mathbf{M}_n, \mathbf{x}_n, \mathbf{z}_n, \alpha_n\}_{n\in\mathcal{N}}$. Furthermore, by noting that $\sum_{n\in\mathcal{N}} \alpha_n = N\gamma$ (from the second equality constraints of (14)), problem (14) can be equivalently expressed as

$$\begin{array}{ll} \text{minimize} & \sum\limits_{n\in\mathcal{N}} \Big(-\dfrac{\alpha_n}{N} + I_n(\mathbf{M}_n, \mathbf{x}_n, \alpha_n) \Big) \\ \text{subject to} & \mathbf{x}_n = \mathbf{z}_n, \quad n \in \mathcal{N} \\ & \alpha_n = \gamma, \quad n \in \mathcal{N}, \end{array} \tag{15}$$

with variables γ and $\{\mathbf{M}_n, \mathbf{x}_n, \mathbf{z}_n, \alpha_n\}_{n\in\mathcal{N}}$.

B. Distributed algorithm via ADMM

In this section we derive distributed algorithm for problem (15). The proposed algorithm is based on ADMM [9].

We start by writing the *augmented Lagrangian* [12] for problem (15) as

$$\begin{array}{l} L_\rho\big(\{\mathbf{M}_n, \mathbf{x}_n, \alpha_n, \mathbf{u}_n, v_n, \mathbf{z}_n\}_{n\in\mathcal{N}}, \gamma\big) = \\ \quad \sum\limits_{n\in\mathcal{N}} \Big(-\dfrac{\alpha_n}{N} + I_n(\mathbf{M}_n, \mathbf{x}_n, \alpha_n) + \mathbf{u}_n^T(\mathbf{x}_n - \mathbf{z}_n) \\ \qquad + v_n(\alpha_n - \gamma) + \dfrac{\rho}{2}\|\mathbf{x}_n - \mathbf{z}_n\|_2^2 + \dfrac{\rho}{2}\|\alpha_n - \gamma\|_2^2 \Big), \end{array} \tag{16}$$

where $\mathbf{u}_n$ [6] and v_n are the dual variables associated with the first and second equality constraints of (15), respectively, and $\rho > 0$ is a *penalty parameter* that adds the quadratic penalty to the standard Lagrangian L_0 for the violation of the equality constraints of problem (15).

Each iteration of ADMM [9] consists of the steps (17)-(20). Note that the first step in (17) is completely decentralized. Each BS $n \in \mathcal{N}$ updates the local variables $(\mathbf{M}_n^{i+1}, \mathbf{x}_n^{i+1}, \alpha_n^{i+1})$ by solving the following optimization problem

$$\begin{array}{ll} \text{minimize} & \Big(-\dfrac{\alpha_n}{N} + I_n(\mathbf{M}_n, \mathbf{x}_n, \alpha_n) + \mathbf{u}_n^{iT}(\mathbf{x}_n - \mathbf{z}_n^i) \\ & + v_n^i(\alpha_n - \gamma^i) + \dfrac{\rho}{2}\|\mathbf{x}_n - \mathbf{z}_n^i\|_2^2 + \dfrac{\rho}{2}\|\alpha_n - \gamma^i\|_2^2 \Big), \end{array} \tag{21}$$

with variables α_n, $\mathbf{M}_n$, and $\mathbf{x}_n$. Let $\mathbf{v}_n = (1/\rho)\mathbf{u}_n$ and $\lambda_n = (1/\rho)v_n$, then combining the linear and the quadratic terms[7],

[4] Note that $\mathcal{L}_{\rm int}(n) \subseteq \mathcal{L}(n)$. Hence, $tran(l) = n$ for all $l \in \mathcal{L}_{\rm int}(n)$.

[5] To simplify the presentation, here we have used a slight abuse of notation, i.e., we have considered that the sets in (10) are ordered.

[6] Let $\{u_{k,nl}\}_{k\in\{n,tran(l)\}, l\in\mathcal{L}_{\rm int}, n\in\mathcal{N}_{\rm int}(l)}$ be the dual variables associated with the fourth equality constraints of problem (6), then by following steps (8) to (10), we can easily express $\mathbf{u}_n = \{\{u_{n,nl}\}_{l\in\mathcal{I}_{\rm int}(n)}, \{u_{n,bl}\}_{l\in\mathcal{L}_{\rm int}(n), b\in\mathcal{N}_{\rm int}(l)}\}$ for all $n \in \mathcal{N}$.

[7] For convenience we can combine the terms in problem (21) as a) $\mathbf{u}_n^{iT}(\mathbf{x}_n - \mathbf{z}_n^i) + \frac{\rho}{2}\|\mathbf{x}_n - \mathbf{z}_n^i\|_2^2 = \frac{\rho}{2}\|\mathbf{x}_n - \mathbf{z}_n^i + \mathbf{v}_n^i\|_2^2 - \frac{\rho}{2}\|\mathbf{v}_n^i\|$ and b) $v_n^i(\alpha_n - \gamma^i) + \frac{\rho}{2}\|\alpha_n - \gamma^i\|_2^2 = \frac{\rho}{2}\|\alpha_n - \gamma^i + \lambda_n^i\|_2^2 - \frac{\rho}{2}\|\lambda_n^i\|$.

$$\mathbf{M}_n^{i+1}, \mathbf{x}_n^{i+1}, \alpha_n^{i+1} = \operatorname*{argmin}_{\mathbf{M}_n, \mathbf{x}_n, \alpha_n} L_\rho\big(\mathbf{M}_n, \mathbf{x}_n, \alpha_n, \mathbf{u}_n^i, v_n^i, \mathbf{z}_n^i, \gamma^i\big), \quad n \in \mathcal{N} \tag{17}$$

$$\{\mathbf{z}_n^{i+1}\}_{n\in\mathcal{N}}, \gamma^{i+1} = \operatorname*{argmin}_{\{\mathbf{z}_n\}_{n\in\mathcal{N}}, \gamma} L_\rho\big(\{\mathbf{M}_n^{i+1}, \mathbf{x}_n^{i+1}, \alpha_n^{i+1}, \mathbf{u}_n^i, v_n^i, \mathbf{z}_n\}_{n\in\mathcal{N}}, \gamma\big) \tag{18}$$

$$\mathbf{u}_n^{i+1} = \mathbf{u}_n^i + \rho\big(\mathbf{x}_n^{i+1} - \mathbf{z}_n^{i+1}\big), \quad n \in \mathcal{N} \tag{19}$$

$$v_n^{i+1} = v_n^i + \rho\big(\alpha_n^{i+1} - \gamma^{i+1}\big), \quad n \in \mathcal{N}. \tag{20}$$

problem (21) can be simplified as

$$\begin{array}{ll}\text{minimize} & -\frac{\alpha_n}{N} + I_n(\mathbf{M}_n, \mathbf{x}_n, \alpha_n) + \frac{\rho}{2}\|\mathbf{x}_n - \mathbf{z}_n^i + \mathbf{v}_n^i\|_2^2 \\ & +\frac{\rho}{2}\|\alpha_n - \gamma^i + \lambda_n^i\|_2^2,\end{array} \tag{22}$$

with variables α_n, $\mathbf{M}_n$, and $\mathbf{x}_n$. Note that in the objective function of (22) constant terms $\frac{\rho}{2}\|\mathbf{v}_n^i\|$ and $\frac{\rho}{2}\|\lambda_n^i\|$ are dropped, because they do not affect the solution of the optimization problem.

Problem (22) is not convex, due to the indicator function $I_n(\mathbf{M}_n, \mathbf{x}_n, \alpha_n)$ which is a function of nonconvex set $\mathcal{C}$ (see (12) and (13)). However, for fixed α_n the set $\mathcal{C}$ is convex, and hence problem (22) can be solved easily. Therefore, to solve problem (22), we first find the optimal $\alpha_n^\star$, and then find $\mathbf{M}_n^\star$ and $\mathbf{x}_n^\star$.

For fixed α_n, let us denote the optimal value function of problem (22) as

$$\begin{aligned} p(\alpha_n) &= \inf_{\mathbf{M}_n, \mathbf{x}_n} \Big(-\frac{\alpha_n}{N} + I_n(\mathbf{M}_n, \mathbf{x}_n, \alpha_n) \\ &\quad + \frac{\rho}{2}\|\mathbf{x}_n - \mathbf{z}_n^i + \mathbf{v}_n^i\|_2^2 + \frac{\rho}{2}\|\alpha_n - \gamma^i + \lambda_n^i\|_2^2) \Big) \\ &= \inf_{\mathbf{M}_n, \mathbf{x}_n} \Big(I_n(\mathbf{M}_n, \mathbf{x}_n, \alpha_n) + \frac{\rho}{2}\|\mathbf{x}_n - \mathbf{z}_n^i + \mathbf{v}_n^i\|_2^2 \Big) \\ &\quad -\frac{\alpha_n}{N} + \frac{\rho}{2}\|\alpha_n - \gamma^i + \lambda_n^i\|_2^2, \end{aligned} \tag{23}$$

where (23) follows by noting α_n/N and $\frac{\rho}{2}\|\alpha_n - \gamma^i + \lambda_n^i\|_2^2$ are constants. Then the optimal value of problem (22) is given by

$$p^\star \quad = \quad \inf_{\alpha_n} p(\alpha_n). \tag{24}$$

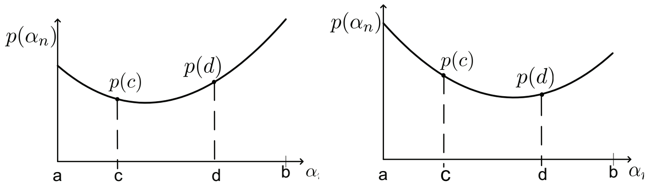
To solve (24), in *Lemma* 1, we first show that the optimal value function $p(\alpha_n)$ on an interval $\alpha_n \in [0, \alpha_n^{\max}]$ is an unimodal function, where $\alpha_n^{\max}$ is any arbitrary positive value. Then we propose a bracketing method (e.g., golden ratio search) [13], [14] to solve problem (24) on the interval $[0, \alpha_n^{\max}]$. For ease of presentation, let us express the optimal value function $p(\alpha_n)$ in (23) as

$$p(\alpha_n) \quad = \quad \tilde{p}(\alpha_n) - \frac{\alpha_n}{N} + \frac{\rho}{2}\|\alpha_n - \gamma^i + \lambda_n^i\|_2^2, \tag{25}$$

where $\tilde{p}(\alpha_n)$ is the optimal value of the following optimization problem

$$\begin{array}{ll}\text{minimize} & \frac{\rho}{2}\|\mathbf{x}_n - \mathbf{z}_n^i + \mathbf{v}_n^i\|_2^2 \\ \text{subject to} & \dfrac{|\mathbf{h}_{ll}^{\mathrm{H}}\mathbf{m}_l|^2}{\sigma_l^2 + \sum_{j \in \mathcal{L}(n), j \neq l} |\mathbf{h}_{jl}^{\mathrm{H}}\mathbf{m}_j|^2 + \sum_{b \in \mathcal{N}_{\mathrm{int}}(l)} x_{n,bl}^2} \geq \alpha_n, \\ & \hfill l \in \mathcal{L}(n) \\ & x_{n,nl}^2 \geq \sum_{j \in \mathcal{L}(n)} |\mathbf{h}_{jl}^{\mathrm{H}}\mathbf{m}_j|^2, \quad l \in \mathcal{I}_{\mathrm{int}}(n) \\ & \sum_{j \in \mathcal{L}(n)} \|\mathbf{m}_l\|_2^2 \leq p_n^{\max},\end{array} \tag{26}$$

with variables $\mathbf{x}_n$ and $\{\mathbf{m}_l\}_{l \in \mathcal{L}(n)}$. Note that to write (26), we have used the notations defined in (12) and (13).



(a) If $p(c) \leq p(d)$, then squeeze from the right and use [a,d].
(b) If $p(d) \leq p(c)$, then squeeze from the left and use [c,b].

Fig. 3: The decision process for finding $\alpha_n^\star$.

Lemma 1: The function $p(\alpha_n)$,

$$p(\alpha_n) \quad = \quad \tilde{p}(\alpha_n) - \frac{\alpha_n}{N} + \frac{\rho}{2}\|\alpha_n - \gamma^i + \lambda_n^i\|_2^2, \tag{27}$$

is an unimodal function on an interval $\alpha_n \in [0, \alpha_n^{\max}]$.

Proof: For any fixed $\alpha_n \in [0, \alpha_n^{\max}]$ the feasible set of problem (26) is closed and convex. Thus, problem (26) is a minimization of a quadratic function over a closed convex set. Therefore, $\tilde{p}(\alpha_n)$ is a continuous and piecewise quadratic function on the interval $\alpha_n \in [0, \alpha_n^{\max}]$ [15, Corollary 4.8]. It follows that $p(\alpha_n)$ is a sum of piecewise quadratic, affine, and quadratic functions. Therefore, function $p(\alpha_n)$ is piecewise convex, and hence, unimodal on the interval $[0, \alpha_n^{\max}]$. ■

In *Algorithm 2*, we summarize the bracketing method (golden ratio search) [13, Section 8.1] to find the optimal $\alpha_n^\star$ for problem (24).

Algorithm 1: Bracketing method to find optimal $\alpha_n^\star$ for problem (24)

1) Initialization: given SINR interval $[0, \alpha_n^{\max}]$, $r = (\sqrt{5}-1)/2$, and $\epsilon > 0$. Set $a = 0$, $b = \alpha_n^{\max}$, $c = ra+(1-r)b$, and $d = (1-r)a + rb$.
2) Compute $p(c)$ and $p(d)$ using (25).
3) Squeeze the search SINR range: if $p(c) \leq p(d)$, set $b = d$, else set $a = c$.
4) Compute $c = ra + (1-r)b$ and $d = (1-r)a + rb$.
5) Stopping criterion: if $b - a < \epsilon$, STOP, and set $\alpha_n^\star = c$. Otherwise, go to step 2.

Figure (3) illustrates the search of $\alpha_n^\star$ on the interval $[a, b]$ by using *Algorithm* 1. The first step initializes the algorithm. Here two interior points c and d are selected such that $a < c < d < b$ as shown in Figure 3a and 3b. Step 2 computes $p(c)$ and $p(d)$ using (25). In step 3 the search interval $[a, b]$ is squeezed by comparing the functional values $p(c)$ and $p(d)$. If $p(c) \leq p(d)$ (see Figure 3a), the minimum occurs in the subinterval $[a, d]$, and we replace b with d. If $p(c) > p(d)$ (see Figure 3b), the minimum occurs in the subinterval $[c, b]$, and we replace a with c. In step 4 interior points c and d are updated. Step 5 checks the stopping criteria, and the algorithm stops if the stopping criteria is satisfied, then we set $\alpha_n^\star = c$. Otherwise the algorithm continues.

Next we find $\mathbf{x}_n^\star$ and $\mathbf{M}_n^\star = \{\mathbf{m}_l^\star\}_{l \in \mathcal{L}(n)}$ associated with $\alpha_n^\star$ by solving problem (26). By writing the problem in the epigraph form, and then following the approach of [6, Section IV-B], problem (26) can be equivalently reformulated in the form of second-order cone program (SOCP) as

$$\begin{array}{ll} \text{minimize} & t \\ \text{subject to} & \begin{bmatrix} t \\ \sqrt{\rho/2}(\mathbf{x}_n - \mathbf{z}_n^i + \mathbf{v}_n^i) \end{bmatrix} \succeq_{\text{SOC}} 0 \\ & \begin{bmatrix} \sqrt{1+\frac{1}{\alpha_n}}\mathbf{h}_{ll}^{\text{H}}\mathbf{m}_l \\ \mathbf{M}_n^{\text{H}}\mathbf{h}_{ll} \\ \tilde{\mathbf{x}}_n \\ \sigma_n \end{bmatrix} \succeq_{\text{SOC}} 0, \quad l \in \mathcal{L}(n) \\ & \begin{bmatrix} x_{n,nl} \\ \mathbf{M}_n^{\text{H}}\mathbf{h}_{jl} \end{bmatrix} \succeq_{\text{SOC}} 0, \quad l \in \mathcal{I}_{\text{int}}(n) \\ & \begin{bmatrix} \sqrt{p_n^{\max}} \\ \text{vec}(\mathbf{M}_n) \end{bmatrix} \succeq_{\text{SOC}} 0, \end{array} \tag{28}$$

with variables t, $\mathbf{x}_n$, and $\mathbf{M}_n$, where $\tilde{\mathbf{x}}_n = \{x_{n,bl}\}_{l\in\mathcal{L}_{\text{int}}(n), b\in\mathcal{N}_{\text{int}}(l)}$ is a subset of $\mathbf{x}_n$ (see (10)), the matrix $\mathbf{h}_{jl}$ in the third set of constraints denotes the channel from BS n to link l (i.e., the index j in the third set of constraints denotes an arbitrary link in $\mathcal{L}(n)$). Note that to write problem (26) in the SOCP form (28), we first took the square root of the objective function of (26). Hence, the optimal value of problem (26) is given by $t^{\star 2}$ (i.e., $\tilde{p}(\alpha_n) = t^{\star 2}$), where $t^\star$ is a solution of problem (28).

We now turn to the second step of ADMM in (18), where the global variables $\{\mathbf{z}_n\}_{n\in\mathcal{N}}^{i+1}$ and γ^{i+1} are updated. By dropping the constant terms which do not affect the solution, problem (18) can be written as

$$\begin{array}{ll} \text{minimize} & \sum_{n\in\mathcal{N}} \Big(\mathbf{u}_n^{iT}(\mathbf{x}_n^{i+1} - \mathbf{z}_n) + v_n^i(\alpha_n^{i+1} - \gamma) \\ & +\frac{\rho}{2}\|\mathbf{x}_n^{i+1} - \mathbf{z}_n\|_2^2 + \frac{\rho}{2}\|\alpha_n^{i+1} - \gamma\|_2^2 \Big), \end{array} \tag{29}$$

with variables $\{\mathbf{z}_n\}_{n\in\mathcal{N}}$ and γ.

Problem (29) is separable in variables $\{\mathbf{z}_n\}_{n\in\mathcal{N}}$ and γ. We first provide solution for $\{\mathbf{z}_n\}_{n\in\mathcal{N}}$, and then for γ. Minimization of problem (29) with respect to $\{\mathbf{z}_n\}_{n\in\mathcal{N}}$ yields the following optimization problem

$$\text{minimize} \quad \sum_{n\in\mathcal{N}} \Big(\mathbf{u}_n^{iT}(\mathbf{x}_n^{i+1} - \mathbf{z}_n) + \frac{\rho}{2}\|\mathbf{x}_n^{i+1} - \mathbf{z}_n\|_2^2 \Big), \tag{30}$$

with variable $\{\mathbf{z}_n\}_{n\in\mathcal{N}}$. Note that the objective function of problem (30) is the sum of linear and quadratic cost incurred by the difference in the first equality constraints of problem (15). Furthermore, the first equality constraints of problem (15) are the compact representation of the fourth equality constraints of problem (6). Hence, by using the fourth equality constraints of problem (6), problem (30) in the components of $\mathbf{x}_n$, $\mathbf{z}_n$, and $\mathbf{u}_n$ can be expressed as

$$\begin{array}{ll} \text{minimize} & \sum_{l\in\mathcal{L}_{\text{int}}} \sum_{n\in\mathcal{N}_{\text{int}}(l)} \sum_{k\in\{n,tran(l)\}} \Big(u_{k,nl}^i(x_{k,nl}^{i+1} - z_{nl}) \\ & \qquad + \frac{\rho}{2}(x_{k,nl}^{i+1} - z_{nl})^2 \Big), \end{array} \tag{31}$$

with variable $\{z_{nl}\}_{l\in\mathcal{L}_{\text{int}}, n\in\mathcal{N}_{\text{int}}(l)}$, where $\{u_{k,nl}\}_{k\in\{n,tran(l)\}, l\in\mathcal{L}_{\text{int}}, n\in\mathcal{N}_{\text{int}}(l)}$ are the dual variables associated with the fourth equality constraints of problem (6) [8].

The problem (31) decouples across z_{nl}, since the objective function is separable in z_{nl} for all $l \in \mathcal{L}_{\text{int}}, n \in \mathcal{N}_{\text{int}}(l)$. Note that the objective function of problem (31) is quadratic in z_{nl}. Hence by setting the gradient of (31) with respect to z_{nl} equal to zero, we can get the solution $z_{nl}^\star$ which can be expressed as

$$z_{nl}^\star = \Big(x_{n,nl}^{i+1} + x_{tran(l),nl}^{i+1} + \frac{1}{\rho}(u_{n,nl}^i + u_{tran(l),nl}^i) \Big)/2 \tag{32}$$

for all $l \in \mathcal{L}_{\text{int}}, n \in \mathcal{N}_{\text{int}}(l)$. Therefore, the update $z_{nl}^{i+1} = z_{nl}^\star$ for all $l \in \mathcal{L}_{\text{int}}, n \in \mathcal{N}_{\text{int}}(l)$. Moreover, by substituting z_{nl}^{i+1} in (19) [9], we can show that the sum of the dual variables $u_{n,nl}^i + u_{tran(l),nl}^i$ is equal to zero, thus the update z_{nl}^{i+1} further simplifies to

$$z_{nl}^{i+1} = \Big(x_{n,nl}^{i+1} + x_{tran(l),nl}^{i+1} \Big)/2, \tag{33}$$

for all $l \in \mathcal{L}_{\text{int}}, n \in \mathcal{N}_{\text{int}}(l)$. Hence the global variable update z_{nl}^{i+1} is simply the average of its local copies $x_{n,nl}^{i+1}$ and $x_{tran(l),nl}^{i+1}$.

We next provide solution $\gamma^\star$ for problem (29). By setting the gradient of problem (29) with respect to γ equal to zero, we can get

$$\gamma^\star = \frac{\sum_{n\in\mathcal{N}} v_n^i + \rho\alpha_n^{i+1}}{\rho N}, \tag{34}$$

hence the update $\gamma^{i+1} = \gamma^\star$. Moreover, by substituting γ^{i+1} in (20) we can show that the sum of the dual variables $\sum_{n\in\mathcal{N}} v_n^i$ is equal to zero, thus the update γ^{i+1} (i.e., (34)) further simplifies to

$$\gamma^\star = \frac{\sum_{n\in\mathcal{N}} \alpha_n^{i+1}}{N}, \tag{35}$$

which is simply the average of the local copies α_n of each BS.

We now summarize the proposed ADMM based distributed algorithm for SINR balancing problem in *Algorithm* 2.

Algorithm 2: Proposed ADMM based distributed algorithm for SINR balancing

1) Initialization: given maximum transmit power $p_n^{\max}$ for all $n \in \mathcal{N}$ and penalty $\rho > 0$. Set $i = 0$, $\{\mathbf{u}_n^0\}_{n\in\mathcal{N}} = 0$, and $\{v_n^0\}_{n\in\mathcal{N}} = 0$.
2) BS $n = 1\ldots N$ update local variables $(\mathbf{M}_n^{i+1}, \mathbf{x}_n^{i+1}, \alpha_n^{i+1})$.
3) Exchange local updates:
   a) BS n and BS $tran(l)$ exchange their local copies $x_{n,nl}^{i+1}$ and $x_{tran(l),nl}^{i+1}$ for all $n \in \mathcal{L}_{\text{int}}, n \in \mathcal{N}_{\text{int}}(l)$.

[8] Note that $\{\mathbf{u}_n\}_{n\in\mathcal{N}}$ are the dual variables associate with the first equality constraint of problem (15). By following steps (8) to (10), we can easily denote $\mathbf{u}_n = \{\{u_{n,nl}\}_{l\in\mathcal{I}_{\text{int}}(n)}, \{u_{n,bl}\}_{l\in\mathcal{L}_{\text{int}}(n), b\in\mathcal{N}_{\text{int}}(l)}\}, n \in \mathcal{N}$.

[9] Note that (19) in the components of $\mathbf{u}_n$, $\mathbf{x}_n$, and $\mathbf{z}_n$ can be expressed as $u_{k,nl}^{i+1} = u_{k,nl}^i + \rho(x_{k,nl}^{i+1} - z_{nl}^{i+1})$ for all $k \in \{n, tan(l)\}, l \in \mathcal{L}_{\text{int}}, n \in \mathcal{N}_{\text{int}}(l)$.

b) BS n transmits local copy α_n^{i+1} to all other BSs for all $n \in \mathcal{N}$.

4) BS $n = 1 \dots N$ update global variables $(\mathbf{z}_n^{i+1}, \gamma)$.
5) BS $n = 1 \dots N$ update dual variables $(\mathbf{u}_n^{i+1}, v_n^{i+1})$.
6) If stopping criteria is satisfied, STOP. Otherwise set $i = i+1$, and go to step 2.

The first step initializes the algorithm. Step 2 updates the local variables of each BS. Step 2 is completely decentralized. In step 3, BSs exchange their local copies to update the global variables. The adjacent BSs that are coupled by variable z_{nl} (i.e., BS n and BS $tran(l)$) exchange the local local copies $x_{n,nl}^{i+1}$ and $x_{tran(l),nl}^{i+1}$. The local copy α_n is broadcasted to all other BSs. Step 4 updates the global variables $\mathbf{z}_n^{i+1}$ and γ^{i+1}. In step 5, the dual variables are updated by each BS. Note that steps 4 and 5 are completely decentralized. Step 6 checks the stopping criteria, and the algorithm stops if the stopping criteria is satisfied[10]. Otherwise, the algorithm continues in an iterative manner.

C. Finding feasible solution at each iteration of Algorithm 2

Note that at each step of *Algorithm 2*, the locally obtained SINR α_n^i at each BS may not be equal before converging the algorithm. So, we take the global variable γ^i as the intermediate solution of *Algorithm 2*. However, due to the difference in the local copies $x_{n,nl}^{i+1}$ and $x_{tran(l),nl}^{i+1}$ for all $n \in \mathcal{L}_{\text{int}}, n \in \mathcal{N}_{\text{int}}(l)$, and the maximum transmit power constraint of the BSs, the intermediate solution γ^i may not be feasible for all BSs.

Therefore to make local copies at each BS equal and check the feasibility of γ^i, we set local variables fixed at the respective global variables at each BS (i.e., we set $\mathbf{x}_n = \mathbf{z}_n^i$ and $\alpha_n = \gamma^i$ for all $n \in \mathcal{N}$). Then solve problem (26), which results into the following SOCP (see [6, Section IV-B]) feasibility problem

$$
\begin{array}{ll}
\text{find} & \{\mathbf{m}_l\}_{l \in \mathcal{L}(n)} \\
\text{subject to} & \begin{bmatrix} \sqrt{1+\frac{1}{\alpha_n}}\mathbf{h}_{ll}^{\mathrm{H}}\mathbf{m}_l \\ \mathbf{M}_n^{\mathrm{H}}\mathbf{h}_{ll} \\ \tilde{\mathbf{x}}_n \\ \sigma_l \end{bmatrix} \succeq_{\text{SOC}} 0, \quad l \in \mathcal{L}(n) \\
& \begin{bmatrix} x_{n,nl} \\ \mathbf{M}_n^{\mathrm{H}}\mathbf{h}_{jl} \end{bmatrix} \succeq_{\text{SOC}} 0, \quad l \in \mathcal{I}_{\text{int}}(n) \\
& \begin{bmatrix} \sqrt{p_n^{\max}} \\ \text{vec}(\mathbf{M}_n) \end{bmatrix} \succeq_{\text{SOC}} 0, \quad l \in \mathcal{I}_{\text{int}}(n)
\end{array}
\tag{36}
$$

with variable $\mathbf{M}_n = [\mathbf{m}_l]_{l \in \mathcal{L}(n)}$, where $\tilde{\mathbf{x}}_n = \{x_{n,bl}\}_{b \in \mathcal{N}_{\text{int}}(l), l \in \mathcal{L}_{\text{int}}(n)}$ is a subset of $\mathbf{x}_n$ (see (10)), the matrix $\mathbf{h}_{jl}$ in the third set of constraints denotes the channel from BS n to link l (i.e., the index j in the third set of constraints denotes an arbitrary link in $\mathcal{L}(n)$).

[10] In ADMM algorithm, standard stopping criteria is to check primal and dual residuals [9]. However, for practical implementation finite number of iteration is more favorable. Thus, we adopt fixed number of iteration to stop the algorithm.

Note that we get a set of $\{\mathbf{m}_l^\star\}_{l \in \mathcal{L}}$ that is feasible for the original problem (4) only if problem (36) is feasible for all $n \in \mathcal{N}$ BSs. Thus, in *Algorithm 2*, we can update the feasible SINR γ_{feas}^i as

$$
\gamma_{\text{feas}}^i = \begin{cases} \gamma^i & \text{if problem (36) is feasiblem for all } n \in \mathcal{N} \\ \gamma_{\text{feas}}^{i-1} & \text{otherwise,} \end{cases}
\tag{37}
$$

where $\gamma_{\text{feas}}^0 = 0$.

IV. Numerical Example

In this section we numerically evaluate the performance of proposed *Algorithm 2*. In our simulations multicell wireless network as shown in Figure 1 is considered. There are $N = 2$ BSs with $T = 4$ antennas at each one. The distance between the BSs is denoted by D_{BS}. We assume BSs have circular transmission and interference regions, where the radius of the transmission region of each BS is denoted by R_{BS}, and the radius of the interference region of each BS is denoted by R_{int}. For simplicity, we assume 4 users per cell. The location of users associated with BSs are arbitrarily chosen as shown in Figure 1.

We assume an exponential path loss model, where the channel matrix between BSs and users is modeled as

$$
\mathbf{h}_{jl} = \left(\frac{d_{jl}}{d_0}\right)^{-\eta/2} \mathbf{c}_{jl},
$$

where d_{jl} is the distance from the transmitter of data stream j (i.e., BS $tran(j)$) to the receiver of data stream l (i.e., user $rec(l)$), d_0 is the *far field reference distance* [16], η is the path loss exponent, and $\mathbf{c}_{jl} \in \mathbb{C}^T$ is arbitrarily chosen from the distribution $\mathcal{CN}(0, \mathbf{I})$ (i.e., frequency-flat fading channel with uncorrelated antennas). Here, we refer an arbitrarily generated set of fading coefficients $\mathcal{C} = \{\mathbf{c}_{jl} | j, l \in \mathcal{L}\}$ as a *single fading realization*.

We assume the maximum power constraint is same for each BS, i.e., $p_n^{\max} = p_0^{\max}$ for all $n \in \mathcal{N}$, and $\sigma_l = \sigma$ for all $l \in \mathcal{L}$. We define the signal-to-noise ratio (SNR) operating point at a distance r as

$$
\text{SNR}(r) = \left(\frac{r}{d_0}\right)^{-\eta} \frac{p_0^{\max}}{\sigma^2}. \tag{38}
$$

In our simulations, we set $d_0 = 1$, $\eta = 4$, $\sigma^2 = 1$, $p_0^{\max}/\sigma^2 = 45$dB, SNR$(R_{\text{int}}) = 0$dB, SNR$(R_{\text{BS}}) = 5$dB, and $D_{\text{BS}} = 1.5 \times R_{\text{BS}}$.

Figure 4 shows the progress of the global variable γ by iteration for SNR = 5 dB. As a benchmark, we consider centralized optimal algorithm proposed in [6, Section V]. For *Algorithm 1*, we set $\epsilon = 0.1$, and $\alpha_n^{\max} = 2 \times 10^{0.1 \times \text{SNR}}$ for all $n \in \mathcal{N}$.

Results show that proposed *Algorithm 2* converges to the optimal centralized solution for all considered penalty parameter ρ. The global variable γ is the average of the locally obtained SINR $\{\alpha_n\}_{n \in \mathcal{N}}$ (see, (35)). Hence, the intermediate values of γ may not be feasible before the algorithm converges (see, Section III-C). For example, the value of γ for $\rho = 0.5$

is clearly infeasible at the iteration step $i = \{4, 5, 6, 7, 8\}$. Therefore, to illustrate the convergence of feasible γ, we define the following metric

$$\gamma^i_{\text{best}} = \max_{t=1,\dots,i} \{\gamma^t_{\text{feas}}\}, \tag{39}$$

where γ^i_{best} is the best feasible SINR value at ith iteration, and γ^t_{feas} is the feasible SINR at tth iteration obtained by (37). Figure 5 shows the behavior of γ^i_{best} by iteration. Results show that the proposed algorithm always generate the feasible SINR γ, when the algorithm converges.

V. Conclusions

We have provided distributed algorithm for signal-to-interference-plus-noise ratio balancing problem in multicell downlink multi-input single-output systems. We have proposed consensus-based distributed algorithms, and a fast solution method via alternating direction method of multipliers. Numerical results show that the proposed distributed algorithm converges to the optimal centralized solution.

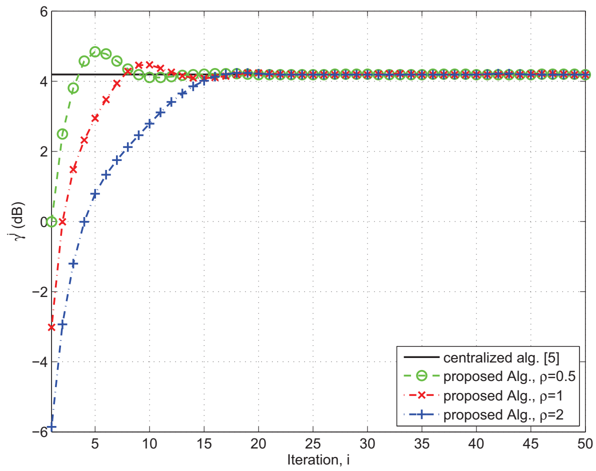


Fig. 4: Progress of global variable γ for SNR = 5 dB.

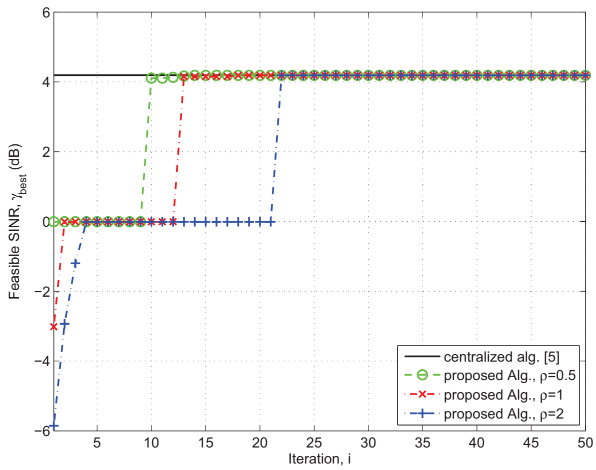


Fig. 5: Feasible SINR γ^i_{best} versus iteration for SNR = 5 dB.

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