A distributed implementation of opportunistic interference alignment for MIMO cognitive radio

Chao Zhang[†], Samson Lasaulce[†], and Sara Berri[‡]

Abstract—**In this paper, we propose a distributed implementation of the opportunistic interference alignment (OIA) technique developed in [1]. Therein, global channel state information (CSI) is assumed at the secondary transmitter that is, the knowledge of all the channel transfer matrices is required, which may be difficult or impossible to obtain in practice. Therefore, we propose to relax this assumption by only assuming local CSI at the transmitters and that the covariance of the received secondary signal is available at the secondary transmitter; this setup is quite similar to the one assumed for the MIMO iterative water-filling algorithm [2]. One of the key ingredients which allows the secondary transmitter to implement the OIA condition of [1] by only having this reduced knowledge is that the primary transmitter reveals information about its local channels to the secondary transmitter by embedding (for one time-slot typically, supposedly among many others) in its pre-processing matrix the information needed at the secondary. The proposed distributed implementation is proved to be effective analytically but simulations are also provided to prove that it seems to be robust against imperfect covariance feedback.**

I. Introduction

Cognitive radio constitutes one possible way of making more efficient the usage of the radio spectrum (see e.g., [3]). One classical and important cognitive radio scenario consists in considering the coexistence of two links: a primary link which has full priority on the spectrum usage and a secondary link which can only exploit the radio resource if it does not create any interference on the primary link. When terminals are equipped with single-antennas and the primary link exploits all the spectrum, the no-interference condition typically implies that the transmission rate vanishes for the secondary link. Interestingly, as observed in [1], when terminals are equipped with multiple-antennas, the secondary transmitter may be able to transmit at a non-negligible rate even when no spectrum is available for the secondary, provided a suitable interference alignment (IA) technique is used. In essence, IA involves constructing signals such that the corresponding interference signal lies in an orthogonal subspace to the signal of interest at the receiver (here at the primary). This technique was introduced independently through several articles [4][5]. It has also become an important tool to study the degrees of freedom of interference channels [5]. IA has been analyzed for feasibility and implementation issues, especially the required channel state information, in [6]. Compared to other famous IA techniques, the technique of [1] relies in part on the key observation that when maximizing its rate, the primary generally leaves some transmission spatial modes unused. For example, to achieve capacity the primary has to water-fill its power over its best singular channels, meaning that the other singular channels are left free. Whereas the opportunistic IA (OIA) technique of [1] has drawn the attention of the signal processing community, it still has a feature which would prevent it from being implemented in practice: it relies on the availability of global channel state information (CSI) at the secondary transmitter (that is, the knowledge of the channel transfer matrices of all the links). Global CSI has also been assumed to be available in other related works on IA such as [7][8] but global CSI availability implies operations such as inter-transmitter communications, which may be difficult or even impossible to implement in some practical scenarios. The purpose of the present paper is precisely to propose a distributed way of implementing the OIA technique of [1]. Indeed, we assume that: each transmitter has local CSI (that is, the knowledge of the links that arrive to its intended receiver); the secondary transmitter has only access to the covariance matrix of the signal observed at the secondary receiver. This type of information assumptions thus makes the implementation distributed information-wise in the sense of famous works such as works using the MIMO iterative water-filling-like algorithms (IWFA) (see e.g., [2]). Indeed, if the receiver makes reliable measurements of the observed signal and sends them through a reliable feedback channel, the transmitter can have access to the covariance matrix of the received signal. At last, note that there exist works where distributed OIA is studied but they don't consider the same OIA technique as here and, additionally, under different assumptions. For instance, [9] assumes channel reciprocity and an OIA technique which differs from the [1]. Additionally, no existing works propose to use the key idea of using the primary pre-processing matrix as a way of communicating information to the secondary as we do in the present paper. This idea is inspired from [10] where power modulation has been proposed to estimate global CSI in multi-band interference channels.

[†] L2S (CNRS-CentraleSupelec-Univ. Paris Sud), Gif-sur-Yvette, France.

[‡] Telecom Paristech, Paris, France.

II. System Model

We consider two unidirectional links simultaneously operating in the same frequency band and producing mutual interference. The first transmitter-receiver pair $(\mathrm{Tx}_1, \mathrm{Rx}_1)$

is the primary link. The pair $(\mathrm{Tx}_2, \mathrm{Rx}_2)$ is an opportunistic link subject to the strict constraint that the primary link must be able to transmit without any interference from the secondary. Denote respectively by $M_i \geq 1$ and $N_i \geq 1$, with $i = 1$ (resp. $i = 2$), the numbers of antennas at the primary (resp. secondary) transmitter and the receiver. Each transmitter sends messages only to its respective receiver and no direct message exchange between the transmitters is allowed.

As generally assumed in the context of interference alignment, it is assumed that the channel transfer matrices evolves slowly over time. They are therefore assumed to be fixed over the whole duration of the transmission which can be seen as a frame comprising a large number of time-slots. The latter quantity will be denoted by $T \geq 1$. The channel transfer matrix from Transmitter $i \in \{1,2\}$ to Receiver $j \in \{1,2\}$ is an $N_j \times M_i$ matrix that is denoted by $\mathbf{H}_{ij}$, which typically corresponds to the realization of a random matrix. No particular distribution needs to be assumed. Only very reasonable and classical invertibility conditions on the channel matrices will be needed and provided further. As [2], we assume that the channel transfer matrices are constant over a period of time (a frame typically) which comprises many time-slots. The information signal or symbol of Transmitter i is represented by the vector $x_i \in \mathbb{C}^{L_i}$, $L_i \geq 1$. In our model, Transmitter i pre-processes its symbols using a matrix $\mathbf{C}_i \in \mathbb{C}^{M_i \times L_i}$ to construct its effectively transmitted signal $\mathbf{C}_i x_i$; the matrix $\mathbf{C}_i$ is called the pre-processing or pre-coding matrix. Following a matrix notation, the primary and secondary received signals, represented by the $N_i \times 1$ column-vectors y_i, with $i \in \{1,2\}$, can be written as

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \mathbf{H}_{11} & \mathbf{H}_{21} \\ \mathbf{H}_{12} & \mathbf{H}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{C}_1 x_1 \\ \mathbf{C}_2 x_2 \end{bmatrix} + \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}, \quad (1)$$

where z_i is an N_i-dimensional vector representing noise effects at Receiver i whose entries are modeled by an additive white Gaussian noise (AWGN) with zero mean and variance σ_i^2, i.e., $\forall i \in \{1,2\}$, $\mathbb{E}\left[z_i z_i^{\mathrm{H}}\right] = \sigma_i^2 \mathbf{I}_{N_i}$, the notation $(.)^{\mathrm{H}}$ standing for Hermitian transpose. The matrices $\mathbf{H}_{ij}$, $(i,j) \in \{1,2\}^2$ are assumed to be realizations of continuous random matrices such that each matrix is full rank with probability 1. We make the classical assumption that x_i corresponds to the realization of a certain random variable which is zero-mean with covariance $\mathbf{P}_i = \mathbb{E}\left[x_i x_i^{\mathrm{H}}\right]$. Additionally, x_1, x_2, z_1, and z_2 are assumed to be independent, which is also a conventional assumption in the context of IA. At transmitter $i \in \{1,2\}$, the power constraints on the transmitted signals $\mathbf{C}_i x_i$ can be written as Trace $\left(\mathbf{C}_i \mathbf{P}_i \mathbf{C}_i^{\mathrm{H}}\right) \leqslant M_i\, P_{i,\max}$, $P_{i,\max}$ being the maximal transmit power for Transmitter i. At Receiver $i \in \{1,2\}$, the signal y_i is processed using an $N_i \times N_i$ matrix $\mathbf{D}_i$ to form the N_i-dimensional vector:

$$\widetilde{y}_i = \mathbf{D}_i y_i. \quad (2)$$

We refer to $\mathbf{D}_i$ as the post-processing or decoding matrix at Receiver i.

III. Short Review of the Opportunistic Interference Alignment of [1]

Here, we briefly review the OIA technique introduced in [1]. The primary link is assumed to implement the following coding and decoding strategies [11]: $\mathbf{C}_1 = \mathbf{V}_{11}$, $\mathbf{D}_1 = \mathbf{U}_{11}^{\mathrm{H}}$, $\mathbf{P}_1 = \mathbf{P}_1^{\star}$. The matrices $\mathbf{V}_{11}$, $\mathbf{U}_{11}$, and $\mathbf{P}_1^{\star} = \mathrm{Diag}(p_{1,1}^{\star}, ..., p_{1,M_1}^{\star})$ are defined by the ordered singular value decomposition (SVD) of $\mathbf{H}_{11}$ and water-filling formula

$$\begin{cases} \mathbf{H}_{11} = \mathbf{U}_{11} \mathbf{\Lambda}_{11} \mathbf{V}_{11}^{\mathrm{H}} \\ \forall n \in \{1, \ldots, M_1\}, \quad p_{1,n}^{\star} = \left(\omega - \frac{\sigma_1^2}{\lambda_{11,n}^2}\right)^{+}, \end{cases} \quad (3)$$

where: $\mathbf{U}_{11}$ and $\mathbf{V}_{11}^{\mathrm{H}}$ are two unitary matrices with dimension $N_1 \times N_1$ and $M_1 \times M_1$ respectively; $\mathbf{\Lambda}_{11}$ is an $N_1 \times M_1$ matrix with main diagonal $\left(\lambda_{11,1}, \ldots, \lambda_{11,\min(N_1,M_1)}\right)$ and zeros on its off-diagonal; ω is the water-level. Suppose $m_1 \triangleq \sum_{n=1}^{M_1} \mathbb{1}_{]0,\infty]}(p_{1,n}^{\star})$, it will be convenient to use the strictly positive matrix $\mathbf{Q}_1^{\star}$ defined as follows:

$$\mathbf{P}_1^{\star} = \begin{array}{c c} & \begin{array}{cc} \overleftrightarrow{m_1} & \overleftrightarrow{M-m_1} \end{array} \\ \begin{array}{r} m_1 \updownarrow \\ M-m_1 \updownarrow \end{array} & \begin{bmatrix} \mathbf{Q}_1^{\star} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \end{array}. \quad (4)$$

where $\mathbf{Q}_1^{\star} = \mathrm{Diag}(p_{1,1}^{\star}, ..., p_{1,m_1}^{\star})$. The idea of OIA [1] is that the secondary uses the singular modes which are not exploited by the primary without creating any interference at the primary. For this, an IA condition needs to be met. This is what we describe here. Define the $N_1 \times M_2$ matrix $\widetilde{\mathbf{H}} \triangleq \mathbf{U}_{11}^{\mathrm{H}} \mathbf{H}_{21}$ and the following block structure:

$$\widetilde{\mathbf{H}} = \begin{array}{c c} & \overleftrightarrow{M_2} \\ \begin{array}{r} m_1 \updownarrow \\ N_1 - m_1 \updownarrow \end{array} & \begin{bmatrix} \widetilde{\mathbf{H}}_1 \\ \widetilde{\mathbf{H}}_2 \end{bmatrix} \end{array}. \quad (5)$$

With the above notations the OIA condition writes:

$$\widetilde{\mathbf{H}}_1 \mathbf{C}_2^{\star} = \mathbf{0}_{m_1 \times k_2}, \quad (6)$$

where k_2 is the dimension of the null space of the matrix $\widetilde{\mathbf{H}}_1$. By choosing a precoding matrix which verifies (6), the secondary transmitter is ensured to create no interference at the primary receiver. However, the secondary receiver undergoes the interference from the primary transmitter. To mitigate the effect of the latter, the authors of [1] proposed to choose the post-processing matrix $\mathbf{D}_2$ as follows:

$$\mathbf{D}_2^{\star} = \left(\mathbf{H}_{12} \mathbf{V}_{11} \mathbf{P}_1^{\star} \mathbf{V}_{11}^{\mathrm{H}} \mathbf{H}_{12}^{\mathrm{H}} + \sigma_2^2 \mathbf{I}_{N_2}\right)^{-\frac{1}{2}}, \quad (7)$$

assuming classical invertibility conditions [1]. At last, the matrix $\mathbf{P}_2$ is chosen to maximize the transmission rate of the secondary link, which amounts to perform a water-filling operation knowing the channel matrix $\mathbf{H}_{22}$.

Summarizing, it is seen that for the secondary to be able to implement the OIA technique as proposed in [1], the knowledge of the matrices $\mathbf{H}_{11}$, $\mathbf{H}_{12}$, $\mathbf{H}_{21}$, and $\mathbf{H}_{22}$ is required. The knowledge of $\mathbf{H}_{21}$ and $\mathbf{H}_{22}$ corresponds to what is called the local CSI assumption, which is generally considered as reasonable since it can be acquired at the secondary receiver e.g., from a training sequence sent by the primary transmitter. However, acquiring the knowledge of $\mathbf{H}_{11}$ and $\mathbf{H}_{12}$ constitutes a much more problematic task especially if the scenario is decentralized. The purpose of this paper is precisely to propose one possible way of implementing (6) and (7) from the knowledge of the received signal covariance matrix feedback from the secondary receiver and the local CSI at the transmitters.

IV. Proposed method to implement OIA

From now on, for the ease of exposition and facilitating the reading, we will assume $M_1 = M_2 = M$ and $N_1 = N_2 = N$, but it can be checked that the proposed technique readily applies to the general case. As mentioned in the preceding sections, we assume the existence of a feedback channel between the secondary receiver and the secondary transmitter. For each of the transmission time-slots, the secondary transmitter is assumed to have feedback measurements from the secondary receiver. This is a setting that is quite similar to the one in [2] in which a MIMO version of the IWFA is developed and analyzed. The covariance matrix of the signal received at the secondary (after post-processing) $\widetilde{y}_2$ is defined by:

$$\mathbf{R}_2 = \mathbb{E}[\widetilde{y}_2\widetilde{y}_2^{\mathrm{H}}]. \tag{8}$$

By indicating explicitly the dependency towards the coding and decoding matrices, it generically expresses as follows:

$$\mathbf{R}_2(\mathbf{C}_2,\mathbf{D}_2,\mathbf{P}_2;\mathbf{C}_1,\mathbf{D}_1,\mathbf{P}_1) = \mathbf{D}_2\mathbf{H}_{22}\mathbf{C}_2\mathbf{P}_2\mathbf{C}_2^{\mathrm{H}}\mathbf{H}_{22}^{\mathrm{H}}\mathbf{D}_2^{\mathrm{H}} + \mathbf{D}_2\mathbf{H}_{12}\mathbf{C}_1\mathbf{P}_1\mathbf{C}_1^{\mathrm{H}}\mathbf{H}_{12}^{\mathrm{H}}\mathbf{D}_2^{\mathrm{H}} + \sigma_2^2\mathbf{I}_N. \tag{9}$$

Now, our goal is to show how to exploit the covariance feedback $\mathbf{R}_2$ to implement (6) and (7), starting with (7).

A. Acquisition of the optimal post-processing matrix

As in [1], assume that the primary operates in the capacity-achieving scenario that is: $(\mathbf{C}_1,\mathbf{D}_1,\mathbf{P}_1) = (\mathbf{V}_{11},\mathbf{U}_{11}^{\mathrm{H}},\mathbf{P}_1^\star)$. To obtain the best pre-processing matrix $\mathbf{D}_2^\star$, the secondary transmitter can implement one very simple acquisition procedure. Assume that for one given time-slot for the secondary transmission, the secondary transmitter chooses to be off that is, $\mathbf{P}_2 = \underline{\mathbf{P}}_2 = \mathbf{0}$ and the secondary receiver chooses a given invertible post-processing matrix $\underline{\mathbf{D}}_2$; the matrix $\mathbf{C}_2$ can be arbitrary. Then, the covariance matrix feedback received by the secondary transmitter corresponds to:

$$\begin{aligned}\underline{\mathbf{R}}_2 &= \mathbf{R}_2(\mathbf{C}_2,\underline{\mathbf{D}}_2,\mathbf{0};\mathbf{V}_{11},\mathbf{U}_{11}^{\mathrm{H}},\mathbf{P}_1^\star) = \\ &\underline{\mathbf{D}}_2\mathbf{H}_{12}\mathbf{V}_{11}\mathbf{P}_1^\star\mathbf{V}_{11}^{\mathrm{H}}\mathbf{H}_{12}^{\mathrm{H}}\underline{\mathbf{D}}_2^{\mathrm{H}} + \sigma_2^2\mathbf{I}_N.\end{aligned} \tag{10}$$

It can then be observed that the optimal post-processing matrix $\mathbf{D}_2^\star$ (which is given by (7)) can be recovered by performing the following operation:

$$\mathbf{D}_2^\star = \left[\left(\underline{\mathbf{D}}_2^{\mathrm{H}}\right)^{-1}\underline{\mathbf{R}}_2\underline{\mathbf{D}}_2^{-1} - \left(\underline{\mathbf{D}}_2^{\mathrm{H}}\right)^{-1}\underline{\mathbf{D}}_2^{-1} + \sigma_2^2\mathbf{I}_N\right]^{-\frac{1}{2}}. \tag{11}$$

Of course, this means that for a given time-slot, the transmission rate for the secondary vanishes but since the transmission is assumed to occur over many time-slots, the corresponding impact would be typically negligible in practice.

B. Acquisition of the pre-processing matrix when $M \leq N$

To implement the OIA condition (6), we use a procedure which is similar to the one used in the previous section for finding the optimal post-processing matrix. The difference this time is that the primary is also put to contribution. Indeed, for a given time-slot, which might be the very first time-slot at which the secondary becomes active, both the primary and secondary transmitters are assumed to make specific choices for their transmission and reception matrices. For this time-slot, the secondary transmitter is assumed to be off again: $\mathbf{P}_2 = \overline{\mathbf{P}}_2 = \mathbf{0}$ and the secondary receiver chooses a given invertible post-processing matrix $\overline{\mathbf{D}}_2$; The matrix $\mathbf{C}_2$ can be arbitrary. For the primary transmitter, the pre-processing matrix is carefully chosen as follows:

$$\mathbf{C}_1 = \overline{\mathbf{C}}_1 = \begin{array}{c} \\ M\updownarrow \end{array}\begin{array}{c} \overset{m_1}{\longleftrightarrow}\overset{M-m_1}{\longleftrightarrow} \\ \left[\begin{array}{cc}\widetilde{\mathbf{H}}_1^{\mathrm{H}} & \mathbf{W}_1\end{array}\right]\end{array}. \tag{12}$$

where $\widetilde{\mathbf{H}}_1$ is given by (5) and $\mathbf{W}_1$ can be any arbitrary matrix with dimensions $M \times (M - m_1)$; note that the primary transmiter knows $\widetilde{\mathbf{H}}_1$ since it has the local CSI knowledge $\mathbf{H}_{11},\mathbf{H}_{21}$. The above choice translates the key idea of implicit communication between the primary and secondary. Indeed, we see that the pre-processing matrix itself conveys the channel information. A priori, any kind of information might be exchanged. Here, the choice we make corresponds to what the primary needs to implement OIA. At last, for the primary we have that: $(\mathbf{D}_1,\mathbf{P}_1) = (\mathbf{U}_{11}^{\mathrm{H}},\mathbf{P}_1^\star)$. Then, for this configuration, the covariance matrix feedback received by the secondary transmitter corresponds to:

$$\begin{aligned}\overline{\mathbf{R}}_2 &= \mathbf{R}_2(\mathbf{C}_2,\overline{\mathbf{D}}_2,\mathbf{0};\overline{\mathbf{C}}_1,\mathbf{U}_{11}^{\mathrm{H}},\mathbf{P}_1^\star) = \\ &\overline{\mathbf{D}}_2\mathbf{H}_{12}\widetilde{\mathbf{H}}_1^{\mathrm{H}}\mathbf{Q}_1^\star\widetilde{\mathbf{H}}_1\mathbf{H}_{12}^{\mathrm{H}}\overline{\mathbf{D}}_2^{\mathrm{H}} + \sigma_2^2\mathbf{I}_N.\end{aligned} \tag{13}$$

Thus the quantity $\widetilde{\mathbf{H}}_1^{\mathrm{H}}\mathbf{Q}_1^\star\widetilde{\mathbf{H}}_1$ can be expressed as:

$$\widetilde{\mathbf{H}}_1^{\mathrm{H}}\mathbf{Q}_1^\star\widetilde{\mathbf{H}}_1 = \mathbf{A}_{12}\mathbf{H}_{12}^{\mathrm{H}}\overline{\mathbf{D}}_2^{-1}\left(\overline{\mathbf{R}}_2 - \sigma_2^2\mathbf{I}_N\right)\left(\overline{\mathbf{D}}_2^{\mathrm{H}}\right)^{-1}\mathbf{H}_{12}\mathbf{A}_{12}, \tag{14}$$

where $\mathbf{A}_{12} = \left(\mathbf{H}_{12}^{\mathrm{H}}\mathbf{H}_{12}\right)^{-1}$.Under the local CSI assumption, the matrix $\mathbf{H}_{12}$ is available to the secondary transmitter. Additionally, it is pseudo-invertible with probability 1 if it corresponds to the realization of a continuous random matrix, as assumed in [1]. The last step is to observe that the null space of $\widetilde{\mathbf{H}}_1^{\mathrm{H}}\mathbf{Q}_1^\star\widetilde{\mathbf{H}}_1$ coincides with the null space of $\widetilde{\mathbf{H}}_1$ when $\mathbf{Q}_1^\star$ is full rank, which is the case by definition. As a conclusion, the OIA condition (6) can be implemented by choosing $\mathbf{C}_2$ to be in the null space of

$$\mathbf{A}_{12}\mathbf{H}_{12}^{\mathrm{H}}\overline{\mathbf{D}}_2^{-1}\left(\overline{\mathbf{R}}_2 - \sigma_2^2\mathbf{I}_N\right)\left(\overline{\mathbf{D}}_2^{\mathrm{H}}\right)^{-1}\mathbf{H}_{12}\mathbf{A}_{12}, \tag{15}$$

which provides $\mathbf{C}_2^\star$, as desired.

C. Acquisition of the pre-processing matrix when $M > N$

Notice that even if $\mathbf{H}_{12}$ corresponds to the realization of a continuous random matrix, it is not pseudo-invertible when $M > N$. It is therefore impossible to reconstruct the matrix $\widetilde{\mathbf{H}}_1^H \mathbf{Q}_1^\star \widetilde{\mathbf{H}}_1$ and therefore the null space of $\widetilde{\mathbf{H}}_1 = \left[\widetilde{h}_{1,1}, \ldots, \widetilde{h}_{1,M}\right]$. To deal with this issue, the proposed idea is to exploit several time-slots to acquire this null space. For each time-slot t, the primary uses a given pre-processing matrix which is denoted by $\mathbf{C}_{1;(t)}$. For the case of interest, that is to say, $M > N > m_1$, the required number of time-slots can be checked to be $K = \lceil \frac{M-N}{N-m_1} \rceil$, as shown next. The case $N = m_1$ would need to be treated separately without bringing a strong added value and is therefore not presented here.

Except for $\mathbf{C}_{1;(t)}$, which is chosen to be time-varying, the coding and decoding matrices for the acquisition procedure are taken to be constant and the same as in the scenario $M \leq N$ for all the acquisition time-slots $t \in \{1, ..., K\}$. More precisely, $\mathbf{C}_{1;(t)}$ is chosen as follows:

$$\mathbf{C}_{1;(t)} = \begin{array}{c c} & \begin{array}{cc} \overset{m_1}{\longleftrightarrow} & \overset{M-m_1}{\longleftrightarrow} \end{array} \\ \begin{array}{c} N \updownarrow \\ M-N \updownarrow \end{array} & \begin{bmatrix} \widetilde{\mathbf{H}}_{1;(t)}^{\mathrm{H}} & \mathbf{X}_1 \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \end{array}. \tag{16}$$

where $\widetilde{\mathbf{H}}_{1;(t)} = \left[\widetilde{h}_{1,1+(t-1)K}, \ldots, \widetilde{h}_{1,N+(t-1)K}\right] \in \mathbb{R}^{m_1 \times N}$ and $\mathbf{X}_1$ can be any arbitrary matrix with dimensions $N \times (M - m_1)$. Therefore, by determining $\mathbf{V}_{2,t}$ such that $\widetilde{\mathbf{H}}_{1,(t)} \mathbf{V}_{2;(t)} = \mathbf{0}$ for every $t \in \{1, \ldots, K\}$, the null space of $\widetilde{\mathbf{H}}_1$ can be reconstructed. Similarly to the case $M \leq N$, determining $\mathbf{V}_{2;(t)}$ amounts to determining the null space of

$$\mathbf{C}_{1;(t)} \mathbf{P}_1^\star \mathbf{C}_{1;(t)}^{\mathrm{H}} = \begin{array}{c c} & \begin{array}{cc} \overset{N}{\longleftrightarrow} & \overset{M-N}{\longleftrightarrow} \end{array} \\ \begin{array}{c} N \updownarrow \\ M-N \updownarrow \end{array} & \begin{bmatrix} \widetilde{\mathbf{H}}_{1;(t)}^{\mathrm{H}} \mathbf{Q}_1^\star \widetilde{\mathbf{H}}_{1;(t)} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \end{array} \tag{17}$$

where the matrix $\widetilde{\mathbf{H}}_{1;(t)}^{\mathrm{H}} \mathbf{Q}_1^\star \widetilde{\mathbf{H}}_{1;(t)}$ is determined by the following relations:

$$\widetilde{\mathbf{H}}_{1;(t)}^{\mathrm{H}} \mathbf{Q}_1^\star \widetilde{\mathbf{H}}_{1;(t)} = \mathbf{H}_{12,\mathrm{L}}^{-1} \overline{\mathbf{D}}_2^{-1} \left(\mathbf{R}_{2;(t)} - \sigma_2^2 \mathbf{I}_N\right) \left(\overline{\mathbf{D}}_2^{\mathrm{H}}\right)^{-1} \mathbf{H}_{12,\mathrm{L}}^{\mathrm{H}^{-1}} \tag{18}$$

with $\mathbf{H}_{12} = \begin{array}{c c} & \begin{array}{cc} \overset{N}{\longleftrightarrow} & \overset{M-N}{\longleftrightarrow} \end{array} \\ \begin{array}{c} m_1 \updownarrow \\ N \updownarrow \end{array} & \left[\begin{array}{cc} \mathbf{H}_{12,\mathrm{L}} & \mathbf{H}_{12,\mathrm{R}} \end{array}\right] \end{array}$. and

$$\begin{aligned} \overline{\mathbf{R}}_{2;(t)} = \mathbf{R}_2(\mathbf{C}_2, \overline{\mathbf{D}}_2, \mathbf{0}; \overline{\mathbf{C}}_{1;(t)}, \mathbf{U}_{11}^{\mathrm{H}}, \mathbf{P}_1^\star) = \\ \overline{\mathbf{D}}_2 \mathbf{H}_{12} \widetilde{\mathbf{H}}_{1;(t)}^{\mathrm{H}} \mathbf{Q}_1^\star \widetilde{\mathbf{H}}_{1;(t)} \mathbf{H}_{12}^{\mathrm{H}} \overline{\mathbf{D}}_2^{\mathrm{H}} + \sigma_2^2 \mathbf{I}_N \end{aligned}. \tag{19}$$

V. Numerical illustration

In the whole paper, we have made similar assumptions to works such as those on IWFA [12] that is: the channel is constant over many time-slots and the secondary received signal covariance matrix is assumed to be available at the secondary transmitter. In this setting, we have proved that the OIA conditions can be perfectly implemented. Although no simulations would be needed to illustrate this, we still wanted to see to what extent the procedure we propose is robust to imperfect covariance matrix feedback (this is an issue which is often neglected in the literature of radio resource allocation and interference alignment). For this, we have assumed that the covariance matrix is estimated from $S \geq 1$ samples. That is, the statistical covariance matrix $\mathbf{R}_2$ becomes $\widehat{\mathbf{R}}_2^{(S)} = \frac{1}{S} \sum_{t=1}^{S} \widetilde{y}_2(t) \widetilde{y}_2(t)^{\mathrm{H}}$. The performance criteria we consider correspond to the primary and secondary transmission rates are respectively denoted by ρ_{P} and ρ_{S}, and defined as:

$$\begin{cases} \rho_{\mathrm{P}} = \log_2 \left| \mathbf{I}_N + \mathbf{D}_1 \mathbf{H}_{11} \mathbf{C}_1 \mathbf{P}_1^\star \mathbf{C}_1^{\mathrm{H}} \mathbf{H}_{11}^{\mathrm{H}} \mathbf{D}_1^{\mathrm{H}} (\sigma_1^2 \mathbf{I}_N + \mathbf{E}_2)^{-1} \right| \\ \rho_{\mathrm{S}} = \log_2 \left| \mathbf{I}_N + \widehat{\mathbf{D}}_2 \mathbf{H}_{22} \widehat{\mathbf{C}}_2 \widehat{\mathbf{P}}_2 \widehat{\mathbf{C}}_2^{\mathrm{H}} \mathbf{H}_{22}^{\mathrm{H}} \widehat{\mathbf{D}}_2^{\mathrm{H}} (\sigma_2^2 \mathbf{I}_N + \mathbf{E}_1)^{-1} \right| \end{cases} \tag{20}$$

where $\mathbf{E}_2 = \mathbf{D}_1 \mathbf{H}_{21} \widehat{\mathbf{C}}_2 \widehat{\mathbf{P}}_2 \widehat{\mathbf{C}}_2^{\mathrm{H}} \mathbf{H}_{21}^{\mathrm{H}} \mathbf{D}_1^{\mathrm{H}}$, $\mathbf{E}_1 = \widehat{\mathbf{D}}_2 \mathbf{H}_{12} \mathbf{C}_1 \mathbf{P}_1^\star \mathbf{C}_1^{\mathrm{H}} \mathbf{H}_{12}^{\mathrm{H}} \widehat{\mathbf{D}}_2^{\mathrm{H}}$, and $\widehat{\mathbf{C}}_2$, $\widehat{\mathbf{P}}_2$, $\widehat{\mathbf{D}}_2$ are the coding and decoding matrices the secondary obtains by using $\widehat{\mathbf{R}}_2^{(S)}$ instead of $\mathbf{R}_2$.

Fig. 1 represents the secondary transmission rate (in bit/s per Hz) for a typical scenario considered in [1]: $M = N = 4$; all the entries of $\mathbf{H}_{ij}$ are independent and identically distributed (i.i.d.) and drawn from a complex Gaussian circularly symmetric entries with zero mean and variance 1; we assume that each transmitter has the same power constraint and same noise variance, i.e., $P_{1,\max} = P_{2,\max}$ and $\sigma_1 = \sigma_2$. The signal-to-noise ratio (SNR) is defined by $\mathrm{SNR} = \frac{P_{1,\max}}{\sigma_1^2}$. The number of realizations is chosen as 10000. There are three curves. The top curve represents the performance obtained when the proposed technique is implemented and the perfect knowledge of the covariance matrix is available; The curve then coincides with the one obtained in [1] by using perfect global CSI. The two other curves are obtained with $S = 100$ (middle curve) and $S = 10$ (bottom curve). For the range of most interest, that is, when $\mathrm{SNR} \in [0\mathrm{dB}, 5\mathrm{dB}]$, the rate loss induced by imperfect feedback is seen to be very reasonable (about 10%) for $S = 100$. When $S = 10$, the estimate is much noisier but even then the secondary rate loss is larger but shows the proposed technique is relatively robust.

Fig. 2 represents the transmission rate for the primary against the SNR when perfect covariance matrix feedback is available at the secondary transmitter (top curve) and when $S = 10$ (bottom curve). When $S = 10$, alignment is not perfect and the secondary creates some interference at the primary receiver. It is seen that even when the covariance matrix $\mathbf{R}_2$ is estimated from a small number of samples, the proposed implementation is of interest since the primary rate degradation is very small. Other simulations, not provided here, have confirmed this behavior. There might exist scenarios where this behavior is not observed, which would necessitate the design of a robust implementation, but this is left as an extension of the present paper.

References

[1] S. Medina Perlaza, N. Fawaz, S. Lasaulce, and M. Debbah, "From Spectrum Pooling to Space Pooling: Opportunistic Interference

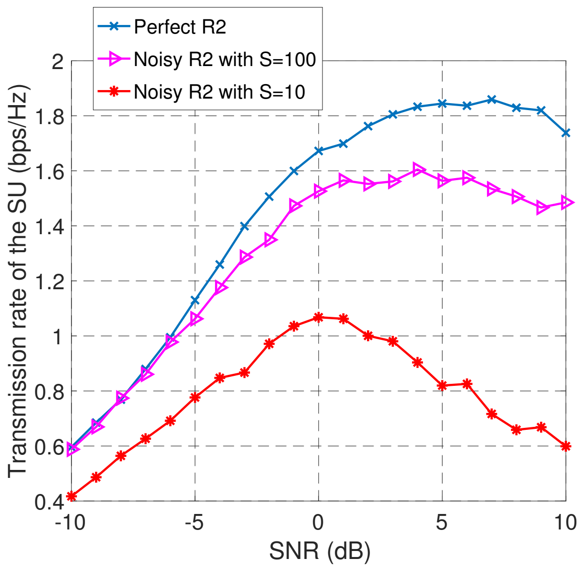

Figure 1. When the covariance feedback is perfect, the proposed implementation leads to the same performance as in [1], but under local information instead of global information. Even with a small number of samples for estimating the covariance matrix of the signal received at the secondary, the proposed distributed implementation of the OIA technique of [1] leads to good performance for the secondary.

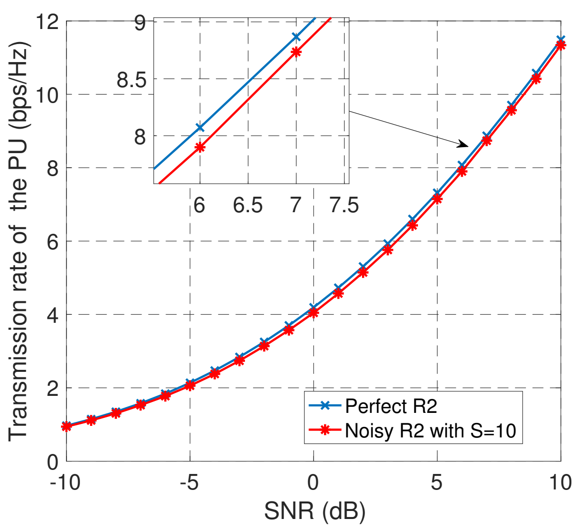

Figure 2. Even with a small number of samples for estimating the covariance matrix of the signal received at the secondary, the OIA condition is typically well verified, which allows the secondary to generate a very small amount of interference at the primary.

Alignment in MIMO Cognitive Networks", IEEE Transactions on Signal Processing, vol. 58, no. 7, pp. 3728-3741, 2010.

[2] G. Scutari, D. P. Palomar, and S. Barbarossa, "The MIMO iterative waterfilling algorithm", IEEE Transactions on Signal Processing, vol. 57, no. 5, pp. 1917-1935, 2009.

[3] S. Haykin, "Cognitive radio: brain-empowered wireless communications", IEEE Journal on Selected Areas in Communications, vol. 23, no. 2, pp 201-220, 2005.

[4] M. A. Maddah-Ali, A. K. Khandani, and A. S. Motahari, "Communication over X channel: Signalling and multiplexing gain", University of Waterloo, Tech. Rep., 2006.

[5] V. Cadambe and S. Jafar, "Interference alignment and degrees of freedom of the K-user interference channel", IEEE Transactions on Information Theory, vol. 54, no. 8, pp. 3425-3441, 2008.

[6] S. W. Peters and R. W. Heath, "Interference alignment via alternating minimization", IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), 2009.

[7] B. Nosrat-Makouei, J. G. Andrews, and R. W. Heath, "MIMO Interference Alignment Over Correlated Channels With Imperfect CSI", IEEE Transactions on Signal Processing, vol. 59, no. 6, pp. 2783-2794, 2011.

[8] O. E. Ayach, A. Lozano, and R. W. Heath, "On the Overhead of Interference Alignment: Training, Feedback, and Cooperation", IEEE Transactions on Wireless Communications, vol. 11, no. 11, pp. 4192-4203, 2012.

[9] C. G. Tsinos and K. Berberidis, "Blind Opportunistic Interference Alignment in MIMO Cognitive Radio Systems", IEEE Journal on Emerging and Selected Topics in Circuits and Systems, vol. 3, no. 4, pp. 626-639, 2013.

[10] C. Zhang, V. S. Varma, S. Lasaulce, and R. Visoz, "Interference Coordination via power domain channel estimation", IEEE Transactions on Wireless Communications, vol. 16, no. 10, pp. 6779-6794, 2017.

[11] E. Telatar, "Capacity of Multi-antenna Gaussian Channels", Eur. Trans. Telecommunication, vol. 10, no. 6, pp. 585-596, Nov. 1999.

[12] W. Yu, G. Ginnis, and J. Cioffi, "Distributed multiuser power control for digital subscriber lines", IEEE Journal on Selected Areas in Communications, vol. 20, no. 5, pp. 1105-1115, 2002.