A Taxonomy of Disposition-Parthood

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Abstract. We propose a taxonomy of mereological relations between dispositions, as well as a series of axioms relating the bearers, triggers or realizations of disposition-complexes and disposition-parts. We apply them on a use case to show how basic dispositions can be combined to express elaborate dispositions. We consider whether those relations of partial ordering satisfy axioms of composition and decomposition.

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1. Introduction

Dispositions are realizable entities, that is, properties that can be realized by certain types of processes, but that are present even when they are not realized [1, 2]. The Basic Formal Ontology (BFO) defines a disposition as a realizable entity "that exists because of certain features of the physical makeup of the independent continuant that is its bearer" ([2], p. 178). To be realized, dispositions need to be triggered by some process: for example, a fragile vase has a disposition to break (its realization) when undergoing a strong shock (its trigger). Dispositions are useful to represent causal relations in BFO, as they provide their bearers with certain causal powers. In particular, when a disposition is triggered by **t** and realized by **r**, this implies that **t** causes **r**.

Röhl and Jansen [3] propose a formalization of dispositions for single-track dispositions, that is, dispositions that have exactly one kind of realization given one specific bundle of triggering events and/or background conditions. Barton and Jansen [4] suggest first steps towards a formalization of multi-track dispositions, taking into account that there could be several kinds of triggers as well as several kinds of realizations. For this, they introduce a relation of parthood between dispositions. The present paper will extend those results by distinguishing several mereological relations that can be defined for dispositions.

In the following, terms for classes will be written in italic (e.g. "*Breaking*"), and terms for instances in bold (e.g. "$\mathbf{vase_0}$"). A statement of the form "disposition **d** of **b** to *R* when *T*" should be interpreted as expressing (in the vocabulary of [3]), when **d** is a single-track disposition, that (1) **d inheres_in b**, that (2) **d realized_in** only *R*, and that (3) **d has_trigger** only *T*. If **a** and **b** are particulars, **a+b** is defined as the mereological sum of **a** and **b**. Similarly, if *A* and *B* are classes or universals, *A+B* is defined as the class of mereological sums of one instance of *A* and one instance of *B*.

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We begin in section 2 by introducing three case studies illustrating different kinds of disposition-parthood which we will axiomatize in section 3. Section 4 will demonstrate the expressive power of the combined use of these kinds of disposition-parthood. Section 5 discusses whether disposition-parthood relations are authentic mereological relations and conclude.

2. Three case studies

The dispositional properties of an object and the way they are realized are often quite complex. In order to distinguish different varieties of such complexity, we introduce three case studies: the fragility of a vase, the solubility of a tablet, and the vulnerability of a human to a certain kind of poison.

2.1. (Vase): Mod-part and fragility

The case study (Vase) serves as an illustration for the kind of disposition-parthood that has been discussed in [4] under the more general label "**d-part_of**". We will call it "**mod-part_of**" here, in order to distinguish it from other varieties of disposition-parthood. This case study involves the fragile vase $\mathbf{vase_0}$, whose fragility can be formalized as a disposition. Three particular dispositions $\mathbf{disp\_break_0}$, $\mathbf{disp\_crack_0}$ and $\mathbf{fragility_0}$ inhere in $\mathbf{vase_0}$, defined as follows:

- $\mathbf{disp\_break_0}$ is the disposition of $\mathbf{vase_0}$ to *Breaking* when *Strong_shock*,
- $\mathbf{disp\_crack_0}$ is the disposition of $\mathbf{vase_0}$ to *Cracking* when *Light_shock*,
- $\mathbf{fragility_0}$ is the disposition of $\mathbf{vase_0}$ to [*Breaking* when *Strong_shock*] and [*Cracking* when *Light_shock*].

We can introduce a relation of disposition-parthood **mod-part_of** such that:

- $\mathbf{disp\_break_0}$ **mod-part_of** $\mathbf{fragility_0}$
- $\mathbf{disp\_crack_0}$ **mod-part_of** $\mathbf{fragility_0}$

We use the modifier "mod", derived from "mode", because $\mathbf{fragility_0}$ has two possible pathways, or modes, of being realized: via $\mathbf{disp\_break_0}$ or via $\mathbf{disp\_crack_0}$. We will now introduce two alternative kinds of disposition-parthood relations.

2.2. (Tablet): Add-part and solubility

The case study (Tablet) involves a medication tablet $\mathbf{tablet_0}$, composed of two halves $\mathbf{half\text{-}tablet_1}$ and $\mathbf{half\text{-}tablet_2}$ – that are both **part_of** $\mathbf{tablet_0}$. Let's introduce the following dispositions:

- $\mathbf{solubility_0}$ is the disposition of $\mathbf{tablet_0}$ to *Dissolving* when *Object_in_solvent*,
- $\mathbf{solubility_1}$ is the disposition of $\mathbf{half\text{-}tablet_1}$ to *Dissolving* when *Object_in_solvent*,
- $\mathbf{solubility_2}$ is the disposition of $\mathbf{half\text{-}tablet_2}$ to *Dissolving* when *Object_in_solvent.*

We can introduce another relation of disposition-parthoood **add-part_of** such that:

- **solubility$_1$ add-part_of solubility$_0$**
- **solubility$_2$ add-part_of solubility$_0$**

The modifier "add" is derived from "additive": the dissolving processes of both **half-tablet$_1$** and **half-tablet$_2$** compose the realization of **solubility$_0$**, and the triggers of the former two compose the trigger of the latter.

2.3. (Poison): Chain-part and vulnerability

The case study (Poison) involves a person Mr. Jones vulnerable to a given kind of poison, that causes cardiac arrest and consequently (cerebral) death. Let's introduce the following dispositions:

- **vulnerability_heart$_0$** is the disposition of **Jones** to *Cardiac arrest* when *Poison ingestion*,
- **heart_need$_0$** is the disposition of **Jones** to *Death* when *Cardiac arrest*,
- **vulnerability_heart_&_life$_0$** is the disposition of **Jones** to *Cardiac arrest* + *Death* when participating in an instance of *Poison ingestion.*

We introduce another relation of disposition-parthoood **chain-part_of** such that:

- **vulnerability_heart$_0$ chain-part_of vulnerability_heart_&_life$_0$**
- **heart_need$_0$ chain-part_of vulnerability_heart_&_life$_0$**

We use the modifier "chain" because the realization of **vulnerability_heart$_0$** triggers **heart_need$_0$**, thus creating a causal chain.

3. Axiomatization

As will become clear later, the relations **add-part_of** and **chain-part_of** share several commonalities. Thus, we introduce a relation **aggreg-part_of**, of which both are subrelations. We also introduce a general relation of dispositional-parthood noted **disp-part_of**, of which **mod-part_of** and **aggreg-part_of** are subrelations. All these relations are subrelations of the general **part_of** relation, which is axiomatized in the Relation Ontology [5] (cf. Table 1).

part_of
- **disp-part_of**
  - **mod-part_of**
  - **aggreg-part_of**
    - **add-part_of**
    - **chain-part_of**

Table 1: Taxonomy of parthood relations

The respective relations of proper parthood will be denoted by adding "**proper_**" in the name – e.g., **proper_mod-part_of**. We define a disp-complex (resp. mod-complex, aggreg-complex, etc.) as a disposition that has some proper disp-part (resp. mod-part, aggreg-part, etc.). I.e., for "[x]"="disp", "mod", "aggreg", "add" or "chain":

d_0 instance_of *[x]-complex* :⇔ **(∃d, d proper_[x]-part_of d_0)**

3.1. Bearer and disp-part_of

The bearer of a disp-part is a (proper or improper) part of the bearer of the disp-complex:

(DISP-BEARER)
d_1 disp-part_of d_0 ⇒ [(d_0 inheres_in b_0 ∧ d_1 inheres_in b_1) ⇒ b_1 part_of b_0]

In (Tablet), the bearer of **solubility**$_1$ (resp. **solubility**$_2$) is **half-tablet**$_1$ (resp. **half-tablet**$_2$), which is a part of **tablet**$_0$. In (Vase), the bearers of **fragility**$_0$, **disp_crack**$_0$ and **disp_break**$_0$ are the same, namely **vase**$_0$. Similarly, in (Poison), the bearers of **vulnerability_heart**$_0$, **heart_need**$_0$, and **vulnerability_heart_&_life**$_0$ are the same, namely **Jones**. However, we will see later in section 4.2 with the domino case study that the bearer of a mod-part (resp. chain-part) is not always identical to the bearer of a mod-complex (resp. chain-complex).

3.2. Realization and disp-part_of

We will now turn to the axioms concerning the realization of a disp-complex. In the general case, if a disp-complex is realized in a process, then at least one of its proper disp-parts is realized in a part of this process:

(DISP-REALIZATION)
(d_0 instance_of *Disp-complex* **∧ d_0 realized_in r_0) ⇒**
(∃d_1, ∃r_1, d_1 proper_disp-part_of d_0 ∧ r_1 part_of r_0 ∧ d_1 realized_in r_1)

As we will now see, both **mod-part_of** and **aggreg-part_of** satisfy more specific axioms. A mod-complex is realized in a process if and only if at least one of its mod-parts is realized in this very same process:

(MOD-REALIZATION)
(d_0 instance_of *Mod-complex* **∧ d_0 realized_in r_0) ⇔**
(∃d_1, d_1 proper_mod-part_of d_0 ∧ d_1 realized_in r_0)

In (Vase), **fragility**$_0$ is realized in a process (instance of *Cracking* or *Breaking*) if and only if either **disp_crack**$_0$ or **disp_break**$_0$ is realized in this process. Things are different for aggreg-parthood. If an aggreg-complex is realized in a process, then all its aggreg-parts are realized in a part of this process:

(AGGREG-REALIZATION)
(d_0 instance_of *Aggreg-complex* **∧ d_0 realized_in r_0) ⇒**
[∀d_1, d_1 proper_aggreg-part_of d_0 ⇒ (∃r_1, d_1 realized_in r_1 ∧ r_1 part_of r_0)]

In (Tablet), if **solubility**$_0$ is realized in **dissolving**$_0$, its aggreg-parts **solubility**$_1$ and **solubility**$_2$ are both realized, respectively in **dissolving**$_1$ and **dissolving**$_2$ (that are both parts of **dissolving**$_0$). In (Poison), if **vulnerability_heart_life**$_0$ is realized in **cardiac_arrest**$_0$+**death**$_0$, its aggreg-parts **vulnerability_heart**$_0$ and **heart_need**$_0$ are both realized, respectively in **cardiac_arrest**$_0$ and **death**$_0$.

Note however that if an aggreg-part is realized, this does not imply anything for the realization of the aggreg-complex. For example, if **half-tablet**$_1$ dissolves, this does not imply the dissolving of **tablet**$_0$ – as **half-tablet**$_2$ may not dissolve (in case it is not put into a solvent). Similarly, in (Poison), Jones might undergo some cardiac arrest which is not due to the ingestion of poison, in which case **heart_need**$_0$ will be realized without **vulnerability_heart_&_life**$_0$ being realized (because it was not triggered by any ingestion of poison).

3.3. Trigger and disp-part_of

We will now turn to the axioms concerning the triggers of a disp-complex. In general, if a disp-complex is triggered, then at least one of its proper disp-parts is triggered.

(DISP-TRIGGER)
(**d**$_0$ **instance_of** *Disp-complex* $\wedge$ **d**$_0$ **has_trigger t**$_0$) $\Rightarrow$
($\exists$**d**$_1$, $\exists$**t**$_1$, **d**$_1$ **proper_disp-part_of d**$_0$ $\wedge$ **d**$_1$ **has_trigger t**$_1$)

Again, both **mod-part_of** and **aggreg-part_of** satisfy more specific axioms. A mod-complex is triggered by a process if and only if at least one of its mod-parts is triggered by this very process:

(MOD-TRIGGER)
(**d**$_0$ **instance_of** *Mod-complex* $\wedge$ **d**$_0$ **has_trigger t**$_0$) $\Leftrightarrow$
($\exists$**d**$_1$, **d**$_1$ **proper_mod-part_of d**$_0$ $\wedge$ **d**$_1$ **has_trigger t**$_0$)

In (Vase), **fragility**$_0$ is triggered by a process (an instance of *Light shock* or *Strong shock*) if and only if either **disp_crack**$_0$ or **disp_break**$_0$ is triggered by this process. However, if an aggreg-complex is triggered, then *all* its aggreg-parts are triggered:

(AGGREG-TRIGGER)
(**d**$_0$ **instance_of** *Aggreg-complex* $\wedge$ **d**$_0$ **has_trigger t**$_0$) $\Rightarrow$
$\lceil$$\forall$**d**$_1$, **d**$_1$ **proper_aggreg-part_of d**$_0$ $\Rightarrow$ ($\exists$**t**$_1$, **d**$_1$ **has_trigger t**$_1$)]

In (Poison), if **vulnerability_heart_&_life**$_0$ is triggered by **ingestion_poison**$_0$, its aggreg-parts **vulnerability_heart**$_0$ and **heart_need**$_0$ are both triggered, respectively by **ingestion_poison**$_0$ and the **cardiac_arrest**$_0$ that followed. More specifically, if an add-complex is triggered by a process, then any of its add-parts is triggered by *a part* of this process:

(ADD-TRIGGER)
(**d**$_0$ **instance_of** *Add-complex* $\wedge$ **d**$_0$ **has_trigger t**$_0$) $\Rightarrow$
[$\forall$**d**$_1$, **d**$_1$ **proper_add-part_of d**$_0$ $\Rightarrow$ ($\exists$**t**$_1$, **d**$_1$ **has_trigger t**$_1$ $\wedge$ **t**$_1$ **part_of t**$_0$)]

In (Tablet), if **solubility**$_0$ is triggered by **tablet**$_0$**_in_solvent**, its add-parts **solubility**$_1$ and **solubility**$_2$ are both triggered, respectively by the processes **half-tablet**$_1$**_in_solvent** and **half-tablet**$_2$**_in_solvent** (that are both parts of the process **tablet**$_0$**_in_solvent**).

Note that replacing **add-part_of** by **chain-part_of** in the axiom (ADD-TRIGGER) does not lead to a true statement: as a matter of fact, when **vulnerability_heart_&_life$_0$** is triggered by **poison_ingestion$_0$**, the disposition **heart_need$_0$** is triggered by **cardiac_arrest$_0$**, which is not a part of **poison_ingestion$_0$**.

However, on top of (AGGREG-TRIGGER), we can state that the trigger of a chain-complex is the trigger of one of its chain-parts:

(CHAIN-TRIGGER)
(d_0 instance_of *Chain-complex* $\wedge$ **d_0 has_trigger t_0)** $\Rightarrow$
($\exists d_1$, d_1 proper_chain-part_of d_0 $\wedge$ d_1 has_trigger t_0)

In (Poison), the trigger of **vulnerability_heart_&_life$_0$** is **poison_ingestion$_0$**, which is a trigger of **vulnerability_heart$_0$**.

Note, however, that if an aggreg-part is triggered, this does not imply anything for the triggering of the aggreg-complex. For example, in (Tablet), if **solubility$_1$** is triggered by **half-tablet$_1$_in_solvent**, this does not imply the triggering of **solubility$_0$** – as the **solubility$_2$** of **half-tablet$_2$** may not be triggered (in case **half-tablet$_2$** is not put into a solvent). Similarly, in (Poison), Jones might undergo some cardiac arrest which is not due to the ingestion of poison, in which case **heart_need$_0$** is triggered without **vulnerability_heart_&_life$_0$** being triggered.

4. Composition of various disposition-parthood relations

We will now show how we can use disposition-parthood relations to combine basic dispositions into mereological complexes, by developing case studies involving dominos.

4.1. Domino arrangement

Let's consider five dominos **domino$_i$** (for any integer $i \in [0,4]$) vertically placed on their short edge, such that they will fall if pushed. Each **domino$_i$** is thus the bearer of a disposition **unstability$_{\leftarrow i}$** that is realized by **domino$_i$** falling to the left (a process that is an instance of the defined class $Fall_{\leftarrow i}$) and triggered when **domino$_i$** is pushed to the left ($Push_{\leftarrow i}$); and the disposition **unstability$_{\rightarrow i}$** realized by **domino$_i$** falling to the right ($Fall_{\rightarrow i}$) when **domino$_i$** is pushed to the right ($Push_{\rightarrow i}$). It also has a disposition **unstability$_{\rightarrow i / \leftarrow i}$** to fall to either side ($Fall_i$ *equivalentTo* ($Fall_{\leftarrow i}$ or $Fall_{\rightarrow i}$)) when pushed ($Push_i$ *equivalentTo* ($Push_{\leftarrow i}$ or $Push_{\rightarrow i}$)). We can characterize this by stating:

- **unstability$_{\rightarrow i / \leftarrow i}$ realized_in** only $Fall_i$
- **unstability$_{\rightarrow i / \leftarrow i}$ has_trigger** only $Push_i$

However, those two relations do not express the information that a $Push_{\leftarrow i}$ always leads to a $Fall_{\leftarrow i}$, and that a $Push_{\rightarrow i}$ always leads to a $Fall_{\rightarrow i}$. Therefore, it is more informative to say that **unstability$_{\rightarrow i / \leftarrow i}$** has the two mod-parts **unstability$_{\rightarrow i}$** and **unstability$_{\leftarrow i}$** and to ascribe the respective realizations and triggers to the mod-parts:

- **unstability$_{\rightarrow i}$ mod-part_of unstability$_{\rightarrow i / \leftarrow i}$**
- **unstability$_{\leftarrow i}$ mod-part_of unstability$_{\rightarrow i / \leftarrow i}$**

The five dominos 0 to 4 are arranged as illustrated on figure 1a below (dominos 5 to 8 from figure 1b will be considered later):

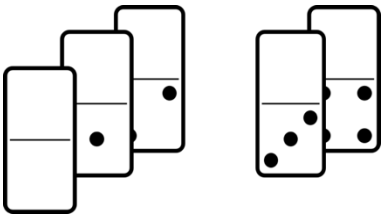

Figure 1a

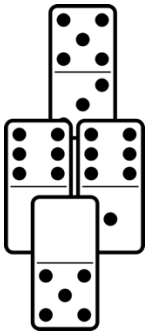

Figure 1b

domino$_2$ is placed to the right (when seen from the domino's side) of **domino$_1$**, such that when **domino$_1$** falls to the right, it pushes **domino$_2$** to the right; therefore, in this configuration, the endphase of an instance of $Fall_{\rightarrow 1}$ is also an instance of $Push_{\rightarrow 2}$.

Let's define **unstability$_{\rightarrow 1,\rightarrow 2}$** as the disposition of **domino$_1$** and **domino$_2$** to fall to the right (a process named $Fall_{\rightarrow 1 \wedge \rightarrow 2} = Fall_{\rightarrow 1} + Fall_{\rightarrow 2}$) when **domino$_1$** is pushed to the right ($Push_{\rightarrow 1}$). We have:

- **unstability$_{\rightarrow 1}$,$_{\rightarrow 2}$ realized_in** only $Fall_{\rightarrow 1 \wedge \rightarrow 2}$
- **unstability$_{\rightarrow 1}$,$_{\rightarrow 2}$ has_trigger** only $Push_{\rightarrow 1}$

However, those two relations do not express the information that there is a causal chain. Therefore, it is more informative to write that **unstability$_{\rightarrow 1,\rightarrow 2}$** is a chain-complex composed by two chain-parts **unstability$_{\rightarrow 1}$** and **unstability$_{\rightarrow 2}$**:

- **unstability$_{\rightarrow 1}$ chain-part_of unstability$_{\rightarrow 1,\rightarrow 2}$**
- **unstability$_{\rightarrow 2}$ chain-part_of unstability$_{\rightarrow 1,\rightarrow 2}$**

Suppose that **domino$_3$** is placed next to **domino$_1$**, such that when one of these falls, it does not push the other. Then, the system **domino$_{1+3}$ = domino$_1$ + domino$_3$** has a disposition **unstability$_{\rightarrow 1:\rightarrow 3}$** of both dominos falling to the right ($Fall_{\rightarrow 1 \wedge \rightarrow 3} = Fall_{\rightarrow 1} + Fall_{\rightarrow 3}$) when both dominos are pushed to the right ($Push_{\rightarrow 1 \wedge \rightarrow 3} = Push_{\rightarrow 1} + Push_{\rightarrow 3}$):

- **unstability$_{\rightarrow 1:\rightarrow 3}$ realized_in** only $Fall_{\rightarrow 1 \wedge \rightarrow 3}$
- **unstability$_{\rightarrow 1:\rightarrow 3}$ has_trigger** only $Push_{\rightarrow 1 \wedge \rightarrow 3}$

However, those two relations do not express the information that when an instance **push$_{\rightarrow 1}$ + push$_{\rightarrow 3}$** causes an instance of **fall$_{\rightarrow 1}$ + fall$_{\rightarrow 3}$**, it is **push$_{\rightarrow 1}$** that causes **fall$_{\rightarrow 1}$**, and **push$_{\rightarrow 3}$** that causes **fall$_{\rightarrow 3}$**. Therefore, it is more informative to write that **unstability$_{\rightarrow 1:\rightarrow 3}$** is an add-complex with two proper add-parts **unstability$_{\rightarrow 1}$** and **unstability$_{\rightarrow 3}$**:

- **unstability$_{\rightarrow 1}$ add-part_of unstability$_{\rightarrow 1:\rightarrow 3}$**
- **unstability$_{\rightarrow 3}$ add-part_of unstability$_{\rightarrow 1:\rightarrow 3}$**

domino$_4$ is placed to the right of **domino$_3$** (like **domino$_2$** was placed to the right of **domino$_1$**), such that when **domino$_3$** falls to the right, it pushes **domino$_4$** to the right. The disposition to $Fall_{\rightarrow 3 \wedge \rightarrow 4}$ when $Push_{\rightarrow 3}$ is noted **unstability$_{\rightarrow 3,\rightarrow 4}$**.

Finally, **domino$_0$** is placed to the left of **domino$_1$**, such that when **domino$_1$** falls to the left, it pushes **domino$_0$** to the left. The chain-complex disposition to $Fall_{\leftarrow 1 \wedge \leftarrow 0}$ when $Push_{\leftarrow 1}$ is noted **unstability$_{\leftarrow 1,\leftarrow 0}$**.

4.2. Composition of mod-part_of and chain-part_of

We can now combine these dispositions into more complex dispositions. **domino$_{1,2}$ = domino$_1$ + domino$_2$** falls to the right ($Fall_{\rightarrow 1 \wedge \rightarrow 2}$) when **domino$_1$** is pushed to the right ($Push_{\rightarrow 1}$), and **domino$_{1,0}$** falls to the left ($Fall_{\leftarrow 1 \wedge \leftarrow 0}$) when **domino$_1$** is pushed to the left ($Push_{\leftarrow 1}$). We can introduce the corresponding disposition **unstability$_{(\rightarrow 1,\rightarrow 2)\ /\ (\leftarrow 1,\leftarrow 0)}$**, which is the disposition of [$Fall_{\rightarrow 1 \wedge \rightarrow 2}$ when $Push_{\rightarrow 1}$] and of [$Fall_{\leftarrow 1 \wedge \leftarrow 0}$ when $Push_{\leftarrow 1}$] (see figure 2). That is, **unstability$_{(\rightarrow 1,\rightarrow 2)\ /\ (\leftarrow 1,\leftarrow 0)}$** is a mod-complex composed by the mod-parts **unstability$_{\rightarrow 1,\rightarrow 2}$** and **unstability$_{\leftarrow 1,\leftarrow 0}$**:

- **unstability$_{\rightarrow 1,\rightarrow 2}$ mod-part_of unstability$_{(\rightarrow 1,\rightarrow 2)\ /\ (\leftarrow 1,\leftarrow 0)}$**
- **unstability$_{\leftarrow 1,\leftarrow 0}$ mod-part_of unstability$_{(\rightarrow 1,\rightarrow 2)\ /\ (\leftarrow 1,\leftarrow 0)}$**

Moreover, by transitivity of the relation **disp-part_of** (of which **mod-part_of** and **chain-part_of** are subrelations), we can state that **unstability$_{(\rightarrow 1,\rightarrow 2)\ /\ (\leftarrow 1,\leftarrow 0)}$** is a disp-complex composed by the disp-parts **unstability$_{\rightarrow 1}$**, **unstability$_{\rightarrow 2}$**, **unstability$_{\leftarrow 1}$** and **unstability$_{\leftarrow 0}$**.

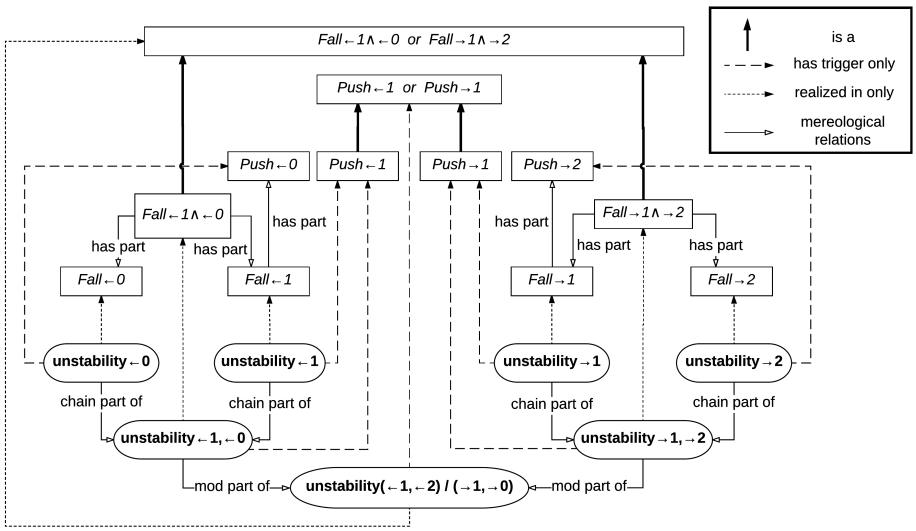

Figure 2. Conjoint use of mod-part_of and chain-part_of

We have pointed out earlier that the bearer of a mod-part (resp. chain-part) is a part of the bearer of a mod-complex (resp. chain-complex), but not necessarily the same bearer; as a matter of fact:

- The bearer of the chain-complex $\mathbf{unstability_{\leftarrow 1,\leftarrow 0}}$ is $\mathbf{domino_{1,0}}$; whereas the bearer of its chain-parts $\mathbf{unstability_{\leftarrow 1}}$ and $\mathbf{unstability_{\leftarrow 0}}$ are $\mathbf{domino_1}$ and $\mathbf{domino_0}$, respectively.
- The bearer of the mod-complex $\mathbf{unstability_{(\leftarrow 1,\leftarrow 0)\,/\,(\rightarrow 1,\rightarrow 2)}}$ is $\mathbf{domino_{0,1,2}}$; whereas the bearers of its mod-parts $\mathbf{unstability_{(\leftarrow 1,\leftarrow 0)}}$ and $\mathbf{unstability_{(\rightarrow 1,\rightarrow 2)}}$ are $\mathbf{domino_{1,0}}$ and $\mathbf{domino_{1,2}}$, respectively.

4.3. Further compositions

We could also express a large variety of dispositions thanks to the various parthood relations introduced here. For example, $\mathbf{domino_{1,3}}$ has a disposition to [$Fall_{\rightarrow 1 \wedge \rightarrow 3}$ when $Push_{\rightarrow 1 \wedge \rightarrow 3}$] and to [$Fall_{\leftarrow 1 \wedge \leftarrow 3}$ when $Push_{\leftarrow 1 \wedge \leftarrow 3}$], which is the mod-complex $\mathbf{unstability_{(\rightarrow 1:\rightarrow 3)\,/\,(\leftarrow 1:\leftarrow 3)}}$, composed by the add-complexes $\mathbf{unstability_{\rightarrow 1:\rightarrow 3}}$ and $\mathbf{unstability_{\leftarrow 1:\leftarrow 3}}$.

To illustrate other configurations, consider the domino arrangement on Figure 1b. $\mathbf{domino_5}$ is placed on the left of $\mathbf{domino_6}$ and $\mathbf{domino_7}$, in-between, such that if $\mathbf{domino_5}$ falls to the right, then both $\mathbf{domino_6}$ and $\mathbf{domino_7}$ also fall to the right. There is an underlying disposition to $Fall_{\rightarrow 5 \wedge \rightarrow 6 \wedge \rightarrow 7}$ when $Push_{\rightarrow 5}$, which is the add-complex $\mathbf{unstability_{(\rightarrow 5,\rightarrow 6):(\rightarrow 5,\rightarrow 7)}}$ composed by the chain-complexes $\mathbf{unstability_{\rightarrow 5,\rightarrow 6}}$ and $\mathbf{unstability_{\rightarrow 5,\rightarrow 7}}$. Now, $\mathbf{domino_8}$ is placed on the right of $\mathbf{domino_6}$ and $\mathbf{domino_7}$, such that if either $\mathbf{domino_6}$ *or* $\mathbf{domino_7}$ falls to the right, then $\mathbf{domino_8}$ also falls to the right. The underlying disposition to [$Fall_{\rightarrow 6 \wedge \rightarrow 8}$ when $Push_{\rightarrow 6}$] and to [$Fall_{\rightarrow 7 \wedge \rightarrow 8}$ when $Push_{\rightarrow 7}$] is the mod-complex $\mathbf{unstability_{(\rightarrow 6,\rightarrow 8)\,/\,(\rightarrow 7,\rightarrow 8)}}$ composed by the chain-complexes $\mathbf{unstability_{\rightarrow 6,\rightarrow 8}}$ and $\mathbf{unstability_{\rightarrow 7\,,\rightarrow 8}}$.

Suppose now that $\mathbf{domino_8}$ is more massive, such that it needs both $\mathbf{domino_6}$ *and* $\mathbf{domino_7}$ to fall to the right for $\mathbf{domino_8}$ to also fall to the right. The underlying disposition to $Fall_{\rightarrow 6 \wedge \rightarrow 7 \wedge \rightarrow 8}$ when $Push_{\rightarrow 6 \wedge \rightarrow 7}$ is the chain-complex $\mathbf{unstability_{(\rightarrow 6:\rightarrow 7)\,,\rightarrow 8}}$ composed by the add-complex $\mathbf{unstability_{\rightarrow 6:\rightarrow 7}}$ as well as by $\mathbf{unstability_{\rightarrow 8}}$.

Finally, we could compose dispositions in more intricate ways, mixing **mod-part_of**, **add-part_of** and **chain-part_of** – consider for example the disposition $\mathbf{unstability_{|(\rightarrow 6:\rightarrow 7)\,,\rightarrow 8|\,/\,|(\leftarrow 6:\leftarrow 7)\,,\leftarrow 5|}}$ to [$Fall_{\rightarrow 6 \wedge \rightarrow 7 \wedge \rightarrow 8}$ when $Push_{\rightarrow 6 \wedge \rightarrow 7}$] and to [$Fall_{\leftarrow 6 \wedge \leftarrow 7 \wedge \leftarrow 5}$ when $Push_{\leftarrow 6 \wedge \leftarrow 7}$].

5. Discussion and conclusion

This paper has presented different ways to compose dispositions together: mod-parthood and aggreg-parthood, and the subrelations of the latter add-parthood and chain-parthood. All of them are subsumed by disp-parthood, which satisfy some axioms (DISP-BEARER), (DISP-TRIGGER) and (DISP-REALIZATION); and each of those subrelations also satisfy more specific axioms.

All these relations of disposition-parthood are partial orders: they are reflexive, transitive and anti-symmetric. To be authentic mereological relations, however, they also need to satisfy axioms of composition and decomposition (see [4] for a few preliminary remarks concerning **mod-part_of**). It can easily be seen that composition axioms can be introduced at least for some disposition-parthood relations. For any two dispositions $\mathbf{d_1}$ and $\mathbf{d_2}$ (with bearers $\mathbf{b_1}$ and $\mathbf{b_2}$) there exists a mod-complex noted ($\mathbf{d_1}$/$\mathbf{d_2}$) and an add-

complex noted ($\mathbf{d_1}$:$\mathbf{d_2}$), which inheres in $\mathbf{b_1}$+$\mathbf{b_2}$. On the other hand, there exist a chain complex ($\mathbf{d_1}$,$\mathbf{d_2}$) (also inhering in $\mathbf{b_1}$+$\mathbf{b_2}$) only if the maximally specified realization class of $\mathbf{d_1}$ has as part the class of triggers of $\mathbf{d_2}$. By introducing such axioms of composition, we commit ourselves to a lot of strange disposition complexes, such as complexes of dispositions of different bearers, that might even exist at different times. This, however, is no special feature of disp-parthood: the same phenomenon occurs in standard mereology. Moreover, the formal commitment to the existence of these mereological sums does not imply any commitment that all of them have causal or explanatory relevance.

Consider now axioms of decomposition. Although this would require further investigations, the following axiom of complementation [6] seems to be satisfied for **P** being any of the five relation of disposition-parthood mentioned above, and **O** the associated relation of overlapping:

$$\neg \mathbf{P}yx \rightarrow \exists z\, \forall w\, (\mathbf{P}wz \leftrightarrow (\mathbf{P}wy \wedge \neg \mathbf{O}wx))$$

Note that our taxonomy of relations of disposition-parthood is not meant to be exhaustive. For example, one could consider the disposition realized by (only) $\mathbf{domino_2}$ falling to the right when $\mathbf{domino_1}$ is pushed to the right – not to be confused with the above disposition $\mathbf{unstability_{\rightarrow 1,\rightarrow 2}}$ realized by *both* $\mathbf{domino_1}$ and $\mathbf{domino_2}$ falling to the right when $\mathbf{domino_1}$ is pushed to the right.

By way of conclusion we point out that we expect disposition-parthood to be a powerful tool to explore relations between medical entities that have been formalized as dispositions, such as diseases [7, 8] or medical risks [9].

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