

A Novel Image Segmentation Method Based on An Improved Bacterial Foraging Optimization Algorithm

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Received July, 2016; revised December, 2016

ABSTRACT. *When some bionic optimization algorithms are used for image segmentation, we find that the search speeds of these algorithms are slow and the local searching abilities of these algorithms need be improved. In order to solve these problems, this paper proposed a new image segmentation method based on the improved bacterial foraging optimization algorithm. Firstly, a dynamic step size is used to instead of the fixed step size of the chemotaxis operator, and a dynamic probability is used to instead of the fixed probability of elimination-dispersal operator. Then, the gray histograms of the images are extracted for the image segmentation. Ultimately, the images are segmented using the improved bacterial foraging optimization algorithm. The image segmentation results show that the accuracy and the speed of the image segmentation based on the improved bacterial foraging optimization algorithm are superior to the ones that based on others traditional bionic optimization algorithms.*

Keywords: Bacterial foraging optimization, Image segmentation, Gray histogram, Feature extraction

1. Introduction. It is well known that image segmentation [14] is a very fundamental and challenging issue of image processing. The results of image segmentation will directly affect the next work in many fields, such as image analysis [15], image classification [16] and image recognition [1]. Currently, many algorithms of image segmentation are proposed by researchers. Simultaneously, image segmentation algorithms based on bionic intelligent optimization, such as genetic algorithm (GA) [2], ant colony optimization (ACO) [3], particle swarm optimization (PSO) [4] and cat swarm optimization (CSO) [5], attract more and more people's attention. In [6], a genetic algorithm is used to segment the skins lesion image, but the speed of the image segmentation is slow. In [7], an ant colony optimization algorithm is used to segment MR brain images. However, the accuracy of the image segmentation needs to be improved further. In [8], a particle swarm optimization algorithm is used for the image segmentation. Although the speed of the image segmentation is fast, the accuracy of the segmentation need be enhanced.

Additionally, as one of bionic intelligent optimization algorithms, bacterial foraging optimization (BFO) [9] which consists of three principal mechanisms, namely, chemotaxis, reproduction, elimination and dispersal, is often used in the field of image segmentation because of its high robustness and good global search capability. For examples, in [11], color lip images are segmented by BFO based on optimal thresholding; in [12], BFO is used to implement the operation of edge detection; in [13], image segmentation is implemented using BFO with varying population; in [17], an improved version of classical bacterial foraging algorithm is proposed for fuzzy entropy optimization when it is applied to the segmentation of gray images. However, the best bacterial individual in BFO algorithms mentioned above, may eliminate at the later evolution stage because of the fixed step size of chemotaxis and the fixed probability of elimination and dispersal.

In order to solve the problems of the traditional bionic optimization algorithms mentioned above and improve the speed and the accuracy of the image segmentation, a new image segmentation method is proposed based on the improved bacterial foraging optimization (IBFO) in this paper.

The structure of this paper is as follows. Section 2 described the overview of the bacterial foraging optimization algorithm. Section 3 proposed an improved bacterial foraging optimization. Section 4 described the image segmentation method based on the improved bacterial foraging optimization. Section 5 shows the experiment results of the image segmentation. Finally, concluding remarks and possible future research directions are given in section 6.

2. Overview of the Bacterial foraging optimization. As a new element of bionic intelligent algorithms, BFO is proposed by K.M.Passino in 2002. The idea of the algorithm is based on the application of group foraging strategies of *Escherichia coli* bacteria. It simulates the behaviors of *Escherichia coli* Bacteria to search for nutrients in a manner to maximize the energy obtained per unit time through methods for locating, handling, and ingesting food. The mechanism of BFO will be reviewed in following subsection.

2.1. Chemotaxis of BFO. In this stage, bacterium starts to tumble in a direction depending on the rotation of the flagellum. If the bacterium finds that the amount of food is better in the new direction, then it begins to swim in that direction. In this way, the bacterium will alternatively tumble and swim in random directions to search for nutrients. So the chemotaxis can be described as follows. Assume that $S^i(j, k, l)$ is the position of the i -th bacterium. This indicates the i -th bacterium at the j -th chemotaxis, the k -th reproduction and the l -th elimination and dispersal. New location of bacterium is adjusted as:

$$\begin{aligned} S^i(j+1, k, l) &= S^i(j, k, l) + R \times step \times \phi(i) \\ &= S^i(j, k, l) + R \times step \times \frac{S^i(j, k, l) - S^{rand}(j, k, l)}{\|S^i(j, k, l) - S^{rand}(j, k, l)\|} \end{aligned} \quad (1)$$

Where R is a number between 0 and 1, $step$ is the size of the step in the random direction specified by the tumble, and the random position $S^{rand}(j, k, l)$ is a neighbourhood of $S^i(j, k, l)$. The bacterial foraging process, such as swimming and tumbling, can be taken as the process of optimization.

2.2. Reproduction of BFO. After the chemotaxis operation has been implemented, about half of the bacterium, which have no advantage of searching for food, will die, but others are healthy. Each healthy bacterium will split into two bacteria which are left at the previous place. This process keeps the good individuals and deletes bad ones, which wildly increase the speed of searching. Simultaneously, it keeps the number of bacteria constant.

2.3. Elimination and Dispersal of BFO. Bacteria will die or be dispersed because of food consumption or temperature. This may disturb the process of chemotaxis, but the dispersion of bacteria can place them in areas with sufficient nutrition. In this stage, bacteria are prevented from being trapped in the local optimum points. For each bacterium in the group, the possibility of elimination and dispersal is equal to p_{ed} . To keep the number of bacteria constant, in case a bacterium is eliminated, another bacterium will be placed randomly in the search space.

Bacteria never satisfy with the amount of food, so they are trying to swim from places with low nutrients to places with high nutrients. Therefore, the three parts of BFO: Chemotaxis, Reproduction, Elimination and Dispersal process are continuous and effective.

3. The Improved Bacterial foraging optimization. It is well known that BFO always use the fixed step size to perform the chemotaxis operation. This may waste time to search back and forth around the optimal location. So, the dynamic step size can be used for the chemotaxis. Simultaneously, the best individual may be lost because the elimination and dispersal operation is implemented for each bacterium using the fixed probability. Hence, the self-adaptive probability will be used for the operation of the elimination and dispersal.

3.1. Improvement of Chemotaxis. The step size of the chemotaxis has a great impact on the search ability of the traditional BFO. If the step size is large, the global search ability of BFO will be strong, but the search precision of BFO will be decreased. If the step size is small, the local optimization ability of BFO will be powerful, but the search efficiency of BFO is reduced. Therefore, in order to increase the performance of BFO, the dynamic step size is used for the chemotaxis as follows:

$$step_c(i, j + 1) = \begin{cases} step_{init}(i) & j = 0 \\ \omega step_c(i, j) & j > 0 \end{cases} \quad (2)$$

Where the reduction coefficient ω of the step size in the $i - th$ step can be obtained according to equation $\omega(t) = \frac{1}{t \cdot C_a + C_b}$ $t \geq 0$, $step_c(i, j)$ is the step size of the $j - th$ chemotaxis operation of the $i - th$ bacterium, C_a, C_b are random constants, and $step_{init}(i)$ is the initial step size of the chemotaxis operation of the $i - th$ bacterium.

3.2. Improved of the Elimination and Dispersal. The migration probability P_{ed} in the elimination-dispersal operation of BFO is used to control the elimination and migration of the bacteria, and to prevent BFO from falling into the local optimum. However, the migration probability P_{ed} in the traditional BFO is a constant. In this case, if the bacteria around the optimal solution perform the elimination-dispersal operation, the population may lose the individuals with high fitness. So the self-adaptive probability will be used for the elimination and dispersal operation of BFO. It can be represented as follows:

$$P_{ed}^{i,j} = \begin{cases} \frac{J_c^{\max} - J_c^{i,j}}{J_c^{\max} - J_c^{\min}} \times P_{ed} & J_c^{i,j} \geq (J_c^{\max} + J_c^{\min})/2 \\ P_{ed} & J_c^{i,j} < (J_c^{\max} + J_c^{\min})/2 \end{cases} \quad (3)$$

Where $P_{ed}^{i,j}$ is the migration probability of the bacterium i in process of the $j - th$ elimination and dispersion operation, $J_c^{i,j}$ is the fitness value of the bacterium i in the $j - th$ elimination and dispersion, J_c^{\min} is the smallest fitness value of the individuals, J_c^{\max} is the largest fitness value of the individuals, P_{ed} is the initial probability.

4. Image segmentation based on IBFO. In this section, the principle and the detailed process of image segmentation are described in following subsection.

4.1. The principle of image segmentation. After the histograms $X = \{X_i, i = 1, 2, \dots, L-1\}$ of the image $f(x, y)$ are obtained, it can be used for the image segmentation. In order to segment $f(x, y)$, we will find an optimal partition $C = \{C_1, C_2, \dots, C_k\}$ which meets the conditions that $C_1 \cup C_2 \cup \dots \cup C_k = C$ and $C_i \cap C_j = \phi$ ($i \neq j, 0 < i, j \leq k$), that is to say, the optimal partition C make J_C minimum. J_C is the sum of total within-class scatter and it can be computed according to the following equation.

$$J_c = \sum_{j=1}^k \sum_{x_i \in C_j} \|X_i - C_j\|^2 \quad (4)$$

Where $\|X_i - C_j\|^2$ is the squared Euclidean distance between the feature vector X_i and the centroids of the cluster C_j .

4.2. Feature extraction of images. Before the operation of the image segmentation is implemented using the IBFO algorithm, the feature description of the original images will be obtained. In this paper, the histogram of the images can be used to distinguish between the background pixels and the foreground pixels. For continuous images, the histogram can be defined as:

$$X(D) = \lim_{\Delta D \rightarrow 0} \frac{A(D) - A(D + \Delta D)}{D - (D + \Delta D)} = \lim_{\Delta D \rightarrow 0} \frac{A(D) - A(D + \Delta D)}{-\Delta D} = -\frac{d}{dD}A(D) \quad (5)$$

, where $A(D)$ is the area function: an area of the image is enclosed by the outlines whose gray level are D , and the area can be computed by the number of pixels with the gray level D . For discrete functions, ΔD is fixed to 1, so the function can be rewritten as $X(D) = A(D) - A(D + 1)$ Where $D = 0, 1, \dots, L - 1$ and L is the number of gray levels.

4.3. The specific steps of image segmentation based on IBFO. The specific steps of the image segmentation based on IBFO can be described as follows.

Step 1 Input the image $f(x, y)$ and obtain the histograms $X = \{X_i, i = 1, 2, \dots, L-1\}$ of $f(x, y)$. Set parameters of IBFO, such as *PoSize* (Population size), *CenNum* (the number of the clusters), N_c (Chemotaxis steps), N_s (Swim steps), N_{re} (Reproductive steps), N_{ed} (Elimination and dispersal steps), P_{ed} (Elimination Probability), S_r (Reproduction Probability). Initialize all the bacteria randomly: each bacterium $B_i = \{l_i, f_i\}$ consists of two parts, namely, the location $l_i = [C_1, C_2, \dots, C_k]$ and the fitness value f_i

Step 2 Calculate the fitness value f_i of each bacterium according to the equation $f_i = 1/J_C$, and record the best bacterium.

Step 3 The bacteria implement the operation of chemotaxis.

Step 4 Determine whether the number of bacterial chemotaxis operation is greater than N_c . If the answer is yes, go to step 5, otherwise, return to step 3.

Step 6 Calculate the fitness value f_i of each bacterium in the population again. Then the bacteria will be sorted by their fitness values in descending order.

Step 7 The bacteria implement the operation of reproduction. Determine whether the number of reproduction operation is greater than N_{re} . If the answer is yes, go to step 7, otherwise, return to step 3.

Step 8 The bacteria implement the operation of elimination and dispersion. Determine whether the number of elimination-dispersal operation is greater than N_{ed} . If the answer is no, return to step 3, otherwise, output the results of the image segmentation.

5. Experimental Results and Analysis. In experiments, a computer is used. The memory of the computer is 2.0 GB, the processors is *IntelCoreTM2DuoE7500@2.93GHZ*, the operating system is Windows 7, and the simulation software of the experiments is matlabR2012a. The images used in the experiment are selected from Berkeley image database (<http://www.eecs.berkeley.edu/Research/Projects/CS/vision/bsds/>) to evaluate the performance of the image segmentation method proposed in this paper.

5.1. Parameter Settings of different algorithms. In order to verify the validity of the image segmentation method based on IBFO, different algorithms, such as ACO, PSO, CSO, BFO, are used to segment images under the same experiment conditions. The parameters of those algorithms are set as Table 1. Where *iterNum* is the number of iterations of algorithms, *Step* is the step size of the chemotaxis operation, *SMP* represents the memory pool, *SRD* is seeking range of the selected dimension. *CDC* represents counts of dimension to change, *MR* is a mixture ration, *R1*, *R2* are the random data between 0 and 1, *C* is the acceleration coefficient. w_{\max} represents the maximum speed of particles; w_{\min} represents the minimum speed of particles; *C1*, *C2* are the learning factor, *rho* represents pheromones residual coefficients, and *q* represents the pheromone intensity.

TABLE 1. Parameters of different algorithms

Algorithms		Parameters							
IBFO	PoSize	CenNum	N_c	N_s	N_{re}	N_{ed}	C_a	S_r	C_b
	10	3	10	1	1	1	2	0.5	1
BFO	PoSize	CenNum	N_c	N_s	N_{re}	N_{ed}	P_{ed}	S_r	<i>step</i>
	10	3	10	1	1	1	0.25	0.5	2
CSO	PoSize	CenNum	iterNum	SMP	SRD	CDC	MR	R1	<i>C</i>
	10	3	10	5	0.5	1	0.5	[0 1]	2
PSO	PoSize	CenNum	iterNum	w_{\max}	w_{\min}	C1	C2	R1	R2
	10	3	10	2	1	2	2	[0 1]	[0 1]
ACO	PoSize	CenNum	iterNum	rho	q				
	10	3	10	0.1	0.5				

5.2. The global search ability of IBFO. In this section, the Griewank function and the Schaffer function are used to test the global search ability of IBFO firstly. The Griewank function is defined as follows.

$$\min f(x_i) = \sum_{i=1}^N \frac{x_i^2}{4000} - \prod_{i=1}^N \cos\left(\frac{x_i}{\sqrt{i}}\right) + 1 \quad (6)$$

where $x_i \in [-600, 600]$. The function has many local minimum points and the number of the points is related to the dimension of the problem which will be deal with. This function is a typical nonlinear multi-modal function, which has a wide range of search space, and is generally considered to be a complex multi-modal problem which is difficult to deal with for optimization algorithms. When $(x_1, x_2, \dots, x_n) = (0, 0, \dots, 0)$, the global minimum value 0 of the function can be obtained. To test the performance of IBFO, different optimization algorithms, which are mentioned in subsection 5.1, are used to get the solution of equation $Y = \frac{1}{\min f(x_i)+1}$, and the results are shown in figure 1. the Schaffer function is defined as

$$\min f(x_1, x_2) = 0.5 + \frac{(\sin \sqrt{x_1^2 + x_2^2})^2 - 0.5}{(1 + 0.001(x_1^2 + x_2^2))^2} \quad (7)$$

where $-10.0 \leq x_1, x_2 \leq 10.0$. Schaffer function has many local minimum points and it is difficult for optimization algorithms to get the global optimal point $(0,0)$ due to the strong oscillation nature of the function. Different optimization algorithms are used to get the solution of equation $Y = \frac{1}{\min f(x_1, x_2) + 1}$, and the results are shown in figure 2.

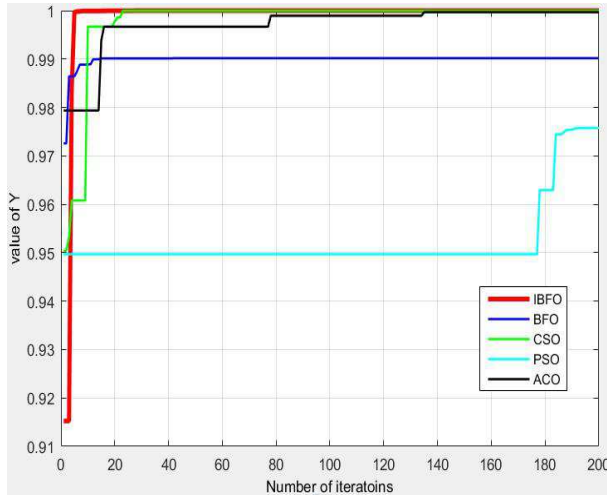


FIGURE 1. Value of Y based on Griewank function

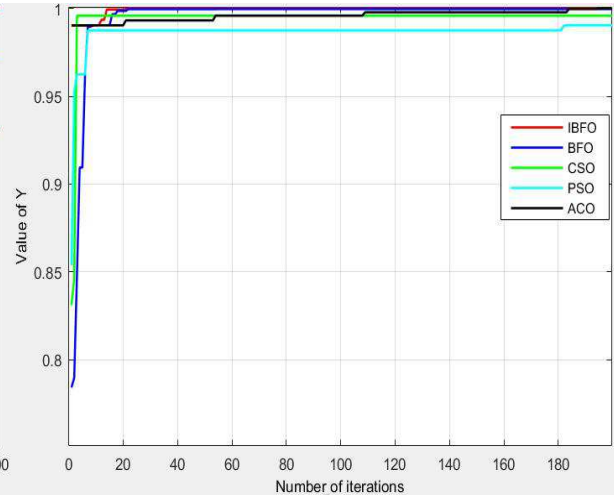


FIGURE 2. Value of Y based on Schaffer function

From figure 1 and figure 2, we can know that IBFO can get the global optimal point faster than other algorithms (such as BFO, CSO, in figure 1), and some optimization algorithms (such as PSO, CSO, in figure 2) never get the global optimal point.

5.3. The performance analysis of the segmented images. The evaluation indicators, such as the regional gray-level contrast, the regional heterogeneity and the time consumption, are used to analyze the performance of the new image segmentation method proposed in this paper.

5.3.1. Regional gray level of the segmented images. The regional gray-level contrast (RGLC) of the segmented images is computed to evaluate the quality of these images [10]. RGLC is defined as follows:

$$RGLC = \sum_{i=1}^k \sum_{j=i+1}^k \frac{|f_i - f_j|}{f_i + f_j} \tag{8}$$

Where k is the number of clusters, f_i, f_j , are the average grey values of cluster i and cluster j respectively. It is well know that the $RGLC$ is more bigger, the quality of th segmented image is more higher. In order to test the quality of the images segmented based on different optimization algorithms mentioned in subsection 5.1, the Mean values of $RGLC$ are computed under different sample sizes. From the results shown as table 2, we can know that the quality of the images segmented using IBFO is best.

TABLE 2. The mean values of $RGLC$ based on different algorithms

Sample sizes	IBFO	BFO	CSO	PSO	ACO
30	1.0853	1.0762	1.0387	1.0486	0.2376
60	1.0891	1.0780	1.0319	1.0363	0.2426
100	1.0871	1.0793	1.0291	1.0309	0.2424

5.3.2. *The regional heterogeneity of the segmented images.* Known from above section, the goal of image segmentation is to find an optimal partition $C = \{C_1, C_2, \dots, C_k\}$ for image $f(x, y)$. Simultaneously, we can know that the gray level distribution in the region $C_i (i = 1, 2, \dots, k)$ is relatively uniform because of the homogeneity of the pixels in it. And good segmentation algorithms will make the region $C_i (i = 1, 2, \dots, k)$ the best uniformity respectively. Therefore, the regional heterogeneity of the segmented images can be used to assess the quality of image segmentation. The regional heterogeneity can be defined as follows.

$$NU = \frac{\sum_{i=1}^k p_i \sigma_i^2}{\sigma^2} \quad (9)$$

where $p_i = \frac{A_i}{A}$ is the proportion of the region $C_i (i = 1, 2, \dots, k)$ to image $f(x, y)$, A_i is the area of C_i and A is the total area of image $f(x, y)$. In order to test the quality of the images segmented based on different optimization algorithms mentioned in subsection 5.1, the Mean values of NU are computed under different sample sizes. From the results shown as table 3, we also can know that the quality of the images segmented using IBFO is best.

TABLE 3. The mean values of NU based on different algorithms

Sample sizes	IBFO	BFO	CSO	PSO	ACO
30	0.1744	0.1769	0.3399	0.1810	0.9481
60	0.1622	0.1629	0.3070	0.1665	0.9414
100	0.1784	0.1817	0.3282	0.1821	0.9448

5.3.3. *Time-consumption of IBFO.* Under the case that the segmentation results of different algorithms, which are mentioned in subsection 5.1, are almost the same, the average time-consumption of different optimization algorithms are computed using different sample sizes to prove the speed of the images segmented based on IBFO is faster than that based on other optimization algorithms. From table 4, we can know that the time-consumption of image segmentation based on IBFO is shorter than that based on other algorithms, such as ACO, PSO, CSO, and BFO.

TABLE 4. The average time-consumption of different optimization algorithms

Sample sizes	IBFO	BFO	CSO	PSO	ACO
30	0.1366	0.2022	1.3477	1.0886	8.3514
60	0.1365	0.2023	1.3504	1.0894	8.3551
100	0.1367	0.2023	1.3518	1.0898	8.3538

5.3.4. *Part of Results of the Image segmentation.* Images are segmented based on different algorithms, and part of the results are shown as figure 4. Known from figure 4, the effect of the image segmentation based on IBFO is best.

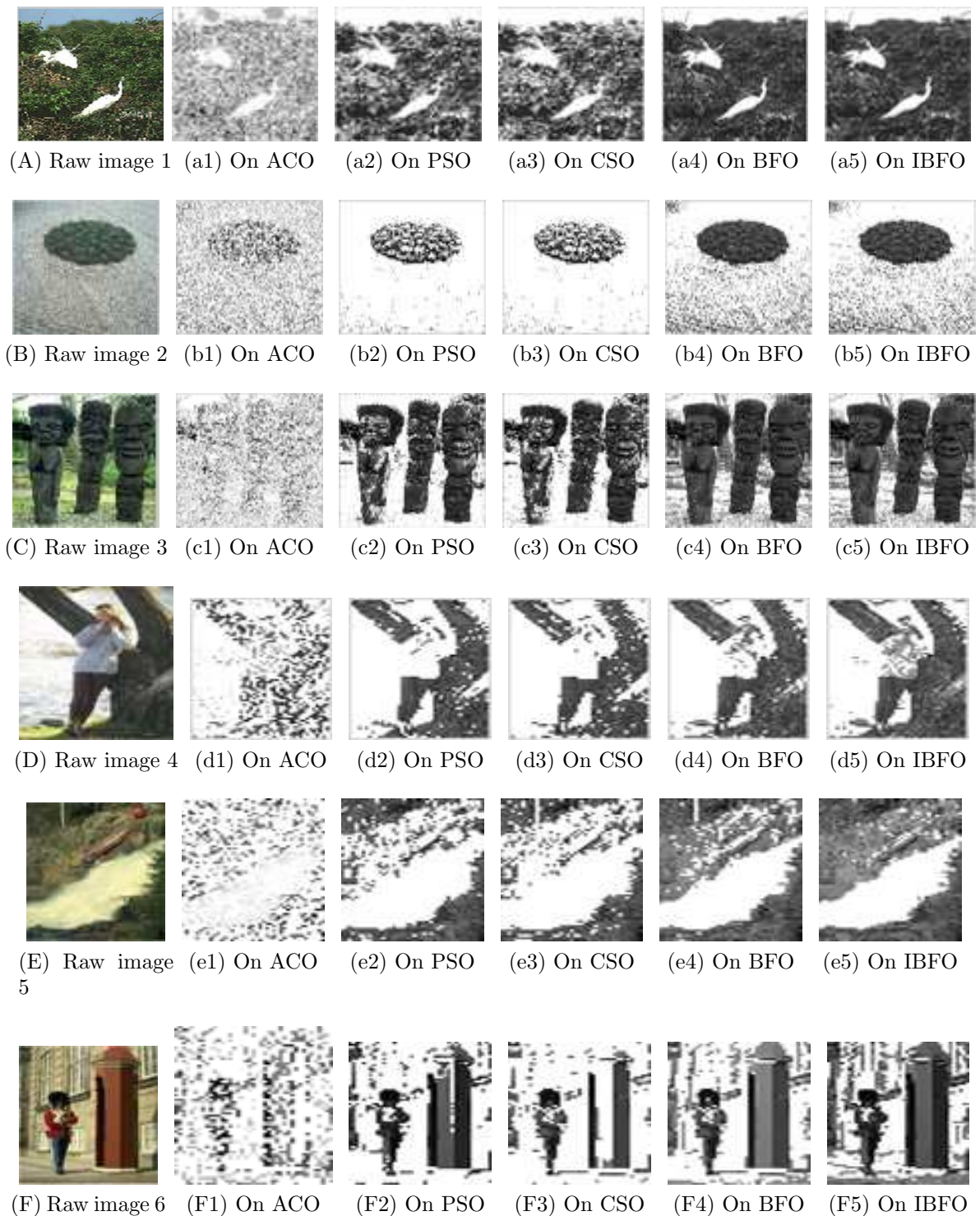


FIGURE 3. Part of the segmentation results based on different algorithms

6. Conclusions. When some traditional bionic optimization algorithms are used for image segmentation, we find that the search speeds of these algorithms are slow and the local searching abilities of these algorithms need be improved. Simultaneously, the accuracy of image segmentation based on these traditional bionic optimization algorithms need be enhanced. In order to solve the problems of these optimization algorithms, an improved

bacterial foraging optimization algorithm (IBFO) is proposed in this paper, and it is applied to image segmentation. The results of image segmentation show that the quality of the image segmentation based on IBFO is superior to the ones that based on others tradition bionic optimization algorithms, such as PSO, ASO, CSO. Simultaneously, we can know from experiments that the speed of image segmentation based on IBFO is faster than that based on any other algorithms. In the future, we will try our best to apply IBFO to the field of high-dimensional data processing.

Acknowledgment. This work is partially supported by the national natural science foundation of China (Grant No. 61170102, 61271140, 61300238), the scientific research fund of Hunan provincial education department, china (Grant No.15A049), the education department fund of Hunan province in china (Research on analysis and modeling of high dimensional data and its application in video image processing(2016)-503-630 and Grant No.15C0402, 15C0395, 13C036), the science and technology planning project of Hunan province in China (Grant No. 2015GK3024) and the project of 701697-CAR-MSCA-IFEF-ST. The authors also gratefully acknowledge the helpful comments and suggestions of the reviewers, which have improved the presentation.

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