A Motion Closed World Asumption

Fusun Yaman and **Dana Nau** and **V.S. Subrahmanian**
Department of Computer Science,
University of Maryland Institute for Advanced Computer Studies (UMIACS),
and Institute for Systems Research (ISR)
University of Maryland
College Park, MD, 20742
{fusun, nau, vs}@cs.umd.edu

Abstract

Yaman et. al. [Yaman *et al.*, 2004] introduce "go theories" to reason about moving objects. In this paper, we show that this logic often does not allow us to infer that an object is *not* present at a given place or region, even though common sense would dictate that this is a reasonable inference to make. We define a class of models of go-theories called *coherent* models. We use this concept to define a *motion closed world assumption* (MCWA) and develop a notion of MCWA-entailment. We show that checking if a go-theory has a coherent model is NP-complete. An in atom checks if a given object is present in a given region sometime in a given time interval. We provide sound and complete algorithms to check if a ground in literal (positive or negative in atom) can be inferred from a go-theory using the MCWA. In our experiments our algorithms answer such queries in less than 1 second when there are up to 1,000 go-atoms per object.

1 Introduction

Reasoning about moving objects is becoming increasingly important. Air traffic controllers in both the US and Europe are facing a dramatically increasing workload as the number of flights increases. Cell phone companies are increasingly interested in knowing where cell phones on their network are located — this is useful for hand-off policies between cell phone towers. Vehicle security systems such as LOJACK and ONSTAR are increasingly being used to determine where vehicles are and where they are not.

[Yaman *et al.*, 2004] proposed the concept of a "go theory" which can be used to make statements of the form "Object o is expected to leave location P_1 at some time point in the interval $[t_1^-, t_1^+]$ and reach location P_2 at some time point in the interval $[t_2^-, t_2^+]$ traveling at a velocity between v_1 and v_2. Go theories can be used, for example, to make statements such as "Plane p22 is expected to take off from Paris at some time between 10 and 12 and land at Boston at some time between 18 and 23 traveling at a speed between 10 to 20." Go theories are sets of such statements. Figure 1 shows the spatial layout of one such go-theory (the go theory is written in text at the top). [Yaman *et al.*, 2004] provides a model theory for go theories,

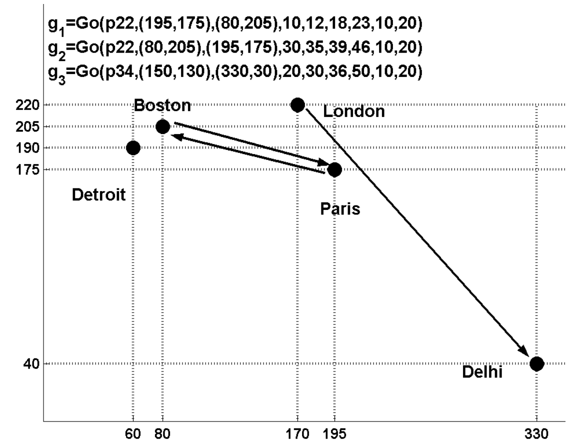

Figure 1: Planes example

together with algorithms to check entailment of certain kinds of atoms: "in" atoms that check if a given moving object is within a given region at a given time, and "near" atoms which are used to check if two objects are within a given distance of each other at some time.

Though the semantics given by [Yaman *et al.*, 2004] is adequate for positive atoms, it is inadequate for negative atoms. For example, suppose we consider the go-theory containing just the two statements above about plane p22. In this case, we would like to infer that plane p22 is not in Detroit at time 30, even though it may be theoretically possible for the plane to make it to Detroit. The goal of this paper is to ensure that intelligent negative inferences of this kind can be made from go-theories.

The contributions and organization of this paper are as follows: in Section 2, we recapitulate the syntax of go-theories from [Yaman *et al.*, 2004]. In section 3, we introduce the concept of a coherent model of a go-theory, and describe the concept of coherent entailment. We also introduce the *Motion Closed World Assumption* (MCWA for short) and show how the MCWA can be used to reason about negative information. Also in Section 3, we show that the problem of checking if a go-theory has a coherent model is NP-complete. In Section 4, we provide algorithms to evaluate "in" literals w.r.t. the MCWA semantics. We are developing algorithms to process other kinds of queries such as the near literals described in [Yaman *et al.*, 2004] — however, space reasons prevent us from presenting them. Section 5 describes a prototype implementation to answer positive and negative "in" queries — the implementation shows that our system is highly scalable. We compare our work with related work in Section 6.

2 Go-Theories: Syntax and Semantics

We first provide a quick overview of the main definitions of [Yaman *et al.*, 2004]. We assume the existence of several sets of constant symbols: $\mathbf{R}$ is the set of all real numbers, $\mathbf{O}$ is the set of names of objects, $\mathbf{P} = \mathbf{R} \times \mathbf{R}$ is the set of all points in two-dimensional cartesian space. We assume the existence of three disjoint sets of variable symbols, $V_{\mathbf{R}}$, $V_{\mathbf{O}}$, and $V_{\mathbf{P}}$, ranging over $\mathbf{R}$, $\mathbf{O}$ and $\mathbf{P}$, respectively. A *real* term t is any member of $\mathbf{R} \cup V_{\mathbf{R}}$. Object terms and point terms are defined similarly. Ground terms are defined in the usual way. We now define atoms as follows.

- If o_1, o_2 are object terms, and d, t_1, t_2 are positive real terms, then $\mathsf{near}(o_1, o_2, d, t_1, t_2)$ is an *atom*. When these terms are ground, this atom says that o_1, o_2 are within distance d of each other during the time interval $[t_1, t_2]$.
- If o is an object term, P_1, P_2 are point terms, and t_1, t_2 are positive real terms, then $\mathsf{in}(o, P_1, P_2, t_1, t_2)$ is an *atom*. When these terms are ground, this atom says that object o is in the rectangle whose lower left (resp. upper right) corner is P_1 (resp. P_2) at some point in the time interval $[t_1, t_2]$.
- If o is an object term, P_1, P_2 are point terms, and t_1^-,t_1^+,t_2^-,t_2^+,v^-, v^+ are positive real terms, then $\mathsf{go}(o, P_1, P_2, t_1^-, t_1^+, t_2^-, t_2^+, v^-, v^+)$ is an atom called a *go atom*. When all these terms are ground, this atom says that object o leaves point P_1 at some time in $[t_1^-, t_1^+]$ and arrives at point P_2 during $[t_2^-, t_2^+]$, traveling in a straight line with a minimum speed v^- and maximum speed v^+.

Ground atoms are defined in the usual way. A *go theory* is a finite set of ground go-atoms.

Notation. If $g = \mathsf{go}(o, P_1, P_2, t_1^-, t_1^+, t_2^-, t_2^+, v^-, v^+)$ then

$$\begin{array}{lll} obj(g) = o, & v^-(g) = v^-, & v^+(g) = v^+, \\ loc_1(g) = P_1, & t_1^-(g) = t_1^-, & t_1^+(g) = t_1^+, \\ loc_2(g) = P_2, & t_2^-(g) = t_2^-, & t_2^+(g) = t_2^+. \end{array}$$

If A is an atom, then A and $\neg A$ are called *literals*. Due to space constraints, we only consider literals in this paper — [Yaman *et al.*, 2004] provide a richer syntax including conjunction and disjunction.

An *interpretation* is a continuous function $\mathcal{I} : \mathbf{O} \times \mathbf{R}^+ \to \mathbf{P}$. Intuitively, $\mathcal{I}(o, t)$ is o's location at time t. We first define satisfaction of a "go" atom w.r.t. a given time interval.

Definition 1 *Let* $g = \mathsf{go}(o, P_1, P_2, t_1^-, t_1^+, t_2^-, t_2^+, v^-, v^+)$ *be an atom and* $\mathcal{I}$ *be an interpretation.* $\mathcal{I}$ ***satisfies*** g *over a time interval* $T = [t_1, t_2]$ *iff:*

- $t_1 \in [t_1^-, t_1^+]$ *and* $\mathcal{I}(o, t_1) = P_1$
- $t_2 \in [t_2^-, t_2^+]$ *and* $\mathcal{I}(o, t_2) = P_2$
- $\forall t \in [t_1, t_2],\ \mathcal{I}(o, t)$ *is on the line segment* $[P_1, P_2]$
- $\forall t, t' \in [t_1, t_2],\ t < t'$ *implies* $\mathsf{dist}(\mathcal{I}(o, t), P_1) < \mathsf{dist}(\mathcal{I}(o, t'), P_1)$ *where* dist *is the function that computes the Euclidean distance between two points.*
- *For all but finitely many times in* $[t_1, t_2]$, $v = d(|\mathcal{I}(o, t)|)/dt$ *is defined and* $v^-(g) \le v \le v^+(g)$.

The above definition intuitively says that $\mathcal{I} \models g$ over a time interval $T = [t_1, t_2]$ iff o starts moving at t_1, stops moving at t_2 and during this interval, the object moves away from P_1 towards P_2 without either stopping or turning back or wandering away from the straight line connecting P_1 and P_2. We are now ready to define the concept of satisfaction of arbitrary literals.

Definition 2 $\mathcal{I}$ satisfies *a ground literal (denoted* $\mathcal{I} \models A$*) in these cases:*

1. $\mathcal{I} \models \mathsf{go}(o, P_1, P_2, t_1^-, t_1^+, t_2^-, t_2^+, v^-, v^+)$ *iff there exists an interval* $[t_1, t_2]$ *such that* $\mathcal{I}$ *satisfies* A *over* $[t_1, t_2]$.
2. $\mathcal{I} \models \mathsf{near}(o_1, o_2, d, t_1, t_2)$ *iff* $\mathsf{dist}(\mathcal{I}(o_1, t), \mathcal{I}(o_2, t)) \le d$ *for all* $t_1 \le t \le t_2$
3. $\mathcal{I} \models \mathsf{in}(o, P_1, P_2, t_1, t_2)$ *iff there are numbers* $t \in [t_1, t_2]$, $x \in [{P_1}^x, {P_2}^x]$ *and* $y \in [{P_1}^y, {P_2}^y]$ *such that* $\mathcal{I}(o, t) = (x, y)$.
4. $\mathcal{I} \models \neg A$ *iff* $\mathcal{I}$ *does not satisfy* A.

$\mathcal{I}$ *satisfies (or is a model of)* a set of ground atoms MT iff $\mathcal{I}$ satisfies every $A \in \mathsf{MT}$. MT is *consistent* iff there is an interpretation $\mathcal{I}$ such that $\mathcal{I} \models \mathsf{MT}$. L is a *logical consequence* of MT, denoted $\mathsf{MT} \models L$, iff every model of MT is also a model of L.

Example 1 *The* Planes *go-theory of Figure 1 is consistent as the interpretations* $\mathcal{I}_1, \mathcal{I}_2$ *below both satisfy it.*

- $\mathcal{I}_1$ *:* $p22$ *leaves Paris at time 11, flies to Boston at a constant speed of 14.85 and arrives in Boston at 19.* $p22$ *waits in Boston until 32, then it departs for Paris with a constant speed of 13.2 arriving in Paris at 41. The other plane,* $p34$ *leaves London at time 25 and flies to Delhi at a constant speed of 18.52, arriving in Delhi at 38.*
- $\mathcal{I}_2$ *:* $p22$ *leaves Paris at time 10, flies to Boston at constant speed of 14.85 and reaches Boston at 18. It waits in Boston until 19, when it takes off for Detroit where it arrives at time 21. It immediately departs and reaches Boston at time 29. At time 30,* $p22$ *leaves Boston and flies to Paris at a constant speed of 11.88, arriving in Paris at time 40. The other plane,* $p34$ *leaves London at time 25 and flies to Delhi at a constant speed of 18.52, arriving in Delhi at 38.*

It is important to note that even though $\mathcal{I}_2$ satisfies the Planes go theory, it is an interpretation that allows plane $p22$ to wander around in ways that were not explicitly stated in the Planes go theory. In particular, it lets the plane wander to Detroit which was never mentioned in the go-theory. We would like to exclude such "wandering" interpretations as they prevent us from making the intuitive (nonmonotonic) inference that Plane $p22$ was never in Detroit.

Throughout the rest of the paper we are going to use the notation $G[o]$ to denote the set of all atoms about an object o in a go theory G.

Definition 3 *Let G be a go theory, o be an object and $G[o] = \{g_1, g_2, \ldots, g_n\}$. Then for every $g_i, g_j \in G[o]$ we define a partial order $\preceq$ such that $g_i \preceq g_j$ iff $t_2^+(g_i) \leq t_1^-(g_j)$. A total order $\sqsubseteq$ on $G[o]$ is compatible with $G[o]$ iff $\sqsubseteq$ is a topological sort of $\preceq$.*

Definition 4 *Suppose G is a go theory, o is an object and $\sqsubseteq$ is a total order compatible with $G[o]$. Then $\mathcal{L}(G[o], \sqsubseteq)$ is set of linear constraints such that*

- *for every $g = \mathsf{go}(o, P_1, P_2, t_1^-, t_1^+, t_2^-, t_2^+, v^-, v^+) \in G[o]$, $\mathcal{L}(G[o], \sqsubseteq)$ contains:*
  - $t_1^- \leq S_g \leq t_1^+$ *and* $t_2^- \leq E_g \leq t_2^+$,
  - $v^- \times (E_g - S_g) \leq \mathsf{dist}(P_1, P_2) \leq v^+ \times (E_g - S_g)$,
- *for every $g, g' \in G[o]$ such that $g \sqsubseteq g'$, $\mathcal{L}(G[o], \sqsubseteq)$ contains: $E_g \leq S_{g'}$.*

Intuitively S_g, E_g are variables that represent the times o starts and stops moving.

Definition 5 *A go theory G is non-collinear iff for each object o there are no $g, g' \in G[o]$ such that*

- *The intersection of line segments $[loc_1(g), loc_2(g)]$ and $[loc_1(g'), loc_2(g')]$ is a line segment $[P, Q]$,*
- *The direction of the movement in g and g' is same, i.e., $\exists k \in \mathbf{R}^+$ such that $\vec{v} = k \times \vec{u}$ where $\vec{v} = loc_2(g) - loc_1(g)$ and $\vec{u} = loc_2(g') - loc_1(g')$*
- *$t_1^-(g) \leq t_2^+(g')$ and $t_1^-(g') \leq t_2^+(g)$, i.e. temporally overlapping.*

The following theorem establishes necessary and sufficient conditions for non-collinear go theories to be consistent.

Theorem 1 *A non-collinear go theory G is consistent iff for every object o there is a total order $\sqsubseteq_o$ compatible with $G[o]$ such that $\mathcal{L}(G[o], \sqsubseteq_o)$ has a solution.*

3 Coherence

In this section, we define the concept of a coherent interpretation. We start by defining precedence of time intervals.

Definition 6 (Precedence) *Let $S = \{T_1, \ldots T_n\}$ be a set of time intervals, where $T_i = [t_{i1}, t_{i2}]$ for each i. T_i **immediately precedes** T_j **in** S if $t_{i2} \leq t_{j1}$ and for every $T_k \in S$, either $t_{k2} \leq t_{i2}$ or $t_{k1} \geq t_{j1}$.*

Intuitively, $\mathcal{I}$ is a *coherent* interpretation of a go theory G if for each object o, there is a time interval T such that for every time point $t \in T$, $\mathcal{I}(o, t)$ either satisfies a go-atom in G or keeps the object at the destination of the last satisfied go-atom in G .

Definition 7 (Coherent Model and Theory) *Let $\mathcal{I}$ be a model of the go theory G. Let $G[o] = \{g_1, g_2, \ldots, g_n\}$ be the set of all go-atoms in G about object o. $\mathcal{I}$ is **coherent w.r.t.** o **and** G iff*

(i) There are time intervals $T_1 = [t_{11}, t_{12}]$, $T_2 = [t_{21}, t_{22}]$, ..., $T_n = [t_{n1}, t_{n2}]$ such that for each i, $\mathcal{I}$ satisfies g_i over T_i and

(ii) For every pair of time intervals T_i, T_j such that T_i immediately precedes T_j in $\{T_1, T_2, \ldots, T_n\}$ the following holds:

$$\forall t \in [t_{i2}, t_{j1}] \;\; \mathcal{I}(o, t) = loc_2(g_i), \text{ i.e. destination of } g_i.$$

*I is a **coherent model of** G iff $\mathcal{I}$ is coherent w.r.t. o and G for all objects o.*
*G is a **coherent go-theory** iff G has a coherent model.*

Example 2 *Let G be the go theory in Figure 1. Let $\mathcal{I}_1$ and $\mathcal{I}_2$ be the two interpretations in Example 1. $\mathcal{I}_1$ is coherent with respect to G and $p22$ because it satisfies g_1 over $[11, 19]$, g_2 over $[32, 40]$ and in between $[19, 32]$ plane $p22$ is in Boston. $\mathcal{I}_2$ is not coherent with respect to G and $p22$ because although it satisfies g_1 over $[10, 18]$, g_2 over $[30, 41]$ during $[18, 30]$, $p22$ does not stay in Boston which is the destination of g_1.*

The following lemma and definition are useful in checking whether a non-collinear go-theory has a coherent model or not.

Definition 8 *Suppose G is a non-collinear go theory, o is an object and $\sqsubseteq$ is a total order compatible with $G[o]$. Let $g_1 \sqsubseteq g_2 \cdots \sqsubseteq g_n$ be the atoms of $G[o]$. $\sqsubseteq$ is **spatially continuous** w.r.t. $G[o]$ iff for every i, $1 \leq i < n$, $loc_2(g_i) = loc_1(g_{i+1})$, i.e., g_i's destination is g_{i+1}'s origin;*

Lemma 1 *Suppose G is a non-collinear go theory. G is coherent if for every object o there is a total order $\sqsubseteq_o$ compatible with $G[o]$ such that $\mathcal{L}(G[o], \sqsubseteq_o)$ has a solution and $\sqsubseteq_o$ is spatially continuous w.r.t. $G[o]$.*

The following theorem shows that checking coherence of a go-theory is NP-complete.

Theorem 2 ***(i)*** *Checking coherence of a non-collinear go theory is NP-complete.* ***(ii)*** *Checking coherence of a go theory is NP-complete.*

The proof is omitted due to lack of space. We now define the concept of coherent entailment.

Definition 9 (MCWA entailment) *Let L be a ground literal and G be a go theory. G **entails** L **via MCWA**, denoted $G \models^{mcwa} L$, iff every coherent model of G also satisfies L.*

The MCWA is inspired by Minker's generalized closed world assumption [Minker, 1982] where a class of models is used to check if a given literal is true. We do the same here. The following example shows that the MCWA can handle examples such as the Planes example.

Example 3 *Let G be the go theory in Figure 1. Let $\mathcal{I}_1$ and $\mathcal{I}_2$ be the interpretations in Example 1. Suppose $a = \mathsf{in}(p22, (75, 200), (85, 210), 23, 30)$. $G \models^{mcwa} a$ since in all coherent models of G, during $[23, 30]$ plane $p22$ is in Boston which is inside the rectangle of the atom a.*

Suppose $b = \mathsf{in}(p22, (55, 185), (80, 200), 23, 30)$. Then $G \models^{mcwa} \neg b$ since in all coherent models of G, during $[23, 30]$ plane $p22$ stays in Boston which is not in the rectangle of the atom b.

Also note that $G \not\models a$ and $G \not\models \neg b$ because according to the semantics in [Yaman et al.*, 2004] plane $p22$ can be anywhere during $[23, 30]$.*

Theorem 3 *Let L be a ground $\mathsf{in}()$ literal and G be a go theory. Checking if $G \models^{mcwa} L$ is co-NP complete.*

Since incoherent theories entail everything the following section describes algorithms for coherent go theories.

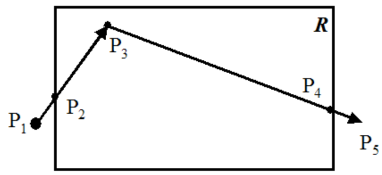

Figure 2: Spatial layout of two go atoms (related to object o) going from P_1 to P_3 and P_3 to P_5 and a rectangle R. In a coherent model, o stays in R between the points P_2 and P_4.

4 MCWA-Entailment Algorithms

This section provides algorithms to check for MCWA-entailment of both positive and negative **ground** literals. Due to space limitations we assume all theories are non-collinear go theories. Extending our algorithm to remove this assumption is straightforward using the methods defined in [Yaman *et al.*, 2004] that combine collinear go atoms into "movements"– our implementation applies to all go theories.

We first introduce some notations. If g is a go atom, then $LS(g)$ is the line segment between the source and destination of g. Let G be a go-theory, o be an object and $\sqsubseteq$ be a total ordering compatible with $G[o]$. Let P be a point on $LS(g)$ where $g \in G$. Then $T^-(G[o], \sqsubseteq, g, P)$ and $T^+(G[o], \sqsubseteq, g, P)$ are the earliest and latest possible times for o to be at P, subject to G,o, $\sqsubseteq$ and g. [1]

4.1 Coherent in()

In this section, we show how to check whether a **ground** atom of the form $a = \mathsf{in}(o, q_1, q_2, t_1, t_2)$ is MCWA-entailed by a *go*-theory G. Let $Rec(a)$ denote the set of points P such that $q_1^x \le P^x \le q_2^x$ and $q_1^y \le P^y \le q_2^y$.

We first consider a non-collinear go theory G=$\{g_1, g_2\}$ about an object o, and an atom $a = \mathsf{in}(o, q_1, q_2, t_1, t_2)$. Assume Figure 2 depicts $Rec(a)$ and the two line segments $[P_1, P_3], [P_3, P_5]$ representing movements defined by g_1 and g_2. In any coherent model of G, g_1 will be satisfied before g_2. Hence the object enters $Rec(a)$ at P_2 and leaves $Rec(a)$ at P_4. If o always arrives at P_2 before t_2 and always leaves P_4 after t_1 subject to the constraints in G, then we can say that $G \models^{mcwa} a$.

For an arbitrary go theory, any object might enter and leave $Rec(a)$ multiple times. We need to identify these entrance and exit points as well as the atoms that contain them.

Definition 10 *Let L be a sequence of line segments $\ell_1 = [P_{11}P_{12}]$, $\ell_2 = [P_{21}P_{22}]$,..., $\ell_n = [P_{n1}P_{n2}]$ such that for $1 \le i < n$, $P_{i2} = P_{(i+1)1}$. Let R be a rectangular region. An* entry-exit *of L for R is (i, j) iff*

- $\ell_i \cap R \neq \emptyset$ *and* $i > 1 \implies P_{i1} \notin R$
- $\ell_j \cap R \neq \emptyset$ *and* $j < n \implies P_{j2} \notin R$
- $\forall k \in [i, j)\ P_{k2} \in R$

The following lemma gives necessary conditions for $G \models^{mcwa} a$ when the atoms in G are satisfied in a specific order and the object enters and exits $Rec(a)$ multiple times.

Lemma 2 *Let G be a coherent go theory, o be an object and $\sqsubseteq$ be a total order compatible with $G[o]$ such that $\mathcal{L}(G[o], \sqsubseteq)$ has a solution and $\sqsubseteq$ is spatially continuous w.r.t $G[o]$. Let $g_1 \sqsubseteq g_2 \cdots \sqsubseteq g_n$ be the atoms of G[o]. Let $a = \mathsf{in}(o, q_1, q_2, t_1, t_2)$ be an atom. If $G \models^{mcwa} a$ then there is an entry-exit (i, j) of $LS(g_1) \ldots LS(g_n)$ for $Rec(a)$ such that*

$$T^+(G[o], \sqsubseteq, g_i, P_i) \le t_2 \text{ and } t_1 \le T^-(G[o], \sqsubseteq, g_j, Q_j).$$

where $[P_k, Q_k] = LS(g_k) \cap Rec(a)$,

The following algorithm uses this lemma to check for MCWA-entailment w.r.t. a specific total ordering.

Algorithm CheckCoherentIn($G, \sqsubseteq, a$)
  Suppose $a = \mathsf{in}(o, q_1, q_2, t_1, t_2)$;
  Let $g_1 \sqsubseteq g_2 \cdots \sqsubseteq g_n$ be atoms of G[o]
  if $\sqsubseteq$ is not spatially continuous w.r.t G[o] **then return** true
  if $\mathcal{L}(G[o], \sqsubseteq)$ has no solution **then return** true
 for each entry-exit (i, j) of $LS(g_1) \ldots LS(g_n)$ for $Rec(a)$
   Let $[P_i, Q_i] = LS(g_i) \cap Rec(a)$
   Let $[P_j, Q_j] = LS(g_j) \cap Rec(a)$
   if $T^+(G[o], \sqsubseteq, g_i, P_i) \le t_2$ and $t_1 \le T^-(G[o], \sqsubseteq, g_j, Q_j)$
    then return true
 end for
 return false

Theorem 4 *Suppose G is a coherent* go*-theory and $a = \mathsf{in}(o, q_1, q_2, t_1, t_2)$ is a ground atom. Then: a is entailed by G via MCWA iff for every total order $\sqsubseteq$ compatible with $G[o]$, the algorithm* CheckCoherentIn($G, \sqsubseteq, a$) *returns "true".*

4.2 Coherent ¬in()

We now address the problem of checking whether a literal of the form $\neg\mathsf{in}(o, Q_1, Q_2, t_1, t_2)$ is MCWA-entailed by a *go*-theory G.

Consider a coherent go theory G=$\{g_1, g_2\}$ about an object o, and an in-atom $a = \mathsf{in}(o, q_1, q_2, t_1, t_2)$. As before Figure 2 depicts $Rec(a)$ and two line segments $[P_1, P_3], [P_3, P_5]$ representing the movements defined by g_1 and g_2. Note that in any coherent model of G, g_1 is satisfied before g_2. Hence the object enters $Rec(a)$ at point P_2 and leaves $Rec(a)$ at point P_4. $G \models^{mcwa} \neg a$ iff

- t_1 is greater than or equal to the start time of g_1 in any coherent model of G.
- t_2 is smaller than or equal to the end time of g_2 in any coherent model of G.
- Let T_1 be the earliest arrival time to P_2 and T_2 be the latest arrival time to P_4 in any coherent model of G then $T_1 > t_2$ or $T_2 < t_1$.

[1] $T^-(G[o], \sqsubseteq, g, P)$ is the solution to linear programming problem: **minimize** $X_{P,g}$ **subject to** $\mathcal{L}(G[o], \sqsubseteq) \cup L(P, g)$, where $L(P, g)$ contains the following linear constraints:

- $\frac{dist(loc_1(g),P)}{v^+(g)} \le X_{P,g} - S_g \le \frac{dist(loc_1(g),P)}{v^-(g)}$
- $\frac{dist(loc_2(g),P)}{v^+(g)} \le E_g - X_{P,g} \le \frac{dist(loc_2(g),P)}{v^-(g)}$

where $X_{P,g}$ is the variable that represents the time the object will arrive P while satisfying g. S_g and E_g are the variables associated with g in $\mathcal{L}(G[o], \sqsubseteq)$. $T^+(G[o], \sqsubseteq, g, P)$ can be computed in the same way, using maximization instead of minimization.

The following lemma gives necessary conditions for $G \models^{mcwa} \neg a$ to hold w.r.t. a specific total ordering $\sqsubseteq$ even if the object enters and exits $Rec(a)$ multiple times.

Lemma 3 *Let G be a coherent go theory, o be an object and $\sqsubseteq$ be a total order compatible with $G[o]$ such that $\mathcal{L}(G[o], \sqsubseteq)$ has a solution and $\sqsubseteq$ is spatially continuous w.r.t. $G[o]$. Let $g_1 \sqsubseteq g_2 \cdots \sqsubseteq g_n$ be the atoms of G[o]. Let $a = \mathsf{in}(o, q_1, q_2, t_1, t_2)$ be a ground atom. If $G \models^{mcwa} \neg a$ then the following hold*

- $T^+(G[o], \sqsubseteq, g_1, loc_1(g_1)) \leq t_1$
- $T^-(G[o], \sqsubseteq, g_n, loc_2(g_n)) \geq t_2$
- $\forall$ *entry-exit* (i, j) *of* $LS(g_1) \ldots LS(g_n)$ *for* $Rec(a)$, $T^-(G[o], \sqsubseteq, g_i, P_i) > t_2$ *or* $T^+(G[o], \sqsubseteq, g_j, Q_j) < t_1$ *where* $[P_k, Q_k] = LS(g_k) \cap Rec(a)$.

The following algorithm checks if $G \models^{mcwa} \neg \mathsf{in}()$ w.r.t. a specific total ordering.

```
Algorithm CheckCoherentNotIn(G, ⊑, ¬a)
  Suppose a = in(o, p1, p2, t1, t2);
  Let g1 ⊑ g2 ··· ⊑ gn the atoms of G[o].
  if ⊑ is not spatially continuous w.r.t G[o] then return true
  if L(G[o], ⊑) has no solution then return true
  if t1 < T+(G[o], ⊑, g1, loc1(g1)) return false
  if t2 > T−(G[o], ⊑, gn, loc2(gn)) return false
  for each entry-exit (i, j) of LS(g1) ... LS(gn) for Rec(a)
    Let [Pi, Qi] = LS(gi) ∩ Rec(a)
    Let [Pj, Qj] = LS(gj) ∩ Rec(a)
    if T−(G[o], ⊑, gi, Pi) ≤ t2 and t1 ≤ T+(G[o], ⊑, gj, Qj)
    then return false
  end for
  return true
```

Theorem 5 *Suppose G is a coherent* go*-theory and $L = \neg\mathsf{in}(o, q_1, q_2, t_1, t_2)$ is a ground literal. Then L is entailed by G via* MCWA *iff for every total order $\sqsubseteq$ compatible with $G[o]$, the algorithm* CheckCoherentNotIn$(G, \sqsubseteq, a)$ *returns "true".*

5 Implementation

Determining MCWA-entailment is co-NP complete because the number of orderings spatially continuous w.r.t. $G[o]$ can be exponential. However, in the real world, we expect a go-theory to allow only a small number of orderings compatible with $G[o]$. In other words, the respective order of movements an object is going to perform is mostly known. For example we might not know exactly when the plane $p22$ will land but we usually know where it is going to fly next. Thus, in practice there is a bound on the number of compatible total orderings per object.

For our experiments we generated random go theories with at most 256 spatially continuous orderings. This is not a hard-coded limit of our implementation. Generating random go-theories such that more than one spatially continuous ordering exists is a little bit tricky. Here is one method to generate a go theory $G = \{g_1, g_2, g_3, g_4, g_5\}$ with two spatially continuous orderings.

- Randomly pick points P_1, P_2, P_3 and P_4
- Set $loc_1(g_1) = P_1$ and $loc_2(g_1) = P_2$,
- Set $loc_1(g_2) = P_2$, $loc_2(g_2) = P_3$ and $loc_1(g_3) = P_3$, $loc_2(g_3) = P_2$,
- Set $loc_1(g_4) = P_2$, $loc_2(g_4) = P_4$ and $loc_1(g_5) = P_4$, $loc_2(g_5) = P_2$,
- Set temporal and speed intervals of every g_i so that g_1 is always first and the rest can be done in any order.

We have generalized the reasoning above to create random go theories with an arbitrary bound on the number of spatially continuous orderings.

We have implemented the two algorithms CheckCoherentIn and CheckCoherentNotIn in Matlab and conducted experiments on a mobile Athlon XP 1800 processor running under Windows XP and having 256MB of memory. Figure 3 shows the computation time of four types of queries for coherent go theories with at most 256 spatially continuous orderings and have the following properties: all points are selected randomly from the rectangle $[(0, 0), (1000, 1200)]$ and the speeds allowed for any object less than 100. The four query templates we used are:

Q1: $\mathsf{in}(o, (500, 500), (550, 600), 0.5 * h, 0.75 * h)$
Q2: $\mathsf{in}(o, (100, 150), (350, 400), h - 100, h - 10)$
Q3: $\neg Q1$
Q4: $\neg Q2$

where h is the latest end time for any atom related to o in the given theory. The data points in Figure 3 are an average of 300 runs.

Our implementation performs very well, executing most queries in less than 0.3 seconds even when there are as many as 1,000 go-atoms per object. In the query $Q1$ where CheckCoherentIn returns true in almost every compatible orderings the algorithm runs in linear time with respect to number of atoms per object and takes up to 0.9 seconds when there are 1,000 go-atoms per object. Consequently $Q3$, the complement of $Q1$, takes almost no time because CheckCoherentNotIn returns false for any compatible ordering.

6 Related work

The Closed World Assumption (CWA) proposed by [Reiter, 1977] holds that anything that cannot be entailed by a theory is false. Minker [Minker, 1982] extended the CWA to a Generalized CWA (GCWA) that accounts for disjunction. GCWA states that a formula is false if it is false in all minimal models of the theory. The go-theories proposed by [Yaman *et al.*, 2004] are disjunctive because the start and end times and object velocities are all known to be within a given range. The notion of a coherent model of a go-theory selects certain models (much like Minker selected minimal models in GCWA) and uses these to make closed world inferences.

[Intille, 1994; Intille *et al.*,] have used the CWA to track moving objects in football games using computer vision algorithms. They use CWA to adaptively select and weight image features used for correspondence. No motion reasoning of the type we perform in this paper is done.

Numerous *spatio-temporal logics* exist [Gabelaia *et al.*, 2003; Merz *et al.*, 2003; Wolter and Zakharyaschev, 2000;

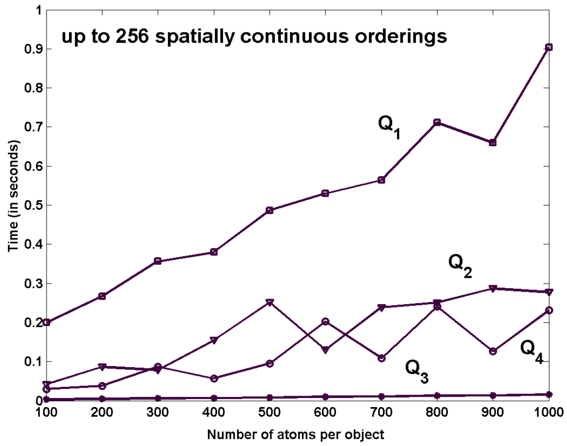

Figure 3: Time to answer queries $Q1, Q2, Q3$ and $Q4$ when total number of spatially continuous orderings is at most 256.

Cohn *et al.*, 2003] have proposed spatio-temporal logics. These logics extend temporal logics to handle space. By and large, these frameworks use discrete rather than the continuous representations we use. Moreover, these works focus on qualitative aspects of spatio-temporal reasoning, rather than deal quantitatively with the dynamics of motion. Our methods are rooted in a mix of geometry and logic, rather than in logic alone. Cohn [Anthony G. Cohn, 2001] provides an excellent survey of spatio-temporal logics but the survey sheds little light on reasoning above motion. A notable exception is the work of Muller [Muller, 1998a; 1998b] who describes a formal *qualitative* theory for reasoning about motion. The expressive power of the theory allows for the definition of complex motion classes. The work however is purely symbolic not quantitive.

7 Conclusions

Yaman et. al. [Yaman *et al.*, 2004] introduce a logic based on "go theories" for reasoning about moving objects. In this paper, we show that this logic often does not allow us to infer that an object is *not* present at a given place or region, even though common sense would dictate that this is a reasonable inference to make. We define a class of models of go-theories called *coherent* models. We use this concept to define a *motion closed world assumption* (MCWA) and develop a notion of MCWA-entailment. We show that checking if a go-theory has a coherent model is NP-complete. An in atom checks if a given object is present in a given region during a given time interval. We provide sound and complete algorithms to check if an in literal (positive or negative in atom) can be inferred from a go-theory using the MCWA. In our experiments our algorithms executed queries in less than 1 second even when there are as many as 1,000 go-atoms per object.

Acknowledgements

This work was supported in part by ARO grant DAAD190310202, ARL grants DAAD190320026 and DAAL0197K0135, the ARL CTAs on Telecommunications and Advanced Decision Architectures, NSF grants IIS0329851 and IIS0412812 and 0205489 and UC Berkeley contract number SA451832441 (subcontract from DARPA's REAL program). The opinions expressed in this paper are those of authors and do not necessarily reflect the opinions of the funders.

References

[Anthony G. Cohn, 2001] Shyamanta M. Hazarika Anthony G. Cohn. Qualitative spatial representation and reasoning: An overview. *Fundam. Inform.*, 46(1-2):1–29, 2001.

[Cohn *et al.*, 2003] A. G. Cohn, D. Magee, A. Galata, D. Hogg, and S. Hazarika. Towards an architecture for cognitive vision using qualitative spatio-temporal representations and abduction. In C. Freksa, C. Habel, and K.F Wender, editors, *Spatial Cognition III*. 2003.

[Gabelaia *et al.*, 2003] David Gabelaia, Roman Kontchakov, Agi Kurucz, Frank Wolter, and Michael Zakharyaschev. On the computational complexity of spatio-temporal logics. In *Proceedings of the 16th AAAI International FLAIRS Conference*, pages 460–464, 2003.

[Intille *et al.*,] S. Intille, J. Davis, and A. Bobick. Real-time closed-world tracking. In IEEE CVPR, pages 697–703, 1997.

[Intille, 1994] S. Intille. Tracking using a local closed-world assumption: Tracking in the football domain, 1994. Master's Thesis, M.I.T. Media Lab.

[Merz *et al.*, 2003] Stephan Merz, Júlia Zappe, and Martin Wirsing. A spatio-temporal logic for the specification and refinement of mobile systems. In *Fundamental Approaches to Software Engineering*, volume 2621, pages 87–101. Springer-Verlag, 2003.

[Minker, 1982] J. Minker. On indefinite databases and the closed world assumpion. In *Proceedings of the* 6^{th} *Conference on Automated Deduction*, volume 138 of *Lecture Notes in Computer Science*, pages 292–308. Springer Verlag, 1982.

[Muller, 1998a] Philippe Muller. A qualitative theory of motion based on spatio-temporal primitives. In *Proceedings of KR'98*, pages 131–141, 1998.

[Muller, 1998b] Philippe Muller. Space-time as a primitive for space and motion. In *FOIS'98*, pages 63–76, Amsterdam, 1998.

[Reiter, 1977] Raymond Reiter. On closed world data bases. In Hervé Gallaire and Jack Minker, editors, *Logic and Data Bases*, pages 55–76, 1977.

[Wolter and Zakharyaschev, 2000] Frank Wolter and Michael Zakharyaschev. Spatio-temporal representation and reasoning based on RCC-8. In *Proceedings of KR2000*, pages 3–14, 2000.

[Yaman *et al.*, 2004] Fusun Yaman, Dana Nau, and V S Subrahmanian. A logic of motion. In *Proceedings of KR2004*, pages 85–94, 2004.