The Transactions Demand for Money in Chile*

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Abstract

This paper examines the transactions demand for money in Chile over the period from 1986 to 2000. Using systems cointegration methods suggested by Johansen (1995), we find that although macroeconomic data for Chile exhibit strong trend-stationarity during this period it is possible to recover relatively robust single-equation specifications for the transactions demand for money. Error-correction models in which money demand is conditioned on real wealth, the level of economic activity, and the nominal Central Bank policy rate provide robust basis for inference. Controlling for a shift in velocity in the end of 1998 the models exhibit a high-degree of out-of-sample predictive power over the period from 1998 to mid 2000.

Key words: Money demand; wealth; vector error correction models; out-of-sample forecasting.

1 Introduction

The objective of this paper is to examine the transactions demand for money in Chile. To add to the already extensive literature on this topic requires some justification. The most prosaic justification is simply that since the Central Bank's policy making relies heavily on assumptions about the demand for money, regular review of the properties of this central behavioural

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relationship is a rather natural activity. However the paper is also motivated by two additional features of the last decade. The first is that since the establishment of an independent central bank in 1989[1], monetary policy in Chile has been framed in terms of an "inflation-targeting" approach, in which the authorities have pursued an inflation target through manipulation of an indexed policy rate of interest (the so-called "policy rate"). Hence it is important for the design of policy to examine the extent to which the private sector's demand for money responds to this policy rate: this represents a key line of investigation running through the paper. The second additional feature is that following a extended period of un-interrupted and rapid growth, the economy experienced a sharp recession from the final quarter of 1998 until mid-1999, before returning back on track in 2000. This recession, the first experienced in the new regime, provides an interesting opportunity to examine whether models estimated over the period of growth are capable of forecasting money demand over this period.

These concerns create a natural shape to the paper. I start with a general discussion of the question of specification, before examining the dynamic money demand equation used in the Bank's own research (see for example, Herrera and Caputo, 1998).[2] As shall be seen, this specification appears to systematically over-estimate money demand, particularly in the late 1990s. The first objective of this paper is therefore to investigate the causes of this systematic predictive failure. I do so by pursuing two lines of enquiry. The first is to re-examine the dynamic specification of the Central Bank money demand function, and the second is to re-visit the underlying structure of money demand, paying particular attention to the role of net-wealth and the specification of the effect of interest rates, inflation and the return on foreign assets.

This stage in the investigation is carried out over the period up to mid 1998, during which time the economy enjoyed uninterrupted growth, holding in reserve the period from 1998 to mid-2000. This latter period is used to conduct out-of-sample forecast analysis and, as a result to revisit a full-sample specification.

[1]The Central Bank's constitutional independence is enshrined in the Political Constitution of Chile and codified in the "Constitutional Organic Act of the Banco Central de Chile" (October 1989). Under this charter, the Bank is charged with "pursuing the stability of the currency against the background of an orderly functioning of the payments system".

[2]Much of the recent debate evaluating the efficacy of stabilization policies in Chile has been conducted within a VAR setting where the demand for money function (or occasionally an inflation function) plays a central role (see Calvo and Mendoza, 1997, Valdes, 1997, Schmidt-Hebbel, 1997, Corbo 1998).

Four principal facts characterize the evolution of the Chilean macroeconomy in since the mid-1980s (Figure 1). The first is the fall in inflation. Since the mid-1980s inflation has fallen consistently from a maximum of 30 percent per annum to less than 4 percent per annum by mid-2000. Second, this has occurred against a background of steady growth in real output and falling unemployment. Growth averaged 7.3 percent per annum from 1985 until mid-1998, making Chile one of the fastest growing economies in the world over the period. This growth contrasts with an average of less than two percent per annum in the 1980s but, more significantly, the recent growth performance has been extremely stable: the coefficient of variation for the period being 0.37 compared to 15.4 for the decade preceding 1985. The recession of 1998/99 saw output decline by around 3 percent year-on-year, but by the final quarter of 1999 output returned to its earlier trend. Third, following a steady depreciation during the early 1980s, the real exchange rate has appreciated at an average of just under 4 percent per annum since the 1989. Even allowing for underlying productivity growth relative to the US (assumed by the authorities to be of the order of 2 percent per annum), and a sharp depreciation towards the end of the period, this represents a significant cumulative real revaluation of the peso during the 1990s. Fourth, the real exchange rate appreciation reflects, at least in part, a consistently tight domestic monetary policy. The key domestic interest rate – the *Tasa Effectiva Politica (TEP)*[3] averaged over 6 percent per annum in real terms from 1985 to 1998 and rose as high as 13 percent per annum during the second half of that year. As a consequence, indexed real lending rates (on medium-term borrowing), for example, have averaged 9.4 percent per annum during the decade and have never fallen below 7 percent.

Against this macroeconomic background of rapid growth and declining inflation, the velocity of circulation for M1A has declined steadily, especially since 1991 (Figure 2).[4] Plotted against developments in the domestic nominal interest rate, shown in the lower panel, it appears that *prima facie* evidence –

[3]Prior to 1987 intervention in money and bond markets occurred across a range of indexed instruments of different maturities. From 1987 the Bank set the price for short-dated (90 day) paper while selling longer dated securities on a tender basis. This continued until April 1995 at which time the Bank focussed on the daily interbank interest rate, influencing its level through market operations (repos and reverse-repos). This rate is referred to as the *Tasa Effectiva Politica* (TEP).

[4]The (quasi) velocity of circulation is defined as the ratio of the index of real economic activity to the money stock. As discussed below, we use the the index of economic activity excluding agriculture and copper. Throughout the paper we restrict our attention to the M1A definition of money which consists of only non-interest bearing components of money, namely currency in circulation plus current account deposit accounts plus other sight deposits (all non-interest bearing).

supported by the econometric analysis which follows – indicates that the fall in M1A velocity in fact moves very closely in line with the decline in nominal interest rates, at least until the second half of 1998. This suggests that, at least in the long run, a simple model defined in terms of economic activity (i.e. income) and nominal interests is likely to have relatively good explanatory power. What is particularly interesting, however, is that from around late 1998 the velocity of circulation *rises*, even though the interest rate falls sharply at this time. This rise in velocity, which indicates a counter-intuitive shift away from money in the face of a falling opportunity costs, may reflect uncertainty about inflation within the region following the stabilization crises in Brazil and elsewhere in the region following the collapse of the East Asian economies in 1997. Whatever the cause, it is clear from Figure 2 that this post-1998 behaviour of the velocity of circulation will pose an interesting challenge to the econometric analysis. It is to the formalization of this relationship I now turn.

Figure 1

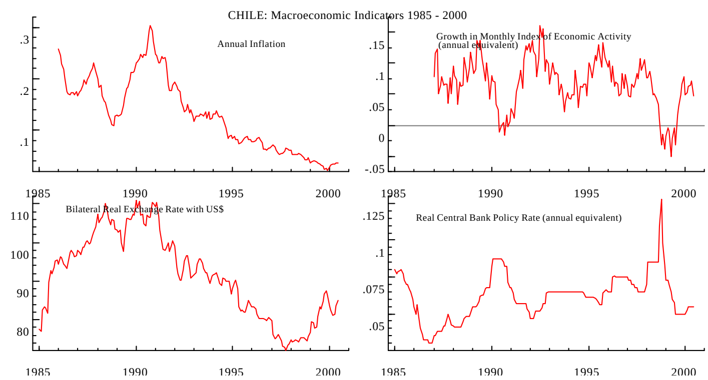
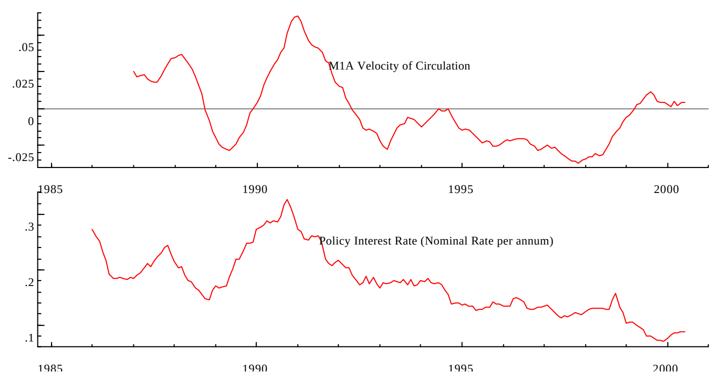


Figure 2: M1A and Central Bank Policy Rate



2 Modelling the demand for money

A standard portfolio approach to the demand for money in an open economy starts with a representative private agent with a four-asset portfolio consisting of claims on government and the banking sector (represented by money), real capital, and the rest of the world (see for example, McCallum and Goodfriend (1987) , Arrau *et al* (1995), McNellis(1998)). Money is assumed to enter directly the utility function of the representative agent, reflecting cash-in-advance constraints and/or transactions technology. Inter-temporal utility is defined as

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} \left(u(c_s), v(\frac{M_s}{P_s}, \frac{E_s M_s^*}{P_s}) \right) \tag{1}$$

where c denotes real consumption, M denotes money (currency and non-interest bearing deposits), M^* is foreign money, suitably defined, E is the nominal exchange rate, and P is the domestic price level. Equation (1) assumes that utility is separable in consumption and money and that domestic and foreign money are substitutes in providing liquidity services to the agent. Assuming for simplicity that the representative agent holds no foreign bonds and interest is paid in arrears on opening asset stocks, the inter-temporal budget constraint is defined as

$$c_t = y_t - \tau_t + r_{t-1}^b b_{t-1} + \frac{i_{t-1}^d D_{t-1}}{P_t} - \frac{\dot{M}_t}{P_t} - \frac{E_t \dot{M}_t^*}{P_t} - \frac{\dot{D}_t}{P_t} - \dot{b}_t \tag{2}$$

where $y - \tau$ denotes real disposable income, b real bonds which earn a real return r, and D are interest-bearing deposits earning a nominal return i^d. A dot denotes the time derivative. The wealth constraint, w, is simply

$$w_t = \frac{M_t}{P_t} + \frac{E_t M_t^*}{P_t} + \frac{D_t}{P_t} + b_t. \tag{3}$$

Maximizing (1) by choosing end-of-period portfolio allocations subject to (2) and (3), results in standard asset demand functions of the form

$$\frac{M^d}{P} = l\left(y^*, w, \pi, i^d, i^b, e\right) \tag{4}$$

where y^* denotes real disposable income (gross of net interest income), i^b denotes the nominal return on bonds, π denotes inflation and e the depreciation of the exchange rate. Importantly, underpinning (4) is the view that current asset demands are conditioned on *expected* income, wealth, and asset prices. Thus the regressors are potentially endogenous. These issues can be handled in a number of ways: in this paper I adopt an approach developed by Johansen (1992) in which the conditional demand functions are derived as restrictions on a generalized vector error correction model.

2.1 Specification and estimation

Equation (4) represents the most common form of conditional demand function, although the literature embraces a wide range of approaches to the estimation of the parameters of this function. One tradition, for example, derives this portfolio choice from an explicit demand system in which asset shares (including money) are defined in terms of wealth and income and *relative* asset prices.[5] It is more common to estimate the parameters from *conditional* asset-by-asset demand functions where asset returns are typically specified in nominal terms. Researchers in this tradition have tended to adopt relatively simple specifications based on a linear approximation of (4), which is the strategy adopted here, although this approach is not universal. Easterly *et al* (1995) adopt a non-linear least-squares estimator to allow explicitly for variable inflation semi-elasticities for high-inflation economies, while McNellis (1998) uses artificial neural network methods to allow for unobservable non-linearity in the long-run demand for money function.

In the current context, two specification issues are of particular interest. The first is the role of real financial wealth. Relatively few studies of the demand for money include wealth as a regressor, mainly because of data limitations. However in those instances where reliable data do exist, researchers have found net wealth to play an important role. In the UK, for example, a number of papers have identified wealth effects in money demand functions (for example, Grice and Bennett (1985), Adam (1991), and Thomas (1997) on broad money aggregates, and Jansen (1998) for narrow money). In these models the wealth effect captures two offsetting processes. The first is a classical income effect arising from the non-neutrality of real financial wealth: assuming money is a normal good this effect is expected to raise money demand, *ceteris paribus*. This may be offset by a second effect where rising wealth typically allows for greater portfolio diversification (particularly if there are non-convex adjustment costs) away from non-interest bearing money. The net effect of rising wealth on the demand for money is therefore strictly ambiguous although the empirical work for the UK and elsewhere typically finds positive but low effects on money demand.

[5]This approach has been popularized by Barr and Cuthbertson (1991), and allows for direct testing of fundamental axions of demand such as, for example, that asset demands are homogeneous of degree zero in prices.

The second issue concerns the specification of asset returns. Here the literature is replete with alternative specifications, reflecting different institutional constraints on portfolio choices. At one extreme, in the case of low-income open economies where financial intermediation is limited, or where financial repression fixes domestic interest rates, the standard measure of the return to holding money is (negative) the rate of inflation (Easterly *et al*, 1995), or the rate of nominal depreciation of the (official or parallel) exchange rate (Domowitz and Elbadawi, 1986). In economies characterized by moderate inflation rates and more developed domestic financial sectors, researchers tend to examine a broader vector of asset returns including the return on interest bearing money (as proxied by the deposit rate of interest),the return on alternative domestic assets (such as bonds), or on foreign-denominated assets. In addition various measures of price volatility (typically backward-looking standard deviation measures) are included to reflect the risk aversion implicit in speculative or portfolio allocation models. Thus, for example, Hendry and Ericsson's (1991) study of money demand in the UK and US include short- and long-term interest rates as well as a measure of inflation (in their case a GNP deflator); Johansen (1995) excludes inflation but includes two interest rates (a representative deposit rate of interest and the bond rate); Jensen's (1998) model for UK base money prefers a specification which includes the short-run interest rate, inflation and inflation variability. Closer to home, Ahumada's (1992) study of Argentina considered inflation and a single interest rate, Arrau *et al* (1995) use the rate of interest on short term deposits, while the body of VAR-based work carried out within the Central Bank of Chile, rather naturally given the empirical methodology, concentrated on inflation and the short-term policy rate as discussed above (see Valdes, 1997 and Herrera and Caputo, 1998).

The work reported below summarizes an extensive evaluation of alternative asset market specifications. As I show below, the data accepts a number of rival specifications which reflect the relative stability of relationship between asset prices. It is not my intention here to examine the term structure within Chilean asset markets, but rather I focus on the key asset market information used by the private sector in determining its demand for money. Since my focus in this paper is the demand for transactions balances only (M1A), which is entirely non-interest bearing, the *ex post* opportunity cost to holding M1A should in principle be the (weighted) return to the portfolio of other assets (real assets, central bank paper, foreign-currency deposits or, more reasonably, interest bearing deposits with the banking sector). The closest substitute to non-interest bearing bank deposits are less-liquid but interest bearing deposits with the banking sector suggesting that for M1A the appropriate interest rate would be the return on short-term deposits with the banking sector, which is the rate used in the Central Bank's money demand studies. However, under the inflation-targeting regime the key interest rate is the policy rate or *TEP* which is an indexed rate denominated in terms of the indexation unit of account, the *unidad de fomento* (UF).[6] In the empirical work reported below I convert the *TEP* to an *ex post* nominal interest rate (the real policy rate adjusted for actual inflation), but also examine whether there is a differential response to the 'policy' and inflation components of the nominal interest rate.

3 Empirical Analysis

The natural starting point for my empirical investigation is the Central Bank's existing model for the demand for money which adopts a traditional partial-adjustment specification of the following form:[7]

$$(m-p)_t = \alpha_0 + \alpha_1 y_t + \alpha_2 i_t + \alpha_3 \Psi_t + \alpha_4 (m-p)_{t-1} + \varepsilon_t \qquad (5)$$

where $(m-p)$ denotes the log of real money balances, $M1A$, deflated by the consumer price index; y_t is a proxy for real income defined by the log of the Central Bank's monthly indicator of economic activity ($IMAE$); i is the domestic interest rate on short-term domestic deposits; and Ψ denotes a vector of monthly seasonal dummy variables, a dummy variable capturing national holidays, and a single-period dummy variable capturing a large impulse to reserve money (and thus $M1A$) in March-April 1992.[8] Using monthly data for the period February 1985 to May 1998 (t-statistics are reported in parentheses) the Central Bank's basic model is:

$$\begin{array}{rcl} (m-p)_t & = & \underset{[10.93]}{1.945} + \underset{[8.69]}{0.314} y_t - \underset{[13.41]}{0.034} i_t + \underset{[26.33]}{0.726} (m-p)_{t-1} \end{array} \qquad (6)$$

$T = 160 \qquad \bar{R}^2 = 0.998 \quad s.d(m-p) = 0.363 \quad \sigma = 0.017$
$DW = 1.751$
Forecast χ^2 : $\chi^2(25) = 98.184$ [0.0000]**
Forecast Chow-test : $F(25, 132) =$ 2.386 [0.0008]**

[6] The UF index is equal to the previous month's consumer price inflation so that the policy rate is (to a close approximation) equal to the *ex post* real interest rate.The equivalence is only approximate because collection lags means that the UF 'month' does not exactly coincide with the calendar month.

[7] This model was operational as of April 1999. I am grateful to Rodrigo Caputo of the Macroeconomic Division for providing details on this specification.

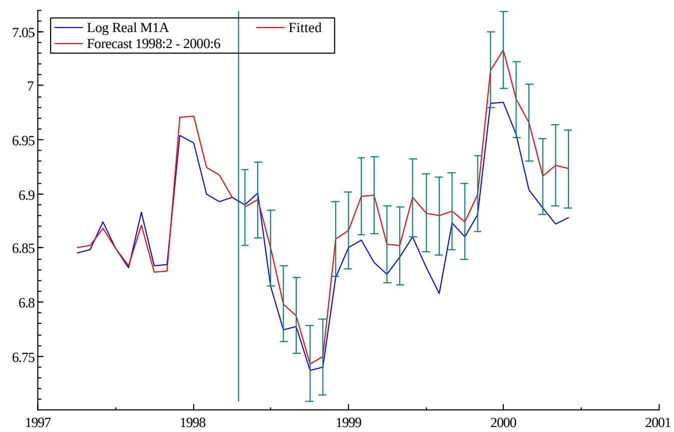
[8] Base money increased by 72 percent between February and May 1992 but returned to its original level in May 1992.

Solving out, this generates an implied long run or equilibrium demand function of the form

$$(m-p)_t = 7.09 + 1.145y_t - 0.124i_t. \tag{7}$$

The model tracks past behaviour of real money balances extremely well but, as the Forecast χ^2 and break-point Chow test statistics reported under equation (6) indicate, the out-of-sample forecast performance of the model over the period from the mid-1998 to mid-2000 is poor. It appears that although the model successfully forecasts the decline in real money demand during the second half of 1998, it systematically (and statistically significantly) *over-predicts* demand for money during the recovery in 1999 and the decline in the first half of 2000: every single forecast error $(m - \hat{m})$ is negative (see Figure 3).

Figure 3: Central Bank Money Demand 25 month forecast



3.1 Re-specification of the Central Bank model

It should also be noted that the point estimate implied greater than unity long run income elasticity in (7) is higher than theory would suggest. One possible explanation is that this reflects the trend decline in the velocity of circulation, but it is also possible that it is due to the well known biases in long-run parameter estimates that arise using partial-adjustment specifications in the presence of non-stationary data. My empirical approach therefore commences with an analysis of the dynamic specification of the above model before proceeding to review issues more general issues in the specification of the model. As noted above, I shall conduct this analysis over the period from 1986 to 1998 only, leaving the most recent history for out-of-sample analysis.

3.1.1 Time-series characteristics of the data

The first step is to examine the time-series characteristics of the data. In view of the well-known low power of these tests, I subject the data to a battery of alternative tests. The data and tests are reported in Appendix I and can be arranged in three groups. The first set derive from the original Dickey-Fuller tests and are defined against the null hypothesis that a time-series has a unit root under different (un-tested) assumptions concerning deterministic components in the data: these are the *Augmented Dickey-Fuller* and *Phillips-Perron t-tests*. The second set of tests reverses the null hypothesis to test the null of stationarity against the alternative of non-stationarity (*Kwiatkowski, Phillips, Schmidt and Shin - KPSS*). The third group focuses on testing joint hypotheses about the stochastic and deterministic components of the time-series (*Dickey-Fuller, Phillips-Perron F-tests*).

Taken together these unit root tests paint an interesting, if slightly confusing, picture. The fundamental issue is not whether the macroeconomic time series on money aggregates, income, and wealth proxies exhibit trending behaviour over time, since they clearly do, but whether this trend is best characterized as stochastic or deterministic. To illustrate the problem, consider the data on the log of the real money stock ($LRM1A$). Allowing only for a drift in the process then the tests suggest that real money is clearly non-stationary: the null that the series has a unit root cannot be rejected for the Augmented Dickey-Fuller and Phillips-Perron tests in columns 1 and 3 of Appendix Table 2, while the null of stationarity is decisively rejected under the KPSS test with drift in column 5. However, from another perspective the data are also fully consistent with a trend-stationary representation (the null is rejected for the PP test with trend (column 4) and is just accepted for the KPSS test with trend (column 6)). Following the reduction sequence suggested by Dolado, Jenkinson and Sosvilla-Rivero (1990), reported in column 7, confirms in this case that a deterministic trend-stationary representation is marginally more likely than a stochastic trend one.[9] A similar analysis for the indicators of economic activity ($LIMAE$ and $LIMAEX$), and real net financial wealth ($LRNW$) reveals a similar degree of ambiguity: while the balance of evidence is marginally in favour of a stochastic trend (i.e. unit root) representation for the measure of wealth, the two measures of economic activity appear to contain a deterministic trend.

An immediate problem is how one should interpret these results, bearing in mind the relatively low power of this class of test. Should the obvious

[9] This reduction sequence uses both F- and t-tests to test for unit roots under progressively restrictive assumption concerning the deterministic components of a time-series. This routine is automated under the RATS source code **urauto.src.**

trend in the data be treated as deterministic (a linear trend) or stochastic (a unit root)? There are strong reasons for favouring the latter. From a theoretical perspective, the assumption that the long-run evolution of the data is deterministic – so that shocks impact only the deviation around the trend and not the trend itself – seems unreasonably restrictive. Moreover, while a trend-stationary description may adequately describe well the current, relatively short, sample, it needs to be recognized that this period is unusual. Viewed over a longer historical time-span, Chile has seen large permanent movements in the level of real variables consistent with a unit-root process. It is therefore unreasonable to assume that money and income in Chile can be characterized over any forecast horizon beyond the short-run as stationary around a *deterministic* linear trend. In the light of these reservations, I therefore proceed under the assumption that the trends in these time-series are stochastic rather than deterministic so that the series can be characterized as random walks with drift. As shall be seen in the next section, this has important implication for the interpretation of the long-run cointegrating characteristics of the data.

Matters are somewhat clearer with the price level ($CPI89$), the exchange rate (NER), and the various interest rates. Prices and the nominal exchange rate are I(1) with drift so that (monthly) inflation and the depreciation of the nominal exchange rate are I(0), while the real (annual) and nominal (monthly) interest rates are borderline non-stationary.

3.1.2 Cointegration analysis

Even though the data appear to be a mixture of trend and difference stationary processes, I start by assuming that the vector of variables in (5), denoted $X_t = \{(m-p)_t, y_t, i_t\}$, consist of at least some non-stationary components. I proceed to a cointegration analysis following a standard Johansen analysis, starting with an analysis of the Central Bank model before extending the discussion to consider alternative specifications of the basic model.

The general vector error-correction model (VECM) which provides the basis for our estimation can be expressed as follows (see Johansen, 1995):

$$\Delta X_t = \alpha\beta' X_{t-k} + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \Phi t + \Psi_t + \varepsilon_t \tag{8}$$

where β' denotes the parameters of the cointegrating vectors (if such relationships exist), and α denotes the matrix of feedback, or error-correction, effects of the dynamic equations in ΔX_t to the long-run relationships. Γ_i represents the vector of short-term parameters, and the vector Ψ_t consists of seasonal dummy variables. This representation thus describes the evolution of the variables X as being driven partly by the cointegrating or long-run equilibrium relations between the variables (assuming they exists), and partly by the evolution of the so-called "common trends" component, ΔX_{t-i}.[10] At this stage a key decision concerns how to characterize the deterministic components of the model, Φ_t, particularly in the light of the slightly ambiguous evidence from the unit root tests. The deterministic components consist of a constant, μ, and a linear trend t. In principle, either or both components can enter the cointegrating and the common-trends components of the model. To reflect this possibility I rewrite equation (8) as

$$\begin{aligned} \Delta X_t &= \alpha \begin{pmatrix} \beta \\ \mu_1 \\ \delta_1 \end{pmatrix}' X_{t-k} + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \alpha_{\perp}\mu_2 + \alpha_{\perp}\delta_2 t + \Psi_t + \varepsilon_t \quad (9) \\ \varepsilon_t &\backsim niid(0, \Sigma) \end{aligned}$$

The parameters $\alpha\mu_1$ and $\alpha\delta_1$ measure the effect of the deterministic components on the long-run properties of the model, while $\alpha_{\perp}\mu_2$ and $\alpha_{\perp}\delta_2$ measure the effect on the common-trends, or growth-rate, components.

Five alternative specifications for the deterministic components of the model are embedded in (9), depending on the values of μ_i and δ_i.[11] These are: (i) μ_i and δ_i (where $i = 1, 2$) are unrestricted so that there may be linear trends in the growth rate of X_t (i.e. $\delta_2 \neq 0$) which implies that there are *quadratic* trends in the levels of the vector X_t ; (ii) $\delta_2 = 0$ which excludes quadratic trends in the levels relationship but allows for the possibility of a deterministic linear trend in the cointegrating vector(s); (iii) $\delta_1 = \delta_2 = 0$ which implies no linear trend in the cointegrating relationships but allows for linear trends in the level of the data itself (through $\mu_2 \neq 0$ operating on ΔX_t), and non-zero intercepts in the cointegration vector; (iv) $\delta_1 = \delta_2 = 0$ and $\mu_2 = 0$ which implies that the only deterministic components in the data are the intercepts in the cointegrating relationship; and (v) $\mu_1 = \mu_2 = \delta_1 = \delta_2 = 0$ which assumes no deterministic components in the model.

I distinguish between these rival specifications (allowing for alternative specifications of the income and interest rate) by comparing the trace statistic under each restriction.[12] At this stage I also consider two other specifica-

[10] See Johansen (1995, chapter 3).

[11] See Hansen and Juselius (1995).

[12] The Trace statistic is defined $-T \sum \ln(1 - \hat{\lambda}_i)$ where λ_i are the eigenvalues of maximization problem underlying the estimation of the vector $\alpha\beta'(4)$. We assume a lag-length of 6 throughout.

tion issues. First, I examine the properties of two alternative versions of the income variable, y_t. The basic measure, the monthly index of economic activity (denoted $LIMAE$) is currently employed by the Bank's Macroeconomic Division. I also consider an alternative index, (denoted here as $LIMAEX$), which is the same index excluding the agriculture and copper mining sectors, the two sectors subject to greater short-run price volatility in world prices, independent of domestic economic factors. Second, I consider both a logarithmic and semi-log specification for the nominal policy interest rate (money and income measures are both defined in logs). The summary trace statistics for the alternative specifications are presented in Table 1.

Table 1: Trace Statistic for Alternative Specifications of Deterministic Components in the VECM

Sample:1986(1)-99(1) Scale Variable	Interest Rate	Model 1 $\delta_i \neq 0$ $\mu_i \neq 0$	Model 2 $\delta_2 = 0$ $\mu_i, \delta_1 \neq 0$	Model 3 $\delta_i = 0$ $\mu_i \neq 0$	Model 4 $\delta_i = 0$ $\mu_2 = 0$	Model 5 $\delta_i = 0$ $\mu_i = 0$
LIMAE	log	57.129	59.013	43.144	57.854	36.701
LIMAEX	log	56.973	58.927	49.186	61.984	51.738
LIMAE	semi-log	53.721	55.132	38.114	54.522	36.026
LIMAEX	semi-log	52.106	53.617	43.028	55.883	51.889

The trace statistics indicate that the most likely specification for the deterministic components in the model is either Model 4, where the only deterministic component is the constant of cointegration, or Model 2, where the cointegrating vector itself contains a deterministic trend. Given the evidence from Appendix Table 2 on the unit root properties of the data, it follows naturally that the trend-stationary representation under Model 2 will explain the data well, at least within our sample period. However, for the reasons discussed above, the alternative given by Model 4 is a more appealing specification. Adopting this latter characterization, the results suggest that the dominant specification is in terms of the narrower definition of the scale variable, and a logarithmic specification of the interest rate.

The Cointegrating Rank To determine and evaluate the cointegrating vectors I next estimate the restricted VECM, equation (10), where $k = 6$ is sufficient to ensure that the error term, $\varepsilon_t$is approximately Gaussian.

$$\Delta X_t = \alpha \begin{pmatrix} \beta \\ \mu_1 \end{pmatrix}' X_{t-k} + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \Psi_t + \varepsilon_t \qquad (10)$$

$$\varepsilon_t \backsim niid(0, \Sigma).$$

Table 2 reports the residual diagnostics of model (10) and Table 3 the eigenvalues and associated statistics for the cointegrating vectors. The residual diagnostics suggest that there is no serious misspecification, either for the vector X_t as a whole, or the components, except in terms of the violation of the Jarque-Bera test for error normality which reflects outlier observations due to the jump in reserve money discussed in the context of equation (5) above. Conditioning the cointegration analysis on a dummy variable for this event eliminates the skewness / kurtosis causing the violation of error normality.

Table 2: VECM Residual diagnostic statistics

	AR(1)	LB(12)	ARCH(7)	H	JB
X	1.209	75.947		0.677	80.227**
m	0.861	3.932	0.081		52.347**
y	0.842	21.337	0.137		5.038
i	1.156	6.498	0.424		8.512

Notes: AR(1) denotes LM test for vector (series) first-order autocorrelation LB(12) is Ljung-Box Portmanteau test against residual autocorrelation; ARCH is Engle's test against conditional heteroscedasticity; H is White's vector test for heteroscedasticity, and JB is Jarque-Bera test for vector (series) normality. All tests distributed χ^2; ** denotes significant at 1% level, * significant at 5%.

Table 3 Eigenvalues (λ_r), Maximal Eigenvalue and Trace Statistics

		Maximal Eigenvalue		Trace Statistic	
r	λ_r	$-(T-nm)\ln(1-\lambda_r)$	$5\%c.v.$	$-(T-nm)\sum\ln(1-\lambda_r)$	$5\%c.v$
0	0.4922	87.42**	22.0	110.00**	34.9
1	0.1499	20.95**	15.7	22.58**	20.0
2	0.0125	1.63	9.2	1.63	9.2

Statistics reported with Reimers (1992) small-sample correction. Critical Values from Osterwald-Lenum (1992).

Table 3 suggests that the cointegrating rank in this case is two, although the second eigenvalue is only one-third the size of the largest, and is only just significant. The two significant β' eigenvectors and α feedback vectors are reported in Table 4. As is well known, cointegrating vectors are only unique in terms of the space they span (i.e. the first and second rows of the β' matrix are unique stationary linear combinations of the variables) so that the normalization chosen in Table 4 is arbitrary: it does not necessarily follow that the first cointegrating vector represents the demand for money. However, in this case interpreting it as such has a *prima facie* plausibility. It suggests an income elasticity of the demand for money of just under unity and an elasticity with respect to the monthly interest rate of -0.189, which seem plausible, both in terms of theory and relative to the existing partial-adjustment based model, equation (7). The second piece of evidence in support of this interpretation is the that the feedback coefficient on the first vector is negative, strongly significant, and quite high, indicating a mean lag response to monetary shocks of around 4 months.[13] The first vector is therefore, plausibly, our money demand function, where c denotes the restricted constant, $\alpha\delta_1$.

Table 4
β' Eigenvectors [Normalized on Diagonal]

	$m-p$	y	i	c
(1)	1.000	-0.931	0.218	-0.670
(2)	-0.891	1.000	-0.006	-0.215

α Loadings (Adjustment/Feedback) Vectors
standard errors in parenthesis

	(1)	(2)
$\Delta(m-p)$	-0.178	0.196
Δy	-0.013	-0.187
Δi	0.442	-2.673

There is, however, the second vector to consider. Although it too could represent a money demand function this seems unlikely. The vector suggests a positive relationship between money and income but that this is not influenced by the interest rate (the coefficient on i is low at -0.006 and not statistically significant). This relationship, which traces the trend decline in velocity as seen in Figure 2, would appear to reflect the evidence from Table A1 on the unit root statistics. There I noted that both real money balances and real economic activity were borderline trend-stationary processes. If the underlying deterministic trend in these processes is similar (as it is in this instance) then such a pair of series would naturally generate a stationary linear combination and hence would lead to a second significant eigenvalue statistic in Table 3, even though it may not represent an economically meaningful cointegrating relationship.[14]

[13]The mean lag is defined as $\mu = (1-\alpha)/\alpha$

[14]Each (trend) stationary variable in the vector X necessarily creates an additional stationary combination: this can be tested by considering whether the stationary combination remains significant in the face of restriction of the non-stationary variables out of the vector. This is what I do with the identification restrictions here. Notice that this interpretation, namely that the second cointegrating vector is a product of the short-

Given that the focus in this paper is the money demand function my strategy is to seek a set of restrictions which allow me to derive a unique identification of the first vector as a money demand relationship in a manner that ensures that the second vector is appropriately handled (even though it does not necessarily lend itself to a meaningful economic interpretation). For a cointegrating rank $r = 2$ identification can be achieved by imposing (at least) one restriction per vector. A natural restriction suggested by Table 5 is to impose a unit income elasticity on the money demand function (vector 1) and to restrict the second vector to be defined in terms the money-income relationship only. This implies that the cointegrating space can be defined by the two identified vectors:-

$$\begin{aligned} \beta_1^* &= (1, -1, *, *) \\ \beta_2^* &= (*, 1, 0, 0). \end{aligned} \tag{11}$$

(where a * denotes an unrestricted coefficient). The validity of the restriction is tested with the standard likelihood ratio test which is distributed χ^2 with $\sum_{i=1}^{r}(p - s_i - r + 1)$ degrees of freedom, where p is the dimension of X, r is the cointegrating rank, and $(p - s_i)$ is the number of restrictions imposed on each cointegrating vector.[15] In the above case I find

$$LR(r = 2) : \chi^2(1) = 1.4524[0.2281]$$

which indicates that the restriction can comfortably be accepted, allowing a unique long-run cointegrating relationship underpinning the demand for money to be identified.

However, as Johansen (1992) shows, it is only valid to move from this result direct to a single-equation representation of the demand function (in the presence of single or multiple cointegrating vectors) if the following "partial system" restriction is also accepted by the data (conditional on the restrictions imposed on the β' matrix)

$$\begin{aligned} \alpha_1' &= (\alpha_{11}, 0, 0) \\ \alpha_2' &= (\alpha_{21}, 0, 0). \end{aligned} \tag{12}$$

This restriction implies that the cointegrating vectors feedback onto the $\Delta(m-p)_t$ equation only, allowing us to ignore, without loss of likelihood, the dynamic equations for Δy_t and Δi_t. Failing to accept this restriction implies

sample characteristics of the data, is consistent with findings from other research on Chile using longer time-periods which typically find only a single cointegrating vector for money demand models of this form.

[15](see Johansen, 1995, Theorem 7.5)

that Δy_t and Δi_t cannot be treated as weakly exogenous with respect to the parameters of a single-equation error correction model for $\Delta(m-p)_t$.

In this case of Chile the restriction is rejected at standard probability levels, but the weaker restriction

$$\begin{aligned} \alpha_1' &= (\alpha_{11}, 0, 0) \\ \alpha_2' &= (0, \alpha_{22}, \alpha_{23}). \end{aligned} \tag{13}$$

is accepted (in this case $LR(r=2) : \chi^2(4) = 7.7084[0.1029]$).This restriction, which is imposed jointly with the restrictions on the β' matrix, implies the following restricted cointegrating vectors and feedback matrix (with asymptotic standard errors reported beneath un-restricted coefficient estimates).

Table 5

β' Eigenvectors [Normalized on Diagonal]

	$m-p$	y	i	c
(1)	1.000	-1.000	0.1185	-0.3571
			[0.0189]	[0.084]
(2)	-0.912	1.000	0.000	0.000
	[0.006]			

α Loadings (Adjustment/Feedback) Vectors

	(1)	(2)
$\Delta(m-p)$	-0.310	0.000
	[0.028]	
Δy	0.000	-0.130
		[0.054]
Δi	0.000	-2.575
		[0.559]

This restriction is quite interesting. It suggests that I can accept that the money demand cointegrating vector impacts only on the dynamic model for money, $\Delta(m-p)_t$, and that the second vector only impacts on the Δy and Δi processes. However, because the second vector is defined in terms of the level of real money balances, it follows that I must allow for the possibility that both income and the short-run interest rate are potentially endogenous regressors in the dynamic money demand model which will, therefore, require to be estimated using an IV estimator.[16]

[16] This arises because the second cointegrating vector is defined in terms of money and income and feeds back on to both income and the interest rate implying a contemporaneous link from these two variables to money.

3.2 Alternative specifications

At this stage it would be natural to proceed directly to the dynamic error-correction model based on the evidence from Table 5.[17] However, it is useful to first return to our general theoretical model (4) to consider whether a more general model, which includes wealth effects and a more comprehensive treatment of asset prices, better fits the data. This kind of specification search is potentially extensive: in order to keep the paper brief I shall report only the main findings of this specification search.

3.2.1 Returns on other assets

I consider three alternative specifications of the vector of opportunity cost variables. First, I consider the consequence of adding a measure of the rate of interest on deposits. Second I examine the effect of decomposing the nominal interest rate into its inflation and real interest rate components. Finally, I consider the role of currency substitution effects by including the (*ex post*) rate of depreciation of the nominal exchange rate. To summarize the findings, it appears that a model defined exclusively in terms of the short-run nominal policy rate of interest is the most efficient specification for the transactions demand for money over this period. Neither adding additional interest rates, nor decomposing the nominal rate into its real interest rate and inflation components, nor allowing for exchange rate effects improves the statistical power of the model, suggesting that for the transactions demand for money, the nominal policy rate of interest is a sufficient statistic for the opportunity cost of money. This is not a particularly surprising finding and is echoed in much of the empirical literature on the transactions demand for money.[18]

The detail of this investigation are summarized in Appendix II so here I limit myself to the details of the investigation strategy and main results. I first consider the own rate of interest. In the previous specification the only opportunity cost variable was the *ex post* nominal policy rate (i.e. the effective nominal rate derived from the real effective policy rate and the inflation rate), based on the assumption that *all* of M1A is non-interest bearing. However, if current accounts attract interest (as is becoming common), or if current accounts are complementary with interest-bearing savings accounts, it may be appropriate to allow for both the policy rate (which measures the cost of holding money for transactions purposes) and the "own-rate" of inter-

[17] Details are available on request from the author but it is possible to show that the error-correction representation readily outperforms the Central Bank specification.

[18] McNelis (1998), for example, finds that only the domestic interest rate is significant in the long-run demand for money estimated over the period from 1983 to 1994.

est on (broad) money.[19] Hence the long-run cointegrating vector is defined $X = \{(m-p)_t, y_t, i_t^p, i_t^d, \}$ where i_t^p id the policy rate of interest and i_t^d is the deposit rate of interest. Estimating this version of the model generates three significant eigenvectors, reported in Table 6

Table 6

β' *Eigenvectors [Normalized on Diagonal]*

	$m-p$	y	i^p	i^d	$cnst$
(1)	1.000	-0.953	0.235	-0.067	-0.709
(2)	-0.882	1.000	-0.022	0.022	-0.250
(3)	-3.839	3.922	1.000	-1.926	-2.287

Allowing for the own interest rate effect does not significantly alter the measured income elasticity or the constant of the vector. However, the coefficient on the policy rate has increased (and is still negative) while that on the own rate has the opposite sign. This indicates that an increase in the deposit rate of interest in the long-run, *ceteris paribus,* increases money demand although with a weaker effect than a corresponding negative effect of an increase in the policy rate. Although the second vector also retains the same structure as before it also hints at a symmetry between the two interest rates, consistent with the stationarity of the spread between the two interest rates. As shown in Appendix II, this symmetry can be used to eliminate i^d in the first vector, in which case the money demand relationship reduces to *exactly* the parameterization found in Table 5 above (where the net interest elasticity = -0.113). From a purely statistical perspective at least, the cointegration between nominal interest rates thus allows us to represent the long-run money demand function solely in terms of the policy rate in the knowledge that other rates are cointegrated with the policy rate. In other words there is no significant loss of information in defining the (long-run) money demand function as a function only of the short-rate.

3.2.2 Differential response to inflation and the real policy rate

Next I consider the role of inflation. Implicit in the original specification is the assumption that agents respond equivalently at the margin to changes in the opportunity cost of holding money, regardless of whether the change in the cost arises from the effect of inflation or from a change in the interest rate. To test this hypothesis I use the same basic model but decompose the

[19] This is the specification examined for Denmark by Johansen and Juselius, 1990).

nominal interest rate into a pure inflation effect and the pure interest rate effect (ignoring the joint effect $r\pi$ which is negligible in monthly data)

$$i = r + \pi + r\pi \approx r + \pi \tag{14}$$

and test whether the coefficient restriction $\beta^r = \beta^\pi$ is accepted in the money demand relationship. As the tables in Appendix II show, the restriction is comfortably accepted by the data and reveals that the restricted interest / inflation elasticity is of the same order of magnitude as in our previous case (-0.145 compared to -0.114). This implies that a model defining the transactions demand for money solely in terms of the nominal policy rate of interest provides an efficient and data-consistent representation of the (long-run) data

3.2.3 Currency substitution

Finally currency substitution is tested by including the depreciation of the nominal peso exchange rate relative to the US dollar in the vector X.[20] Since this variable is strongly stationary, this will generate an additional cointegrating vector. However, even allowing for this effect there is some weak evidence of currency substitution in money demand (a rise in the exchange-rate adjusted foreign interest rate reduced domestic money demand), although the effect is not statistically significant and the coefficient can be restricted to zero, suggesting that in the interests of parsimony I am justified in reverting to the basic specification used above (10).

3.3 Wealth effects

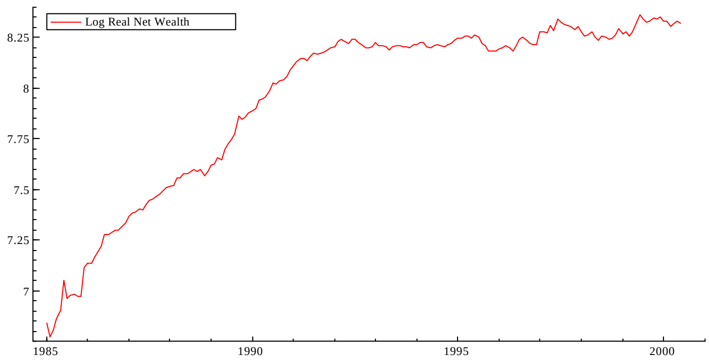
Finally, I turn to the role of wealth in the money demand function. Ideally I would use a measure of the total net wealth of the private sector (i.e. including their claims on real assets including housing and foreign assets). These data are not, however, readily available and I therefore follow common practice elsewhere and use a measure of real net financial wealth (see Jansen, 1998), defined as the total financial claims of the non-bank private sector against the banking sector, government and the rest of the world, net of their liabilities to those sectors (i.e. the net claims of the monetary sector on the non-bank private sector), all deflated by the price index. For Chile wealth is defined as

[20]This specification assumes that currency substitution is a purely transactions activity and that foreign currency deposits are un-remunerated. However allowing for the possiblity that foreign assets are interesting bearing by defining the relevant opportunity cost as the exchange rate adjusted interest differential does not alter the findings reported here.

$$w_t = \frac{(M4 + FXD - NCRED)}{P} \tag{15}$$

where $M4$ equals currency plus demand, sight and time deposits, plus private sector holdings of bonds and bills; FXD equals "on-shore" holdings of foreign currency (valued in current pesos), and $NCRED$ is the net credit from the monetary system to the non-bank private sector.

Figure 4 Log Real Net Wealth (Constant 1989 Prices)



As Figure 4 indicates, on this measure there has been a significant increase in real net wealth over the period from 1985-1991(averaging around 15% - 25% per annum) but a leveling out during the 1990s, and indeed stagnating later in the decade. Table 7 reports the eigenvalues and significant cointegrating vectors (plus their feedback coefficients) for the model represented by (10) but where X is augmented by real net financial wealth.

Table 7 Eigenvalues, Maximal Eigenvalue and Trace Statistics

		Maximal Eigenvalue		Trace Statistic	
r	λ_r	$-(T-nm)\ln(1-\lambda_r)$	5%c.v.	$-(T-nm)\sum\ln(1-\lambda_r)$	5%c.v
0	0.5386	113.70**	28.1	159.90**	53.10
1	0.1761	28.47**	22.0	46.25**	34.90
2	0.0953	14.72	15.7	17.78	20.0
3	0.0206	3.06	9.2	3.06	9.2

Statistics reported with Reimers (1992) small sample correction. Critical Values from Osterwald-Lenum (1992).

β' Eigenvectors [Normalized on Diagonal]

	$m-p$	y	w	i^p	c
(1)	1.000	-0.434	-0.362	0.245	0.762
(2)	-0.766	1.000	-0.108	0.086	0.287

α Loadings (Adjustment/Feedback) Vectors

	(1)	(2)
$\Delta(m-p)$	-0.173	0.263
Δy	-0.0234	-0.192
Δw	0.001	0.083
Δi^p	0.343	-2.919

Including wealth in the demand function has a significant effect on the measured transactions demand for money, lowering it to approximately 0.4. The wealth elasticity is positive and significant (the asymptotic standard error is approximately 0.094), but lower than the income elasticity. The elasticity with respect to the policy interest rate remains negative and significant although is significantly larger than in the baseline case. As above, I seek identifying restrictions on this two-vector system to isolate the wealth-augmented demand function. To do so I restrict the income elasticity to 0.5 and restrict both wealth and the interest rate from the second vector (to allow it the same interpretation as before). I have also imposed the same "partial system" restriction as before on the feedback vector. This gives the following restricted cointegrating vectors where I have included the asymptotic standard error for the β' vectors and their feedback coefficients.

Table 8

β' Eigenvectors [Normalized on Diagonal]

	$m-p$	y	w	i^p	c
(1)	1.000	-0.500	-0.331	0.248	0.610
			[0.032]	[0.042]	[0.298]
(2)	-0.909	1.000	0.000	0.000	0.000
	[0.008]				

α Loadings (Adjustment/Feedback) Vectors

	(1)	(2)
$\Delta(m-p)$	-0.184	0.000
	[0.016]	
Δy	0.000	-0.164
		[0.058]
Δw	0.000	0.022
		[0.046]
Δi^p	0.000	-1.400
		[0.545]

LR Test against restriction (r=2) $\chi^2(6) = 10.137$ [0.0715].

The LR test indicates that I can accept the same partial-system restriction on the vector error correction model as before. Hence I can define the error correction model in terms of the first row of Table 8 but where Δy, Δw, and Δi^p may be endogenous in the dynamic specification for the reasons discussed above. Given this, I next proceed to a single-equation error correction model.

3.4 A single equation error-correction model

I define the deviation from equilibrium money demand directly from the first eigenvector in Table 8 (controlling also for seasonal dummy variables) as

$$\lambda_t = (m - 0.5y - 0.331w + 0.248i^p + 0.610). \tag{16}$$

Since the dynamic model for $\Delta(m-p)_t$ does not adjust to deviations from the second eigenvector, the general over-parameterized error correction model can be defined as:

$$\begin{aligned} A(L)\Delta(m-p)_t &= B(L)\Delta y_t + C(L)\Delta i_t + D(L)\Delta w_t \\ &\quad + E(L)\sigma_{\pi t} + \gamma\hat{\lambda}_{t-1} + \Psi_t + \varepsilon_t \end{aligned} \tag{17}$$

where Δ denotes the monthly difference operator, $A(L)$, $B(L)$, $C(L)$, $D(L)$ and $E(L)$ are lag polynomial parameter matrices, and γ is the error-correction coefficient. The vector Ψ_t denotes the deterministic components as discussed above (monthly dummy variables, a dummy variable controlling for the pattern of public holidays (denoted $DHOL$), and $DM0292$, an impulse dummy variable taking the value of one in March 1992. Equation (17) also includes a measure of inflation volatility denoted σ_π, measured as the change in the 12-month moving standard deviation of the monthly inflation rate.[21]

Table 9 reports three versions of the model. In the first, I estimate the model using OLS and then, given the results from the long-run cointegration analysis, I re-estimate the model instrumenting for Δy_t, Δw_t and Δi_t^p. I also consider a version of the OLS model in which I allow for an asymmetric error-correction mechanism (so that the speed of adjustment to equilibrium differs between positive and negative disequilibria).

[21] An alternative measure of volatility, defined as a moving standard deviation of the measure of the deviation from the long-run equilibrium.

$$vol_t = \sum_{i=1}^{3} \sqrt{(\hat{\lambda}_{t-i} - \bar{\lambda})^2}.$$

Both measures are virtually identical, implying that measured deviation from the long-run equilibrium reflects price rather than income shocks. We therefore only report the version with the traditional inflation volatility measure.

Table 9. Dynamic Error Correction Money Demand Model
Dependent Variable $\Delta(m-p)_t$. Sample 1986(7) - 1998(8)

Model	[1]	OLS	[2]	OLS	[3]	IV	
Variable	Coeff.	HCSE-t	Coeff.	HCSE-t	Coeff.	t-value	Instab
$Constant$	0.037	7.71	0.032	2.76	0.037	5.519	0.10
$\Delta(m-p)_{t-2}$	-0.274	5.17	-0.264	5.08	-0.270	5.946	0.15
$\Delta(m-p)_{t-3}$	-0.160	3.55	-0.183	4.13	-0.153	2.868	0.99*
Δy_t	0.353	1.55	0.393	1.75	0.382¶	1.147	0.04
Δw_t	0.333	4.31	0.317	4.32	0.341¶	2.863	0.74*
Δw_{t-2}	0.242	3.50	0.215	3.22	0.241	3.213	0.32
Δi_t^p	-0.058	10.27	-0.058	10.18	-0.063¶	10.294	0.15
Δi_{t-3}^p	-0.015	2.83	-0.017	3.31	-0.015	2.868	0.03
$\hat{\lambda}_{t-1}$	-0.124	11.63	-	-	-0.129	11.783	0.06
$\hat{\lambda}_{t-1}^+$	-	-	-0.103	5.48	-	-	
$\hat{\lambda}_{t-1}^-$	-	-	-0.139	6.44	-	-	
$\sigma_{\pi t}$	-0.359	2.87	-0.372	3.05	-0.318	2.281	0.24
$DM0292$	0.138	15.86	0.138	16.26	0.138	14.345	0.09
$Dhol$	0.004	2.67	0.004	3.36	0.004	2.879	0.09
	R^2	0.935	R^2	0.943	s.e (SF)	0.0138	
	s.e	0.0138	s.e	0.0135	s.e (RF)	0.0167	
	s.d	0.0488					
	DW	1.96	DW	1.96			
	L_v	0.239	LR:$\chi^2(1)$	1.695	Sp:(17)	9.824	
	L_f	6.114*		[0.1292]		[0.9108]	
	AR(6)	1.14 [0.343]			DM:(21)	1767.8	
	ARCH(6)	1.21 [0.310]				[0.000]	
	White-H	0.81 [0.738]					
	J-B	0.37 [0.832]					

Notes: Coefficients on seasonal dummy variables not reported. ¶ denotes endogenous variable; HCSE-t denotes t-statistics computed using White's correction for heteroscedasticity; Instab denotes Hansen's (1992) for individual parameter stability against the null that the coefficient is stable over the full sample; L_V and L_J denote Hansen's tests for variance and joint parameter stability; and . $\hat{\lambda}^+$ and $\hat{\lambda}^-$ denote positive and negative deviations from equilibrium respectively. AR(6) and ARCH(6) denote LM tests against the null of autocorrelation and autoregressive conditional heteroscedasticity of order 6; J-B denotes the Jarque-Bera test against the null that the equation error is distributed normally; White-H denotes White's test against the null of homoscedastic errors; Sp denotes Davidson and Mackinnon's over-identifying test for the validity of instruments, and DM their test of the efficiency of instruments in the auxiliary regression.

The dynamic error correction model performs well, reducing the unconditional standard error of the dependent variable from around 5 per cent per month to less than 1.4 percent per month. The standard battery of diagnostic tests indicate that the model is free of serious misspecification error. Hansen's tests of stability suggest that over the full sample the model offers a constant-parameter representation of the demand for money, although the joint-parameter test of stability, L_J, is marginally rejected, reflecting in the main instability in one of the seasonal effects.[22]

From an economic perspective, the error-correction model accords with general theoretical priors. The error-correction coefficient itself is negative and significant, suggesting a mean lag adjustment to a shock of around 7 months, slightly slower than corresponding feedback vectors reported in Tables 7 and 8. The significance of the error-correction coefficient supports the difference-stationary assumptions made above: if the variables had been truly trend-stationary as opposed to being difference-stationary then the error-correction coefficient would be low and insignificant (since the whole of the trending component would be "differenced away" in the dynamic model). The second point to notice is that while the short-run income and wealth elasticities are of a similar order of magnitude to their long-run values, the short-run interest elasticity is significantly lower than its long-run value (but remains strongly significant). One possible explanation is that the interest rate effect is conditional on the presence of $\sigma_{\pi t}$, the measure of inflation volatility. Although the collinearity between these variables is low, it is positive, and eliminating $\sigma_{\pi t}$ raises the interest effect slightly, albeit at the cost of lower statistical precision of the model. The significance of $\sigma_{\pi t}$ indicates that the private sector economizes on money holdings in the presence of increased inflation volatility. Given the recent history in Chile, the model suggests that money demand has increased (i.e. velocity has declined) both in response to the falling rate of inflation and also to the increased stability in inflation (i.e. the fall in the variance) that has accompanied the decline in inflation.

The second column in Table 9 allows for asymmetric responses to positive and negative disequilibria. Although, as indicated by the test statistic, I cannot reject the restriction that the feedback effects are equal it would appear that the private sector responds slightly more rapidly to negative shocks (i.e. when agents are holding lower than desired balances) than to negative ones.

Finally, as noted above, it is not appropriate to treat as weakly exogenous the contemporaneous values of the scale variables (Δy_t and Δw_t) and the interest rate (Δi_t). I therefore re-estimate the error-correction model using an IV estimator which is reported in the final column of Table 9. Lagged

[22]These results are confirmed by recursive estimation methods, details of which are available on request.

values of the three variables plus lagged values of inflation were used as instruments. This implicitly assumes agents adopt a feedback or adaptive structure for forecasting expected nominal interest rates and income. As indicated by the over-identifying tests, this characterization appear to be both valid and efficient. In the case of the interest rate and wealth, the instrumenting process does not substantially alter the estimated coefficients (or their significance): the short-run income elasticity there is a marked fall in the significance of the income variable As important, though, is that overall the IV estimation displays the same statistical and forecast properties as the OLS model which underscores the validity of the particular "partial-system" strategy adopted in this paper.

3.5 Comparative performance

Finally, to nail down the argument about specification I undertake a simple within-sample encompassing of the model presented in Table 9 with a similar dynamic error-correction model without wealth effects as presented in Tables 2 - 5 (allowing for dynamic effects). To do so I report a suite of encompassing test statistics reported in Table 10. The first three rows of the table record the results of symmetric encompassing tests of the model with wealth (Model 1) versus the model without wealth (Model 2) where the left hand column is a test of the null than Model 1 encompasses Model 2 and the right hand column that Model 2 encompasses Model 1. The final row (the Joint test) is an F-test against the null that each model encompasses the joint nesting model of the two specifications.

Table 10: Encompassing Performance

Model 1 $\in$ Model 2	Form	Test	Form	Model 2 $\in$ Model 1
-1.196	N(0,1)	Cox	N(0,1)	-9.522**
1.073	N(0,1)	Ericsson IV	N(0,1)	7.418**
2.858	$\chi^2(5)$	Sargan	$\chi^2(6)$	28.359**
0.562	F(5,122)	Joint	F(6,122)	5.787**
[0.7291]				[0.0000]

In this case the results are unambiguous. For the model estimated over the sample to 1998(9), the specification including wealth clearly dominates the baseline model, and decisively encompasses the nested model.

Table 11 concludes this section by reporting the within-sample forecast performance of the model. The table reports four statistics: the Forecast χ^2 is a direct test against the null that the forecast errors are jointly zero and the Forecast Chow test is an F-test measure of parameter stability between the estimation and forecast period. The probability level is reported beside each. The innovation t test indicates whether there is a systematic bias in the forecast errors, and the MSFE is the standard mean-square forecast error. In all cases I report *ex ante* dynamic forecasts which are conditional on the *actual* value of the vector of exogenous and pre-determined variables and instruments, (and hence the estimated values of the endogenous variables) over the forecast horizon.

Table 11. Within-Sample Forecast Performance

Estimation Sample	86(7)-94(9)	86(7)-96(9)	86(7)-97(9)
Forecast Horizon	94(10)-98(9)	96(10)-98(9)	97(10)-98(9)
Forecast periods	48	24	12
Forecast χ^2	79.855 [0.0026]*	31.418 [0.1412]	14.022 [0.2993]
Forecast Chow Test	1.189 [0.2466]	1.125 [0.3313]	1.019 [0.4359]
Innovation -t	1.756	0.162	-0.598
MSFE (percent)	1.713	1.555	1.485

Although there is some evidence of significant forecast errors over the four-year horizon, the model exhibits a reasonable degree of stability (note that the 48-forecast period represents a forecast over almost 30 percent of the usable sample). It may be noted, however, that the direction of the forecast errors as measured by the t-statistics suggest that forecasts have moved from a situation in which the model *under*-predicts actual real money demand on average to one where it *over*-predicts real money on average towards the end of the period (i.e. the forecast error defined as $m-\hat{m}$ is negative). As shall be seen below this tendency increases with out-of-sample forecast performance.

3.6 Interim summary

The error correction model reported above appears to be relatively well specified, in terms of both its long-run and dynamic characteristics, and exhibits satisfactory within-sample forecast stability. The restrictions imposed on the model suggest a relatively simple theory of the aggregate transactions demand for money where the demand for money is determined by income, wealth and the nominal domestic interest rate (equivalently by its components, the real policy rate and domestic inflation). In the long run, the income and wealth elasticities are around 0.5 and 0.3 respectively, while a

5 percent increase in interest rates (per month) reduces real money demand in the long run by around one percent. In the short-run, agents respond rapidly to monetary disequilibrium and also to the volatility of recent inflation: in recent years this has worked to the favour of Chile with real money demand rising not only from lower inflation and hence lower nominal interest rates, but also from the lower variance in inflation that has accompanied this decline in inflation.

Although the current specification is robust and relatively simple, a key test must be its ability to forecast out-of-sample. In the final section I return to this issue.

4 Out-of-sample predictive power

Circumstances in Chile changes dramatically in the second half of 1998 when the economy moved into a short, sharp recession which saw output fall by 3 percent (considerably more relative to trend) within a single quarter. In normal circumstances this might be considered as a regular event, but in Chile this was the first downturn in output since 1985 and since the creation of an independent Central Bank. A well specified money demand function should be able to predict behaviour accurately through this period. Unfortunately, however, the preliminary evidence on our preferred model estimated up to 1998 is not encouraging. Using the error correction model presented in Table 12 as the basis for out-of-sample forecasting and employing the battery of forecasting tests from Table 11 it is clear that not only do the statistics imply a significant rejection of parameter stability but the forecast errors are systematically negative (i.e. the model systematically over-predicts actual money demand): the model chronically fails to correctly predict real money balances (Table 12, column 1). It is certainly true that the performance of Equation (6), the Central Bank partial adjustment specification, shown in column 2, is even worse, but neither can claim much validity as a specification.[23]

There are two potential reasons why this model may perform so poorly. One possibility is that the model may have been "over-fitted" and hence misspecified: in other words it is so tightly calibrated to the sample data that it is unable to explain out of sample. Alternatively, the poor forecast performance reflects a structural break, an obvious candidate being the negative shock to money demand following the stabilization crises in the region in 1998(reflected as an increase in velocity of circulation in Figure 2 above).

[23]The statistics in column 2 of Table 12 correspond to Figure 3 above.

Table 12. Out–of-Sample Forecast Performance

Model	Table 9	Eq(6)	Table 9 (excl ECM)
Estimation Sample	86(7)-98(9)	86(7)-96(9)	86(7)-98(9)
Forecast Horizon	98(10)-2000(6)	98(10)-2000(6)	98(10)-2000(6)
Forecast periods	21	21	21
Forecast χ^2	74.241 [0.0000]**	98.184 [0.0000]**	18.993 [0.5856]
Forecast Chow Test	2.584 [0.000]*	2.368 [0.0008]**	0.794 [0.722]
Innovation -t	-1.994	-8.481	-0.192
MSFE (x 100)	2.633	3.246	1.935

Clements and Hendry (1998) have suggested, one-off structural breaks or shocks of this type will lead to systematic forecast failure in error-correction models if the structural break alters the long-run cointegrating vector. Since the error correction model embodies a feedback effect, then following such a shift the error-correction model will exhibit persistent short-term forecast errors as it tries to "error-correct" towards the old, but inappropriate, long-run equilibrium. This form of equilibrium-correction error will not, however, manifest itself in a more conventionally defined dynamic models (such as a VAR) where there is no link between the common-trends and the cointegration components of the model. As can be seen from the third column of Table 17 simply excluding the error correction term from the model, the out-of-sample forecast performance of the model significantly improves and eliminates any bias in the forecast errors. This suggests rather strongly that the reason for the forecast error in our preferred specification would appear to reside in a shift in the long-run equilibrium, possibly reflecting the jump in the velocity of circulation in late 1998.

Re-estimating the long-run relationship indicates that there is indeed evidence of parameter instability over the forecast period consistent with the collapse of the equilibrium demand for money function. However, by the simple expedient of an intercept-correction (i.e. a dummy variable which interacts with the constant in the cointegrating vector) to pick-up the shift in velocity post-1998) I recover the following long-run cointegration results.

Table 13 Eigenvalues, Maximal Eigenvalue and Trace Statistics

		Maximal Eigenvalue		Trace Statistic	
r	λ_r	$-(T-nm)\ln(1-\lambda_r)$	5%c.v.	$-(T-nm)\sum\ln(1-\lambda_r)$	5%c.v
0	0.5229	106.60**	28.1	181.20**	53.10
1	0.1981	31.79**	22.0	56.88**	34.90
2	0.0806	14.12	15.7	19.78	20.0
3	0.0331	5.66	9.2	5.66	9.2

Statistics reported with Reimers (1992) small sample correction. Critical Values from Osterwald-Lenum (1992).

β' Eigenvectors [Normalized on Diagonal]

	$m-p$	y	w	i^p	c	c'
(1)	1.000	-0.350	-0.397	0.569	0.600	0.195
(2)	-0.737	1.000	-0.131	0.093	0.327	-0.036

α Loadings (Adjustment/Feedback) Vectors

	(1)	(2)
$\Delta(m-p)$	-0.151	0.245
Δy	-0.018	-0.259
Δw	-0.004	0.114
Δi^p	0.283	-2.731

The comparison with Table 9 is instructive: with the addition of the one-off intercept correction (c') I recover almost exactly the two long-run relationships identified for the earlier sample.[24] Hence I have isolated the cause of the predictive-failure to an un-modelled increase in the quasi velocity of circulation around late 1998. Using this revised cointegrating model I finally re-estimate the dynamic model over the full sample (Table 14).

[24] Full details of the re-estimation of the model can be obtained on request from the author.

Table 14. Dynamic Error Correction Money Demand Model
Dependent Variable $\Delta(m-p)_t$. Sample 1986(7) - 2000(6)

Model	[1]		[2]		
	OLS		IV		
Variable	Coefficient	HCSE-t	Coefficient	t-value	Instab
$Constant$	-0.101	6.82	-0.099	10.76	0.10
$\Delta(m-p)_{t-2}$	-0.264	4.44	-0.249	5.69	0.15
$\Delta(m-p)_{t-3}$	-0.169	3.93	-0.155	3.09	0.79*
Δy_t	0.362	1.91	0.342¶	1.35	0.16
Δw_t	0.341	5.13	0.252¶	2.59	0.74*
Δw_{t-2}	0.212	3.27	0.217	3.05	0.43
Δi_t^p	-0.056	10.96	-0.064¶	10.09	0.42
Δi_{t-3}^p	-0.017	3.53	-0.016	3.06	0.03
$\hat{\lambda}_{t-1}$	-0.109	10.71	-0.113	11.71	0.27
$\sigma_{\pi t}$	-0.393	3.02	-0.309	2.17	0.14
$DM0292$	0.140	17.16	0.138	14.35	0.14
$Dhol$	0.004	3.41	0.004	2.88	0.14
	R^2	0.930			
	s.e	0.0139	s.e.(SF)	0.014	
	s.d	0.0528	s.e (RF)	0.016	
	DW	1.85			
	L_v	0.227			
	L_f	5.607*	Sp:(18)	19.024	
	AR(6)	1.74 [0.105]		[0.3903]	
	ARCH(6)	0.88 [0.524]			
	White-H	0.83 [0.716]	DM:(21)	1815.1	
	J-B	0.57 [0.751]		[0.000]	

Notes: See Table 9.

Table 15. Within-Sample Forecast Performance (Full-Sample Model)

Estimation Sample	86(7)-96(6)	86(7)-98(6)	86(7)-99(6)
Forecast Horizon	96(7)-2000(6)	98(7)-2000(6)	99(7)-2000(6)
Forecast periods	48	24	12
Forecast Chow Test	0.9005 [0.6509]	0.7627 [0.7755]	0.7147 [0.7086]
Innovation -t	-0.221	-1.103	1.103
MSFE (x 100)	1.491	1.342	1.163

This dynamic model indicates how closely the basic model has been recovered once the levels-correction is embodied. Not only are the coefficients of the dynamic model statistically indistinguishable from those reported in columns [1] and [3] of Table 9 but the forecast stability of this model (within-sample this time) is now restored (see Table 15).This robust within-sample forecast evidence is fully consistent with the recursive estimation of the model, the plots of which are shown below.

Figure 5

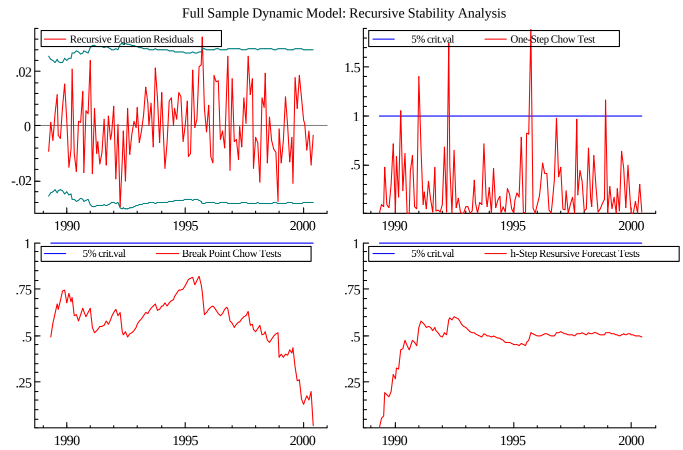
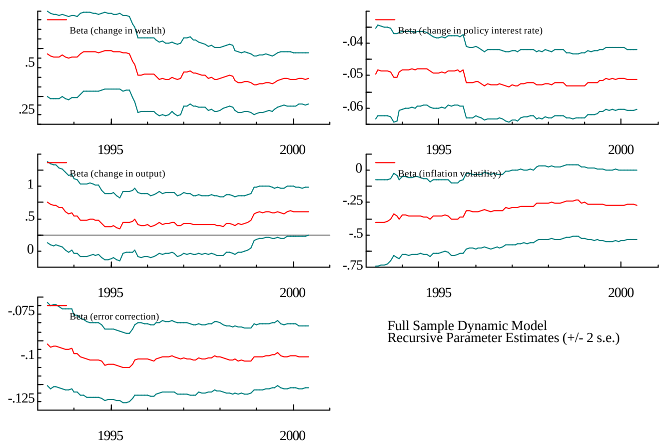


Figure 6



5 Conclusions

The econometric evidence presented in this paper suggests that although macroeconomic time-series in Chile exhibit a strong degree of trend-stationarity it is possible to recover a relatively simple but robust single-equation model for the transactions demand for money which exhibits good out-of-sample properties. The error-correction specification exhibits significantly better within-sample forecast accuracy than the partial adjustment model in use by the Central Bank, partly due to the superiority of this class of model in the presence of cointegration, and partly because of the use of a measure of inflation volatility as a regressor. Moreover once account had been taken for an apparent permanent shift if the velocity of circulation in 1998, the out-of-sample forecast performance was good and the systematic forecast errors present in the partial adjustment model for the period appear to have been eliminated.

Having said this, however, there are two areas of concern which mean that the model remains tentative and which point to future extensions. The first is that the out-of-sample forecast results were obtained only after imposing an 'intercept-correction' to the long-run money demand function. Given the limitations of the data sample it is not possible at this stage to determine whether the sharp rise in velocity from the final quarter of 1998 is a temporary or permanent feature of the data. As more data become available it will be necessary to re-investigate this event and reconsider the nature of the long-run demand function. The second related area of research concerns the appropriate measurement of wealth. The measure of wealth used in this paper is extremely crude and certainly does not capture wealth effects arising from either the accumulation of non-financial assets, such as housing, or from other financial assets such as pensions and life insurance. Efforts should therefore be directed towards the compilation of a comprehensive measure of private sector real wealth. Finally, and building on this, one final direction for research involves broadening the scope of the analysis to examine the demand for interest-bearing financial assets, since it here that the analysis of financial innovation and wealth effects are likely to be of much greater importance.

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6 Appendix I: Data and unit root tests

The data are derived from the Central Bank's Informe Economico y Financiero (various issues).

Appendix Table 1. Data and Sources

Variable	Description	Symbol	Source
M1A	Money stock M1A	$M1A$	IEF Table 12
LRM1A	Log real money stock M1A	$(m-p)$	$ln(\frac{M1A}{CPI89})$
M4	Money stock M4		IEF Table 12
FXD	Foreign currency deposits		Research Dept
NCRED	Credit to private sector		IMF IFS Tables line 32
RNW	Real net financial wealth	w	$\frac{M4+FXD-NCRED}{CPI89}$
LIMAE	Log indicator of monthly economic activity	y	IEF-Table 1
LIMAEX	LIMAE excluding agriculture and copper	y	
CPI89	Consumer price index	p	IEF Table 21
INFLM	CPI inflation	π	$\frac{p_t-p_{t-1}}{p_{t-1}}$
NER	Nominal exchange rate	e	IEF Table 23
DEPR	Depreciation of nominal exchange rate		IEF Table 23
TEP	Real policy rate (TEP)	i^r	IEF Table 19
PNBC	Nominal policy rate	i^p	IEF Table 19
RDSM	Nominal short-term deposit rate (30-90 days)	i^d	IEF Table 20

Appendix Table 2: Unit Root Tests: 1985(1) - 2000(6) T=186

Test	ADF[1]	ADF[1]	PP[1,2]	PP[1,2]	KPSS[1,3]	KPSS[1,3]	DF F[1,4,5]
Null: H_0	I(1)	I(1)	I(1)	I(1)	I(0)	I(0)	I(1)
Variable	with drift	with trend	with drift	with trend	with drift	with trend	sequence
LRM1A	-2.689	-0.169	-1.292	-3.907	3.693	0.143	I(0)+trend
LRNW	-2.792	-1.517	-4.899	-1.143	3.114	0.899	I(0)+trend
LIMAE	-1.343	-1.003	-1.020	-9.178	3.555	0.218	I(0)+trend
LIMAEX	-1.107	-1.963	-1.322	-11.226	3.577	0.332	I(0)+trend
CPI89	-0.948	0.896	-0.714	-1.286	3.705	0.962	I(1)+drift
INFLM	-2.157	-6.551	-7.336	-9.683	2.391	0.136	I(0)+cnst
NER	-0.644	-1.939	-1.731	-3.782	3.508	0.812	I(1)+drift
DEPR	-7.757	-7.923	-9.331	-9.710	0.773	0.137	I(0)+cnst
TEP	-3.406	-3.716	-2.387	-2.836	0.961	0.115	I(0)+cnst
PNBC	-2.437	-3.416	-5.739	-5.637	4.723	0.382	I(0)+cnst
crit.val	-2.88	-3.44	-2.88	-3.44	0.463	0.146	

Notes: [1] Augmented Dickey-Fuller (ADF) with lag-length selected by minimum Aikaike Information Criterion. Monthly dummy variables included in each test. [2] Phillips-Perron (PP) tests estimated with Newy-West correction. [3] Kwiatkowski, Phillips, Schmidt and Shin test against null of stationarity. [4] DF-t and DF-F tests estimated using Phillips-Perron non-parametric correction. [5] Inference based on Dolado et al (1990) reduction sequence.

7 Appendix II: Alternative specifications

The own return on money The dimension of the vector X is four, with an estimated cointegrating rank of three. The following restriction is tested

$$\begin{aligned} \beta_1^{*\prime} &= (1, -1, *, *, *) \\ \beta_2^{*\prime} &= (*, 1, 0, 0, 0) \\ \beta_1^{*\prime} &= (0, 0, 1, -1, *) \end{aligned}$$

generating restricted cointegrating vectors and associated likelihood ratio test on the restriction.

β' Eigenvectors [Normalized on Diagonal]

	$m-p$	y	i^p	i^d	c
(1)	1.000	-1.000	0.284	-0.171	-0.539
(2)	-0.900	1.000	0.000	0.000	0.000
(3)	0.000	0.000	1.000	-1.000	0.000

LR Test (r=3) $\chi^2(3) = 6.975$ [0.0727]

Inflation and the real interest rate Re-estimating the model with inflation and the real interest rate generates two cointegrating vectors

β' Eigenvectors [Normalized on Diagonal]

	$m-p$	y	π	r^p	c
(1)	1.000	-0.961	0.335	0.407	-0.031
(2)	-0.966	1.000	-0.095	0.021	0.008

Imposing the symmetry restriction $\beta_\pi = \beta_{rp}$, I obtain the following restricted eigenvectors and associated likelihood ratio test.

β' Eigenvectors [Normalized on Diagonal]

	$m-p$	y	π	r^p	$cnst$
(1)	1.000	-1.000	0.145	0.145	-0.779
(2)	-0.898	1.000	0.000	0.000	0.000

LR Test against restriction (r=2) is $\chi^2(3) = 6.126$ [0.1056]