QEH Working Paper No 39

Making bad decisions: Firm size and investment under uncertainty*

Alexander Cobham**

This paper presents a 'real options' model of investment under uncertainty, which incorporates the assumption of a financial market characterised by asymmetric information and which can explain the stylised facts of firm growth. The decision-making situation faced by small and medium-sized enterprises (SMEs) features much greater constraints on the ability to gather information in order to reduce uncertainty about their investment opportunities, compared with that faced by large companies (LCs). This necessarily causes relatively poor decision-making by SMEs, and explains their substantial death rates.

** Finance and Trade Policy Research Centre
Queen Elizabeth House
University of Oxford
21 St Giles, Oxford, OX1 3LA
alex.cobham@qeh.ox.ac.uk

First draft: November 1999. This version: April 2000.

* The author would like to thank Valpy FitzGerald, Chris Adam, Richard Mash, Tim Jenkinson, participants at a conference in Athens and especially Andrew Goodall for comments and assistance with the model. All remaining errors are my own.

COMMISSION OF THE EUROPEAN COMMUNITIES
Research Project

Investing in Southern Europe:
Technological and Financial Decisions in Manufacturing Firms in Southern Europe

Actual Cost Contract no. SOE1-CT96-1037 (DG XII - SOLS)

Making bad decisions: firm size and investment under uncertainty

This is a series of four papers which forms the financing section of the above EC project, addressing the problems facing technology-based manufacturing SMEs in Southern Europe with particular reference to the changes initiated by economic and monetary union in Europe (EMU). The first two papers surveyed the literature on technology investments and SME finance, and produced first, a lifecycle model of appropriate financing for different investment stages, and second, the theory that types of financing can affect not only the level but also the nature of technological investment. This is the third paper of the series, and presents a model of firms' investment decisions. In particular, conclusions are drawn for the impact of policy changes concerning firms' business conditions in terms of the likely effect on economic growth. The final paper will assess the likely impact of EMU on the Southern European technology-based SMEs through changes in capital markets, using the approach to financing and technology which has been developed throughout the project.

Contents

Making bad decisions: firm size and investment under uncertainty

The stylised facts of firm size are few. Firstly, small and medium-sized enterprises (SMEs) exhibit a much higher incidence of death than large companies (LCs). Table 1 shows that a figure of 50% survival after five years is fairly robust across European countries and years of birth. The obvious explanation is that SMEs are unable to diversify their investments as widely, and this failure to spread risk causes higher volatility. In effect, the divisions of LCs are equivalent to smaller firms, and the profits of these divisions are not perfectly correlated. Furthermore, LCs are better able to absorb the effect of a loss in one section of the firm through their stronger financial position. The death rates of SMEs result then from a higher volatility of return compared to the firm's overall financial resources.

Table 1: Survival of newly registered or newly established enterprises

Country	Start-up year	Surviving after 5 years (%)
Denmark	1990	58
France	1987	48
Ireland	1985	57
Italy	1987	54
Luxembourg	1980	45
Portugal	1986/87	47
UK	1980	47
Norway	1980	53
Sweden	1988	59

Source: Bank of England, 1999.

The second key stylised fact of firm size concerns growth rates. Gibrat's Law – that firms' growth rates are independent of firm size – has been shown to be empirically unsubstantiated, e.g. Evans (1987), Hall (1987) and Dunne & Hughes (1994). These results are particularly strong among smaller sizes of firm, where the growth rate appears to be highest.

Few studies have explicitly attempted to explain this phenomenon. Cabral (1995) models the investment choices of firms as depending on sunken costs: since small entrants are more likely to exit (and lose the sunk costs) than large entrants, they invest more gradually and hence experience higher growth rates. Cabral & Mata (1996) extend this analysis to include the impact of learning-by-doing (improving the efficiency and thus survival rate of longer-lived firms) and financing constraints (which bind more tightly on smaller, younger firms).

In models which fit the stylised empirical facts, the size distribution of firms has generally been derived either from an underlying distribution of managerial abilities (following the static model of Lucas, 1978) or from a process of firms learning about their abilities from noisy production signals (Jovanovic, 1982). Cooley & Quadrini (1999) present a model which generates some of the stylised facts of firms' investment and production behaviour through heterogeneity of firms in terms of the amount of equity held. However, none of the work done has managed to combine the distributional characteristics of firms' size and investment, with the key stylised fact of small firms – their high death rates.

Analysis of firms' finances has provided perhaps the most convincing explanations of differences in firms' performance by size and age. Fazzari et al's seminal (1988) paper on the determinants of firms' investments highlighted the importance of current period cashflow, and spawned a body of literature using this as evidence of the existence of financial constraints. These constraints have been argued to be the driving force behind the different investment and growth patterns of SMEs, as opposed to larger companies.

The debate about the interpretation of these results has been moved on most usefully by Kadapakkam et al. (1998). They find that smaller firms' investment level is less sensitive to cashflow than that of larger firms. However, they do not interpret this as implying that smaller firms can access capital markets most easily. Instead, since larger firms can be more flexible in the timing of their investments and can wait for the availability of internal funds, the cashflow-sensitivity of their investments will be higher and those of smaller firms relatively less so. The model of Pratap & Rendon (1998) leads to poorer firms' borrowing being an increasing function of their financial position, while this position is reversed for wealthier firms. This finding supports the view of Kadapakkam et al that smaller firms are less able to be flexible about their timing, since they borrow more as soon as they are able to.

In this paper, I aim to model the abilities of firms of different sizes and ages to be flexible in their investment timing, and the resultant quality of the investments undertaken. The model is based on the 'real options' theory of investment under uncertainty, but crucially incorporates the assumption of a financial market characterised by asymmetric information. The main innovation of the model is that it differentiates between firms on the basis of the time available to them for making investment decisions. In this way it models relatively constrained decision-making by small and medium-sized enterprises (SMEs) compared to large companies (LCs), as the result of asymmetric information problems in financial markets and hence a lack of available time in which to analyse investments.

SMEs therefore exhibit higher volatility of investment returns. High death rates of SMEs can be explained in terms of the inability to wait before taking an investment decision, and the resultant higher uncertainty which characterises their decisions. It is not in fact the decision-making processes of SMEs which are 'bad,' but rather the level of constraints on their decisions which is high. The empirical observation of higher growth rates among smaller firms has interesting implications for the model, which are discussed.

Two previous papers within this project have laid the foundations for the model of SMEs' investments which will be presented here. Cobham (1999a) found that the availability of financing could affect not only the level but also the nature of SME technology investment. Cobham (1999b) built on this paper to create a life-cycle model of SMEs' financing and investment, where different types of financing are optimal for different types of investment at different stages of the cycle. The present paper assumes technology is embodied in an investment (although the effect of innovative strength is considered), and focuses instead on the impact of changes in financial markets on general investment levels. Table 2 outlines some findings from the previous papers, which will be used as a basis for discussion of the differences between sizes of firm throughout the paper.Table 2: Typical positions of small and large firms[1]

[1] NB. The values reflect the relative attributes of *typical* small and large firms. There does exist a category of highly innovative, high growth small firms whose access to financing is wider than suggested. However, for the vast majority of small firms, Table 2 is a reasonable representation.

	Small firm	Large firm
Financing		
Credit history	Short, weak	Long, strong
Successful rounds	None/one	Many
Credit risk rating	Low	High
Accessible finance	Bank debt	Bank debt, public equity, corporate bond market
Investment assessment		
Information access	Low	High
Managerial expertise	Low	High

The paper is organised as follows. Section I explains the real options approach to investment under uncertainty, and sets out both a basic model and a variation using a different definition of uncertainty. Section II then focuses on the differing extent to which the investment decisions of different sizes of firms meet the conditions necessary for application of the real options approach, and delineates the algebra of the distinction. The model is then applied in section III, and the key results outlined. Section IV then considers the model's policy implications and derives recommendations for the particular case of Southern Europe's technology-based smaller firms.

I. The real options approach to investment

Investment under uncertainty – a summary

Dixit & Pindyck (1994) characterise firms' opportunities to invest as analogous to a call option. That is, the opportunity, like the financial market instrument, grants the right, but not the obligation, to make a given purchase at a given price within a given time period. The option is described as being 'in the money' when the price per share under the option is less than the current market value - i.e. if the owner exercised the option and sold the shares immediately back to the market, she would be 'in the money.' The focus is therefore on when the value of the option will be maximised, or most deeply 'in the money.'

The opportunity to invest is not explicitly purchased as the financial instrument is, but the costs of identifying the opportunity are analogous to the option price. The firm must have made some commitment – of time and effort at least – to have reached a position where it can consider making the investment. There are three key criterion for an investment decision to be suitable to the options approach: the investment must be at least partly *irreversible*, its future returns must be subject to some *uncertainty*, and the investor must have the ability to *wait*. The extent to which these conditions are met for different sizes of firm will be discussed in section II.

The potential value of waiting and gathering more information about, e.g., expected future demand and supply conditions in a market, or the rate of change of relevant technologies, is in the form of reduced uncertainty of the investment when it is actually made, and is equivalent to an opportunity cost of investing. There is a trade-off between this value, which is foregone when the investment is made, and the value of the investment itself.

The option is only exercised when it is 'deep in the money' - the net present value (NPV) of the investment is sufficiently large that the value of waiting for more information is counterbalanced. This calculation is in stark contrast to 'naïve' use of the discounted cashflow methods (such as the NPV) which analyse the value of investment decisions in terms of estimates of future cost and revenue flows, *as if with certainty*; or with no possibility of delay.

The key finding of Dixit and Pindyck's approach is that, unlike static discounted cashflow analysis, for sufficiently high levels of uncertainty there will exist a real, positive value to waiting and gathering further information. The value of *not* exercising the option is positive when uncertainty over the outcome will be diminished by choosing not to exercise the option *at that precise moment*, but delaying the investment to accumulate information. That is, the internal rate of return (*IRR*) of the investment must exceed not just the nominal interest rate (*i*) but its sum with the option value (*o*): *IRR=i+o*. Thus an option to invest, which by simple discounted cashflow methods would seem to be 'in the money' (i.e. *IRR=i*), should not always be exercised.

A basic real options model

The basic case to consider is that raised by McDonald & Siegel (1986), who studied the optimal timing of paying a sunk cost *I* for a project of value *V*, where *V* follows a geometric Brownian motion. Specifically:

$$dV = \quad Vdt + s\,Vdz \qquad \ldots(1),$$

where *dz* is the increment of a Wiener process, and a and s are drift and variance parameters, respectively. Equation 1 implies that the current value of the project is known, but future values are lognormally distributed with a variance that grows linearly over time [var(V) = $s^2 t$]. The function trends upwards over time, according to the first term on the RHS, but the firm's uncertainty over market conditions (s) increases the deviation of the path from its drift path (determined by the parameter a).

The project has payoffs equal to the value in the chosen period net of the investment cost. The value of the investment opportunity, denoted by *F(V)*, will be maximised in terms of the payoff at some unknown time *T* by:

$$F(V) = \max E\left[\left(V_T - I\right)e^{-rT}\right] \qquad \ldots(2),$$

subject to equation 1 for *V* and where r is the discount rate. Because *V* is a stochastic process, it is not possible to determine the time *T*; instead the investment rule will generate some critical value *V** such that investment is optimal for $V \geq V^*$. A key feature of the real options approach is that *V** can be two or three times as large as *I* for reasonable parameter values, as Dixit & Pindyck (1994) show, while for a net present value decision rule, *V*=I.*

It is assumed that $a < r$ (the firm's discount rate exceeds the drift parameter), since otherwise the integral in equation 1 could be made indefinitely larger by choosing a larger *T*, and hence waiting longer would always be preferred so no optimum would arise. By defining $d = r - a$, this assumption becomes $d > 0$.

The dynamic programming solution is given by equations (3)-(6):

$$F(V) = AV^{b_1} \quad \ldots(3)$$

$$V^* = \frac{b_1}{b_1 - 1}.I \quad \ldots(4)$$

$$A = (V^* - I)/(V^*)^{b_1} = (b_1 - 1)^{b_1 - 1}\Big/\left[(b_1)^{b_1} I^{b_1 - 1}\right] \quad \ldots(5),$$

where $b_1 = \frac{1}{2} - (r - d)/s^2 + \sqrt{\left[(r-d)/s^2 - \frac{1}{2}\right]^2 + 2r/s^2} \quad \ldots(6).$

Equations (3)-(6) define the critical value of *V* which triggers the decision to make the investment. As this value follows its path (equation 1) over time, it may eventually reach this critical value and the investment will be made. Although there exist considerably more complex variations of this model, the basic intuition remains the same.

Using Dixit & Pindyck's criteria for suitable investment situations, it is possible to argue that not all investment decisions are equally well-suited to application of the model. As will be made clear below, there are good reasons for believing that the extent to which this maximisation process can be followed will vary systematically with firm size. First, however, I outline a variant of the McDonald & Siegel model.

A variation: uncertainty as a function of time waited

This section outlines a variation to the McDonald & Siegel (1986) model. Specifically, it is useful to consider uncertainty as a function of the time available to the firm in which to gather information. Equation 1 represented *V* as a geometric Brownian motion with drift, but as has been said the literature on real options extends to rather more complicated functions. Equation 1' represents *V* as a generalised Brownian motion:

$$dV = a(V,t)Vdt + b(V,t)Vdz \quad \ldots(1').$$

Setting $a(V,t) = aV$ allows the drift parameter to operate as in McDonald & Siegel's model, but a time-dependency may be incorporated into the underlying level of uncertainty as follows:

$$b(V,t) = s(W)V \quad \ldots(7),$$

where *W* denotes the time waited before investment. Equations 1' and 7 state that, while the variance of *V* is again growing linearly with the time horizon, the value of the underlying constant is a function of the time in which the firm was able to gather information *prior* to investing.

Uncertainty, $s(W)$, is reduced by the (time-consuming) collection of information, and so will fall as waiting time (*W*) increases, in a standard exponential decay function. The strength of this relationship will be denoted by b_s, which itself depends on two factors. Firstly, managerial expertise (*m*) dictates the efficiency with which the firm uses information gained during additional waiting time in order to reduce uncertainty. Maximum expertise implies a value for m of unity, minimum expertise a value of zero.

Secondly, the strength of the relationship between uncertainty and waiting time depends on the speed of flows of information about market conditions to the firm (g). Again, unity represents maximum speed and zero the minimum.[2] b_s will determine the rate of the standard (exponential) informational decay process with which marginal uncertainty reduction from additional time waited falls, c is a scaling factor:

$$s(W) = ce^{-b_s W}; \quad b_s = gm; \quad 0 < g, m < 1 \qquad \ldots(8)$$

Equation 8 shows the relationship between time waited and uncertainty, for a given firm considering a given decision. Note that both the speed of information flow and especially the level of managerial expertise are constants – for a given project, they do not change over time. Firms gain expertise through learning-by-doing, from one project to the next, but *not* within a given project process.

II. Firm size and appropriateness of the real options approach

Firms' ability to wait

This section will detail how the approach adopted differs from the simple real options model outlined above. As has been stated, the real options approach requires investments to exhibit three main characteristics: at least partial irreversibility (by way of a sunk cost), uncertainty about future values, and the ability of the investor to wait. It is a key characteristic of SMEs that their ability to wait is significantly less than that of LCs. Greater constraints on SMEs, through relatively less obtainable financing reduce their potential waiting time. This reduces their ability to eliminate uncertainty over investment prospects, and thus forces them into the position of making 'bad' decisions – 'bad,' that is, in so far as they are relatively more constrained.

This results in greater uncertainty of investment returns, and therefore explains the observed higher volatility of smaller firms' investments, and their higher death rates. The 'insufficient diversification' argument for SME death rates made earlier is based on the variance of returns which SMEs are relatively less able to diversify. The approach here shows instead how smaller firms face higher *absolute* volatility of any given investment.

On the face of it, the real options approach is well suited to the case of SME's technological innovation investments (with which we are particularly concerned), where uncertainty over future revenues resulting from the investment will be great and the SME lacks information about developing market conditions. However, firms will be differentiated by one of the key criteria for application of the real options approach - the extent to which they are actually able and willing to *wait*; and this difference makes the real options approach less important than the net present value method in many cases. The ability to wait is tempered by the risk firms face of losing their investment option through a financial market channel. This risk encapsulates the differences between SMEs and LCs under the Dixit-Pindyck approach.

---

[2] NB. This information flow speed is concerned with the flow of (primarily public) information about industry- and economy-wide conditions for the firm's market. Private information enters the firm's maximisation differently, as will be seen below.

A firm cannot be considered to own a call option on an investment until it actually has a guarantee of funding which will allow it to exercise the option when it is sufficiently in the money. The value of a call option to buy shares in, say, Intel at $1 each, is in effect limited by the number of shares the owner can afford. In the case of a small firm with limited assets, a short track record of success and little credit history, financing cannot be expected to be available continuously.

That is, while a bank may be willing to lend today, the SME cannot rely on the offer staying open for any great length of time. Banks are not given to making indefinite offers of finance, and if they are assumed to ration credit to SMEs (see Stiglitz & Weiss, 1981), then a SME granted access to finance in any given time period may randomly fall into the 'rationed' category when next it requests finance. The risk that an offer of financing will be withdrawn increases the costs of waiting; indeed under certain conditions the cost will reach the point where waiting time is so limited that the use of a static (net present value) approach is appropriate. A large company, on the other hand, is likely to have a track record of success and a long history of financing. These factors send strong signals to potential financiers and make the prospects for the future availability of financing much more stable, and thus reduce the risks, and hence cost, of waiting.

The same arguments can be applied to equity financiers as to banks. Cho (1986) and de Meza & Webb (1987) show that rationing disappears in the original Stiglitz-Weiss model if entrepreneurs are able to access an equity market. However, Myers & Majluf (1984) and Greenwald, Stiglitz & Weiss (1984) present different models in which equity rationing is a feature. More recently, Hellman & Stiglitz (1995) have shown that credit and equity market rationing are compatible.[3]

If banks and equity financiers are considered rational, then arguably waiting and gathering more information on the investment should not prejudice firms' chances of receiving funding. Indeed, if potential financiers act in awareness of the Dixit-Pindyck approach, they should realise that the firms are improving their chances for success. Even if financiers did adjust their supply schedules to take this into account, however, the existence of rationing on the basis of incomplete and asymmetric information would still generate the possibility for losing financing. For smaller firms in particular, where a decision to provide finance may have been long awaited, there will exist an incentive to take what is offered and not risk a reversal of the decision at a later date.

Whereas LCs with long credit histories and track records of success may consider a window of finance which extends into semi-perpetuity, for SMEs this window may open only at certain times, with certain financiers, and thus the potential costs of waiting are higher.

[3] NB. This result relies on entrepreneurs having informational advantages about both the expected return and the risk of the project.

Financial risk

Financial risk (r_f) represents the risk of losing the investment opportunity due to financial market imperfections and problems of informational asymmetry. r_f depends on the interaction between two factors – the information contained in the firm's financial history and the informational efficiency of the financial market in which the firm operates. The extent of credit rationing in a market will depend on these two factors, since they represent the firm's ability to cheaply and effectively transmit credible information to financial markets, and the financial market's ability to cheaply and effectively use information about a firm to assess its creditworthiness.

A financial market's informational efficiency reflects how well information is managed by the market, and thus how flows of information from the firm are used. The firm's financial transparency encompasses its financial history and its level of accounting. The former signals to financial markets the firm having previously been a good risk. It indicates the extent to which the market's informational inefficiency will reduce the firm's prospects of obtaining financing, since this depends on the informational asymmetries relevant for the firm and a long and successful history with repeated rounds of financing will effectively reduce information problems.

Financial history reflects the length of the firm's successful trading (its age), asset value (as collateral or indicator of depth of resources) and previous rounds of financing – these may each send strong signals about the firm's creditworthiness. The accounting standard of the firm affects the credibility of the financial data which the firm provides to the financial market.

Let d_{fm} and d_{ft} represent the level of 'perfection' of the financial market and the firm's financial transparency respectively, defined between zero and unity where zero represents complete 'imperfection' and unity perfection. As either of these factors tends to zero (i.e. the firm's financial position becomes more precarious), the risk of losing the investment option due to these imperfections is increasing. The strength of the firm's financial position is a product of these imperfections, and is denoted by:

$$b_f = d_{fm}.d_{ft} \quad \text{where} \quad 0 < d_{fm}, \quad d_{ft} < 1 \qquad \ldots(9)$$

The more imperfect the financial market's informational efficiency and the firm's financial transparency, the closer is b_f to zero – the greater the effective imperfection and hence the associated risk. To complete the picture, it is useful to examine the growth of risk over time. Consider a small, young firm with little track record or credit history. It perceives a risk of losing the option to invest which escalates rapidly from some point where financing has been agreed throughout the process of gathering information. The slope of this risk function will be declining in the later stages as the risk reaches unity. After being initially 'rationed in,' the firm's risk of not having financing rises sharply as they must choose to wait and thereby expose themselves to going through the lottery of rationing again.

After the first rise, risk continues to increases but only slowly – the incremental effects of moving further from the point of being 'rationed in' are relatively small. Even a large firm with an established credit history and successful track record will face an increasing risk of losing the option through financing, as long as there is some uncertainty over the availability of future debt, equity or retained profits for investment. Finally, r_f is bounded at unity, so the following form is specified:

$$r_f = 1 - \left[\ln\left(e + b_f^{-1} W\right)\right]^{-1} \qquad \ldots(10).$$

III. Operation of the model and key results

Financial risk as a time constraint on the real options approach

Figure 1 illustrates how different values of b_f drive the growth of risk over time waited. It shows the path of risk where b_f = 0.25, b_f = 0.5 and b_f = 0.75. The dotted horizontal line provides an example of a risk threshold, where firms are assumed to be uniform in their attitudes to risk.[4] Given such a threshold – a refusal to wait when the risk of losing the option by doing so is fixed (in this case at 50%) - the division of firms according to their ability to wait is immediately clear.

A firm with b_f of 0.25 – i.e. a small, young firm in a less efficient financial market, e.g. Greece – will exhibit an ability to wait which is approximately one third that of a firm with b_f of 0.75 – i.e. a large, well-established firm in a stronger financial market, e.g. Germany.

This risk in equation 13 forms a constraint on firms' ability to wait. The way this is expressed here is to assume that firms have some threshold level of risk, a critical level at which they will not tolerate further waiting. The risk of losing financing rises over time, and one of two scenarios will emerge. Either the value of the investment reaches V^* and the firm makes its investment on a real options approach basis, or at some point, the level of risk becomes too much for the firm to continue waiting, and at this point they take a net present value investment decision.

This generates a solution of the following form: each firm's maximum waiting time, W^*, is determined as the time at which their risk function (equation 10) reaches the threshold risk level, r_f^* - which is where:

$$r_f = r_f^* = 1 - \left[\ln\left(e + b_f^{-1} W^*\right)\right]^{-1} \qquad \ldots(11).$$

The firm then faces a two part decision. If, within the resultant maximum waiting time, the value of their investment exceeds V^* then the investment is made in accordance with the real options approach – the solution is again given by equations 3-6. Note that the variance parameter has been respecified as a function of time waited, however, and substituting equation 8 into equation 6 gives:

---

[4] Risk-neutrality is the general assumption of the investment under uncertainty literature, and implicitly a uniform attitude. Such assumptions may be less appropriate here, so firms' attitudes to risk will be considered explicitly below.

$$b_1 = \frac{1}{2} - (r-d)\Big/\left(ce^{-b_s W}\right)^2 + \sqrt{\left[(r-d)\Big/\left(ce^{-b_s W}\right)^2 - \frac{1}{2}\right]^2 + 2r\Big/\left(ce^{-b_s W}\right)^2} \quad \ldots(6')$$

If the value of the investment does not reach V^* within the time the firm has available to wait, however, they are forced to make an investment decision at W^*. In this case, without the ability to wait, the decision becomes once more suitable to the application of NPV methods. The firm's decision concerns the expected value of V and no longer takes account of the possible value of waiting since waiting is no longer feasible. However, the uncertainty associated with the future values of V – the resulting volatility of investment returns - is set by the firm's characteristics and the waiting time W^*.

Investment scenarios for small and large firms

Figure 2 depicts the possible scenarios for two firms which face the same competitive risk (i.e. have the same drift parameters) but different financial risks – i.e. their (identical) threshold level is reached at different stages. The maximum waiting times for each firm are simply those from the example given in Figure 1.

Scenario 1: Both firms make investments by the 'real options' approach
For a low value of V^*, i.e. below the smaller firm's threshold waiting time V_s, both firms make the investment on a real options basis. If the function V had no trend variable, the chances of finding this situation might be considered proportional to the ratio of the small firm's threshold time to the large firm's; however, the function grows over time, so even this ratio would overestimate the chances of such a situation. In any event, when this case occurs, the firms make the same investment, but crucially, with different levels of uncertainty about the returns.

The relationship between uncertainty and time waited before investing contains a firm-specific variable, and this variable will exhibit a positive relationship with firm size (according to the relative abilities to assess investment set out in Table 2 in the introduction). When both firms invest according to the real options approach – i.e. $V^* \leq V_s$ - the smaller firm still faces higher values of uncertainty, and hence higher volatility of returns.

Scenario 2: small firm invests by NPV, large firm by 'real options'
The next scenario is that $V_L > V^* > V_s$, where the critical value occurs between the threshold waiting times of the two firms. In this case, the smaller firm has been forced to make an investment decision at its threshold waiting time, while the larger firm – by virtue of being able to wait longer – can make a more informed decision.

The smaller firm invests according to the NPV decision rule: this states that the firm will invest when the value of the investment exceeds the cost, I. Since V^* may be two or three times greater than I for "reasonable parameter values" (Dixit & Pindyck, 1994, p.136), there remains a high chance that the investment will be made even though V has not reached V^*. As is apparent from equations 3-6, the size of the wedge between the NPV decision rule ($V^*=I$) and the real options rule is increasing in uncertainty, s , and this will increase the likelihood of the investment being made.

Two important results can be inferred. First, smaller firms invest first. Although larger firms are more likely to make the investment – that is, they *will* make the investment while the small firms' decisions *may* be negative – the smaller firms are making the decision (to invest or not) in a substantially shorter time, perhaps half. Second, the small firm faces greater uncertainty about its investment returns.

Scenario 3: both firms invest by NPV

The second possibility when $V^* > V_s$, is that the critical value does not occur during the larger firm's waiting time threshold either. In this case, the larger firm makes a net present value decision at its own *W**. Although both firms are applying the same decision rule, the smaller firm is making its decision after a much shorter period of time and hence with greater uncertainty and a greater spread between the investment cost *I* and the real options critical value *V**. The smaller firm is therefore more likely to invest.

Key results

Over each scenario, then, the effect of the risk of losing financing for investment options is two-fold – smaller, younger firms will (i) make investment decisions more quickly, and (ii) face higher volatility in their investment returns, than larger, older companies. The latter effect is responsible for the key stylised fact of small firms, their high death rates: the greater volatility of the smaller firms' investments will result directly in higher death rates. This effect will be compounded by the standard explanation discussed in the introduction, the relatively shallow finances of SMEs (compared to LCs) relative to their investment exposure.

To explain the higher growth rates of smaller firms requires further research, but it would seem to imply that 'Scenario 3,' where smaller firms are more likely to invest than larger firms, dominates the other two (in which smaller firms are less likely to invest). In other words, this implies that larger firms too are often restricted from fully following the real options approach by the frailties of financial markets. Both sets of firms are constrained then, but the effect is to produce a higher investment rate among smaller firms and hence they invest systematically more than their larger rivals.

The Cabral (1995) finding – that small firms invest in smaller stages, as a response to sunk costs, thus causing higher growth – is also consistent with the model's results, as long as the size of the investments is proportional to the size of the firm when the comparison is made here. In other words, the investments of smaller firms (which are more quickly decided upon in this model) must represent smaller stages of an investment than those of larger firms.

IV. Some policy conclusions and directions for further research

A broader example

It is helpful to consider an example which covers a wider range of firms and financial markets. Consider firms of three sizes, operating in financial markets of three different efficiency levels. In particular, consider a small, young firm, a medium-sized firm with (say) a 5-year track record of success, and a large company with (say) a 10-year history. Next, consider a firm in each of these categories operating in the following financial market settings: a highly developed and relatively efficient financial market, e.g. the USA, a developed financial market with relatively greater inefficiencies, e.g. France, and a less developed financial market in a Southern European country, e.g. Portugal.

Based on the typical firm characteristics of different sizes of firm given in Table 2, reasonable values for the variables which make up uncertainty and financial risk are given in Tables A1-A3 in Appendix A, and Figure 3 shows the resultant values.[5] Figure 4 graphs the maximum waiting times (for a risk threshold of 50%) and illustrates clearly the problem which faces smaller firms operating in less developed financial markets. The combination of relatively small size and relatively underdeveloped financial market structure make SMEs in Southern Europe doubly weak.

Because of financial markets' asymmetric information characteristics, firms are distinguished by their ability to wait. As a result, firm size is a key determinant of both the speed of the investment decision, and of the volatility of investment returns. Smaller, younger firms in general, and especially those from weaker financial markets such as Southern Europe, face an inability to wait which greatly exacerbates their investments' volatility. European economic and monetary union will have two particular effects on the banking sector, which will act in different directions.

Firstly, there will be a 'skill' effect: the consolidation of the banking sector should lead to a higher average standard of informational efficiency (though not necessarily market efficiency). This will reduce the rationing problems which generate the high death rates of SMEs. On the other hand, the 'size' effect may act in the opposite direction: Berger et al. (1997, p.34), for example, find that "larger bank mergers are in general associated with a decrease in small business lending."

Equity market changes may have more unambiguously positive effects. Clearly any 'size' effect through the growth of individual European bourses, and the establishment of new pan-European trading floors (e.g. Euro-Nasdaq) should increase the potential for venture capital flows to smaller, younger firms. The 'skill' effect should also operate to reduce rationing , as in the banking sector. However, the strong caveat remains that most small firms are simply *not* the high growth candidates for venture capital that such changes will impact.

Policy conclusions

The model presented here leads to policy conclusions of two distinct types. On the one hand are policies to mitigate the key factor driving different waiting times - financial market inefficiency. On the other hand there are policies to mitigate the *effects* of imperfect financial markets. In the first group, policies are primarily concerned with the efficient use and provision of information from firms to the financial market. On the market side, the trends are positive. Costs of information transfer and storage, and data manipulation are lower than at any time in the past. The market is involved in a process of streamlining risk management procedures which should (ultimately at least) lead to superior evaluation and monitoring of risks.

---

[5] NB. To avoid entering the arguments about the relationship between firm size and innovation capacity (see, e.g., the debate between van Dijk *et al.*, 1997, Brouwer, 1998, and Mendeveld & Thurik, 1999), d_{in} is set to 0.5 in all cases.

On the firm side, firms' ability to collate and transfer credible information requires attention. The old adage that Portuguese firms keep three sets of books – one for the tax authority, one for the bank and one with the real figures – is a dangerous indicator. Policies to encourage improved accounting standards, e.g short-term subsidies to firms or a joint scheme with accountancy organisations to expand their client base among smaller firms in particular, might be the most effective means to address the problem.

The second set of policy conclusions is concerned with reducing the impact of smaller firms' time constraints. This can take two main forms – either reducing the differential in competitive risk and hence drift parameters of investment values, or reducing the differential in firms' uncertainty about their investment returns by enabling them to make better use of time waited. The first of these involves altering not the difference in waiting time of firms of different sizes, but trying to ensure that the relatively more time-constrained firm does not also have an inferior chance of obtaining the critical value V^* in any given time period. The policies this suggests are primarily concerned with the firms' ability to generate and maintain *private* information flows. Specifically, this calls for an end to public R&D campaigns, for example, or the non-commercial business support institutions which form part of national strategies for improving innovation and competitiveness. The model implies that generating freely available information may well have a negative impact on investment by driving down the value of firms' options, through increased competitive pressure.

The final set of policies is that concerned with improving firms' ability to reduce uncertainty over time. This involves increasing either the level of managerial expertise within firms, through improved training or support institutions, or the flow of information about market conditions to firms. The implied generation of public information might seem at odds with the previous recommendation, but if policy is considered to favour private information where firm-specific factors of the investment decision are concerned, but public information provision or support where general investment and industry-specific factors are involved, the conflict is resolved. Again, for the latter, the trend in the speed of flows of information is positive.

Avenues for further research

Firstly, empirical research should be carried out to assess the strength of the model's predictions. This should include not only testing to see whether observed distributions of firm size can be generated for reasonable parameter values, but to what extent changes in levels of investment can be predicted for different sizes and ages of firm. Simulations to establish patterns of firm death should also be undertaken. The second aim of research should be to ascertain the relative values of differences (generated by firm size and age) in uncertainty and financial risk. In terms of the causality between financial market development and economic growth, financial market characteristics could perhaps be shown to drive the formation of industrial structure, and hence relative levels of economic growth.

An outstanding issue is the different way in which the availability of debt and equity act on the decisions of firms of different sizes. The extent to which the financial risk argument of credit rationing applies to equity investors such as venture capitalists as well as banks has been discussed here, although further work in this direction might yield interesting results – for example, applying the two-dimensional asymmetric information model of rationing proposed by Hellman & Stiglitz (1995).

In particular, the relative attractiveness of debt and equity to different sizes of firm should be examined. Since the costs of uncertainty (borne by smaller firms especially) will be mitigated by equity as opposed to debt financing, the model implies a greater relative preference for equity among smaller firms. This in turn suggests a role for third-party non-market equity investors, but the supply side of the market for this form of financing has not been sufficiently explored. The question of rationing should therefore be investigated, along with the likely impact of investment on firms' managerial expertise and incentive structure.

An issue which has not been dealt with here is that of competitive pressure on firms. Hartman (1972) and Abel (1983) found that increased uncertainty can raise investment levels (as seems to happen here with smaller firms), through a positive impact on the value of a marginal unit of capital to a *competitive* firm. Empirical findings suggest strongly however that this effect is dominated by the option value effect acting in the opposite direction by making investment less attractive (see Carruth et al., 1998, for a useful survey of results). In particular, the level of competitive pressure on a firm seems to influence the effect of uncertainty. Future work should address this issue.

Finally, the next paper in this project will consider explicitly and in more detail the impact of EMU on the financial markets accessed by Southern European SMEs. This will be combined with both the model of their investment decisions presented here, and the results of surveys conducted in Portugal, Greece and Spain. A better understanding of the role of financial markets in firms' investment decisions will allow the effects of the changing nature of those financial markets to be fully examined.

Bibliography

Abel, A., 1983, 'Optimal Investment under Uncertainty,' *American Economic Review* 73, pp.228-33.

Bank of England, 1999, *Quarterly Report on Small Business Statistics* (October).

Berger, A., A. Saunders, J. Scalise & G. Udell, 1997, 'The Effects of Bank Mergers and Acquisitions on Small Business Lending,' *Federal Reserve Bank Board of Governors Working Paper Series* 28 [http://www.bog.frb.fed.us/pubs/feds/1997/199728/199728pap.pdf].

Brouwer, M., 1998, 'Firm Size and Efficiency in Innovation: Comment on van Dijk et al.,' *Small Business Economics*, 11(4), pp.391-393.

Cabral, L., 1995, 'Sunk Costs, Firm Size and Firm Growth,'* Journal of Industrial Economics *18, 161-172.

Cabral, L. & J. Mata, 1996, 'On the Evolution of the Firm Size Distribution: Facts and Theories,' *Research Paper* [http://www.london.edu/faculty/lcabral/papers/firmsize.pdf].

Carruth, A., A. Dickerson & A. Henley, 1998, 'What do we know about Investment under Uncertainty?,' *University of Kent Studies in Economics* 98-04.

Cho, Y., 1986, 'Inefficiencies from Financial Liberalization in the Absence of Well-Functioning Equity Markets,' *Journal of Money, Credit and Banking* 18(2), 191-199.

Cobham, A., 1999a, 'The financing and technology decisions of SMEs I.: Finance as a determinant of investment,' *Queen Elizabeth House Working Paper Series* 24 [http://www2.qeh.ox.ac.uk/].

Cobham, A., 1999b, 'The financing and technology decisions of SMEs II.: Technology and policy,' *Queen Elizabeth House Working Paper Series* 25 [http://www2.qeh.ox.ac.uk/].

Cooley, T. & V. Quadrini, 1999, 'Financial Markets & Firm Dynamics,' Stern Business School Working Paper EC-99-14.

de Meza, D. & D. Webb, 1987, 'Too Much Investment: A Problem of Asymmetric Information,' *Quarterly Journal of Economics* 102(2), 281-292.

DTI, 1999, "Small and Medium Enterprise Statistics for the United Kingdom, 1998" *DTI Statistical Bulletin*, London: HMSO.

van Dijk, R., R. den Hertog, B. Menkveld & R. Thurik, 1997, 'Some New Evidence on the Determinants of Large and Small Firm Innovation,' *Small Business Economics* 9(4), 335-343.

Dixit, A.K. & S. Pindyck, 1994, *Investment under Uncertainty,* Princeton: Princeton University Press.

Dunne, P. & A. Hughes, 1994, 'Age, Size, Growth and Survival: UK Companies in the 1980s,' *Journal of Industrial Economics*, 42(2), 115-140.

European Observatory for SMEs, 1999, *Fifth Annual Report: 1997* [3rd ed. October 1999], Netherlands: EIM Small Business Research & Consultancy.

Evans, D., 1987, 'The relationship between firm growth, size, and age - estimates for 100 manufacturing industries,' *Journal of Industrial Economics*, 35(4), 567-582.

Fazzari, S., R. Hubbard & B. Petersen, 1988, 'Financial Constraints and Corporate Investment,' *Brookings Papers on Economic Activity* 88, 141-195.

Greenwald, B., J. Stiglitz & A. Weiss, 1984, 'Informational Imperfections in the Capital Markets and Macroeconomic Fluctuations,' *American Economic Review* 74(1), 194-99.

Hartman, R., 1972, 'The Effects of Price and Cost Uncertainty on Investment,' *Journal of Economic Theory* 5, 258-66.

Hall, B., 1987, 'The relationship between firm size and firm growth in the US manufacturing sector,' *Journal of Industrial Economics*, 35(4), 583-606.

Hellman, T. & J. Stiglitz, 1995, 'A Unifying Theory of Credit and Equity Rationing in Markets with Adverse Selection,' *Stanford University Business School Working Paper* 1356.

Jovanovic, B., 1982, 'Selection and the Evolution of Industry,' *Econometrica* 50, 649-670.

Kadapakkam, P-R., P. Kumar & L. Riddick, 1998, 'The impact of cash flows and firm size on investment: the international evidence,' *Journal of Banking & Finance* 22, 293-320.

Lucas, R., 1978, 'On the Size Distribution of Firms,' *Bell Journal of Economics* 9, 508-523.

McDonald, R. & D. Siegel, 1986, 'The Value of Waiting to Invest,' *Quarterly Journal of Economics* 101, 707-728.

Menkveld, B. & R. Thurik, 1999, 'Firm Size and Efficiency in Innovation: Reply,' *Small Business Economics*, 12(1), 97-101.

Myers, S. & N. Majluf, 1984, 'Corporate Financing and Investment Decisions when Firms Have Information that Investors Do Not Have,' *Journal of Financial Economics* 13, 187-221.

Pratap, S. & S. Rendon, 1998, 'Firm Investment in Imperfect Capital Markets: a Structural Estimation,' *Universitat Pompeu Fabra, Working Paper* 274.

Rajan, R.G. & L. Zingales, 1999, 'Financial Systems, Industrial Structure and Growth,' paper prepared for the *Symposium on the international Competitiveness of the Swedish Financial Industry*, available on Rajan's website: http://gsbwww.uchicago.edu/fac/raghuram.rajan/research/finsys.pdf

Stiglitz, J. & A. Weiss, 1981, 'Credit Rationing in Markets with Imperfect Information,' *American Economic Review* 71(3), 393-410.

Tsuru, K., 2000**,** *'*Finance and Growth: some theoretical considerations, and a review of the empirical literature,' *OECD Economics Department Working Paper 228*, available at http://www.olis.oecd.org/olis/2000doc.nsf/linkto/eco-wkp(2000)1

Figure 1: Risk changing over time waited, for firms of different sizes

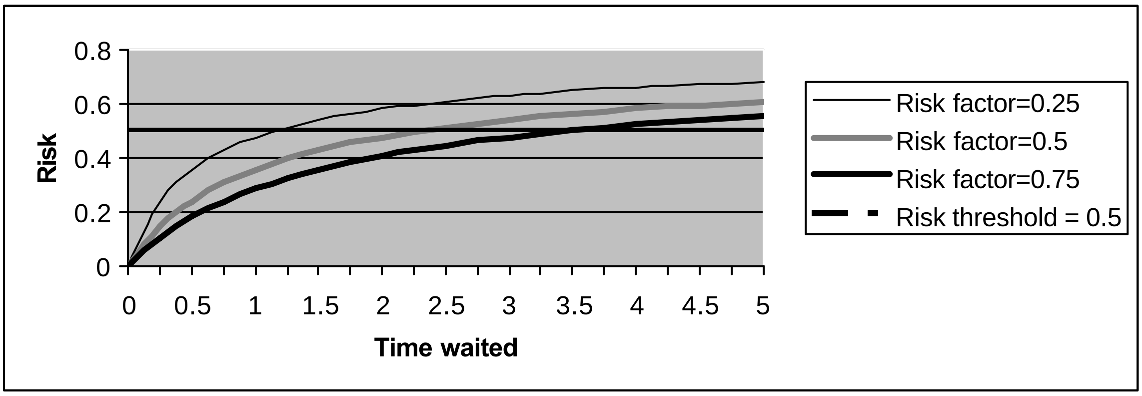


Figure 2: When firms invest – real options or no options

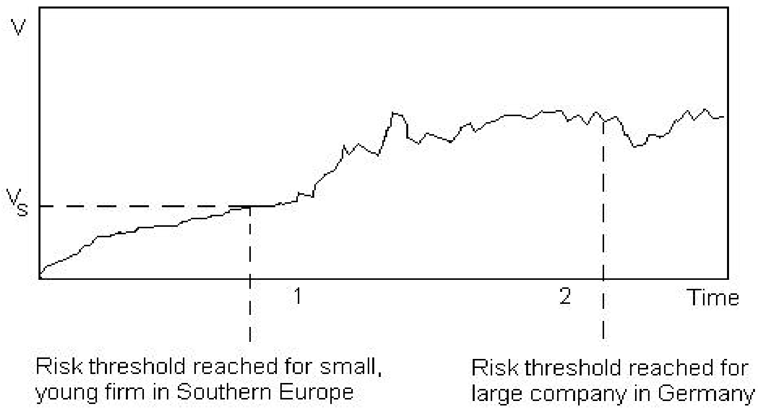


Figure 3: Firm size, financial market and strength of uncertainty-waiting relationship (b_f)

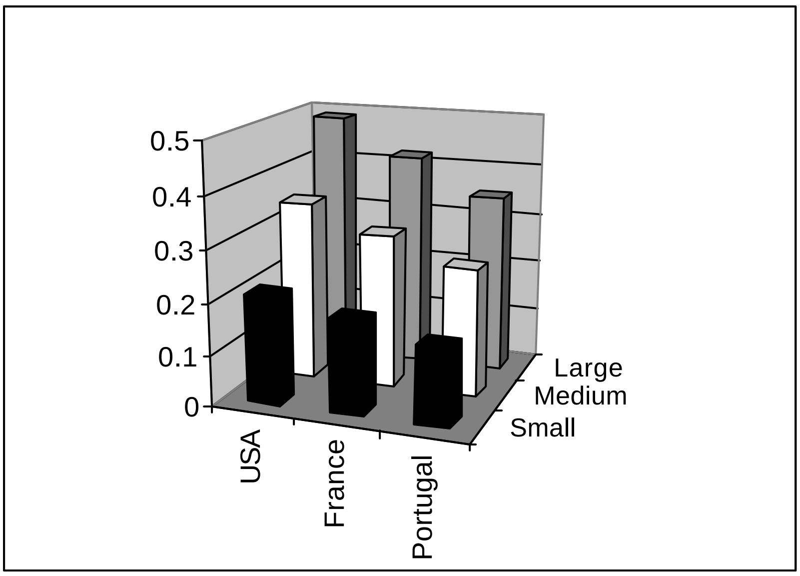
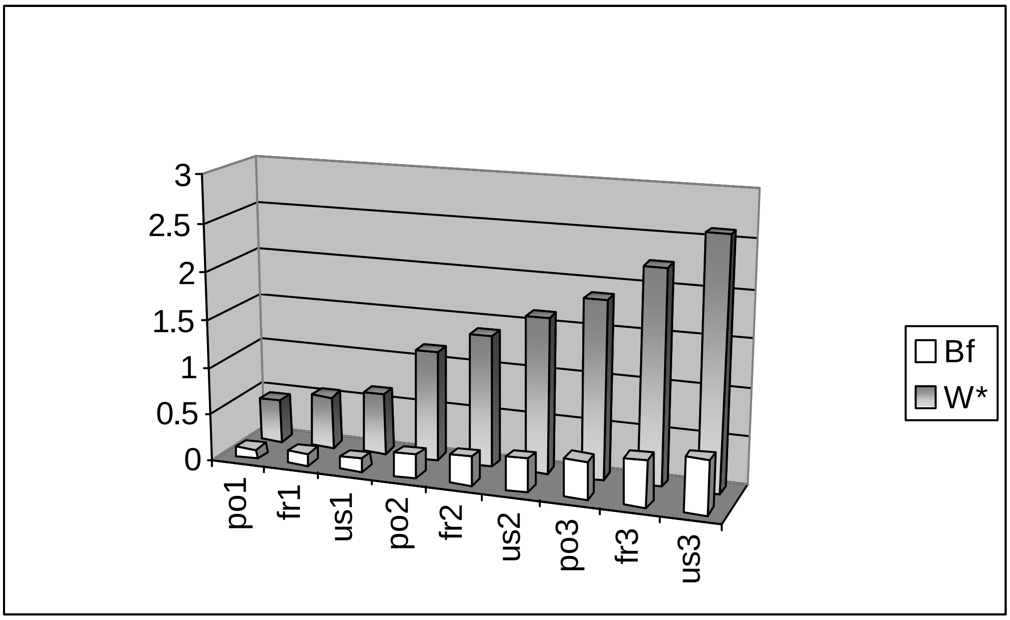


Figure 4: Maximum waiting times for different financial risks, (b_f)



Key: *po* indicates firms from 'Portugal,' *fr* firms from 'France' and *us* firms from the 'USA;' the numbers 1-3 refer to the relative size and age of the firms as defined in the text (1=smallest). "Bf" is b_f .