Working Paper Number 185

Better-behaved Heckscher-Ohlin models through more consistent treatment of trade costs

Adrian Wood*[1]

adrian.wood@qeh.ox.ac.uk

Heckscher-Ohlin logic implies that the relative costs of trading different goods are largely independent of the relative costs of producing them. By attenuating the effects of variation in comparative advantage, the independence of trade costs helps to explain why in reality countries are less specialised, and trade less, than is predicted by the standard Heckscher-Ohlin-Samuelson (HOS) model with iceberg trade costs. This independence similarly helps to explain why the factor prices of countries are more sensitive to their endowments than in HOS, though it also tends to increase the effects of foreign prices on factor prices.

Revised version (with change of title): January 2012

* Department of International Development, Queen Elizabeth House, Oxford

[1] I am indebted for valuable comments and suggestions on earlier versions of this paper to Enrique Aldaz-Carroll, Richard Baldwin, Alberto Behar, Richard Brecher, Mauro Caselli, Alan Deardorff, Peter Neary, Ben Nelson, Dennis Novy, Sherman Robinson, Alasdair Smith, Alan Winters and audiences in Geneva, Oxford and Nottingham.

1. Introduction

The compelling intuition of Heckscher and Ohlin – that what countries trade depends on their endowments and that the earnings of factors are affected by trade – combined with the elegance of Samuelson's formalisation of this intuition has given HO theory a prominent place in every graduate course and textbook. It is often used in academic work, both on its own and with newer theoretical approaches in hybrid models, and is often cited in public debate, especially over the effects of trade on wages.

The standard Heckscher-Ohlin-Samuelson (HOS) model, however, is widely regarded as unsatisfactory. Theorists draw attention to its 'uncomfortable features' (Deardorff, 2006) and find its prediction of factor price equalisation 'a cause of embarrassment' (Bliss, 2007). There is also a history of failed tests of core HO propositions. Recent studies (including Trefler, 1995; Wood and Berge, 1997; Davis and Weinstein, 2001; and Romalis, 2004), have established that HO theory works quite well empirically, as did some earlier studies (Keesing and Sherk, 1971; Leamer, 1984), but only if the HOS model is modified, augmented or abandoned.

This paper shows how more HO-compatible treatment of trade costs can yield better-behaved HO models. The usual specification of trade costs in the HOS model is 'ad-valorem' – the cost of trading each good is a fixed proportion of its production cost – often in the 'iceberg' framework. But this specification is inconsistent with HO logic: for example, if goods produced with different factor proportions use the same trade services, produced with the same factor proportions, changes in factor prices will alter their relative production costs but leave their relative trade costs unchanged, so that ratios of trade costs to production costs cannot be fixed.

In a more consistent HO model, in which relative trade costs are largely independent of relative production costs, the relative purchaser prices of goods (production costs plus trade costs) vary by proportionally less than their relative production costs. The relationship between proportional changes in relative purchaser prices and in relative production costs will be labelled the 'price-ratio elasticity'. It is roughly equal to the complement of the average share of independent (non-ad-valorem, or 'per unit') trade costs, internal as well as international, in the purchaser prices of the goods concerned. This share varies among goods and countries, and is not yet well documented, but is probably typically about one-half (evidence on this point is cited later), so that price-ratio elasticities are low – typically also about one-half.

Low price-ratio elasticities matter because they reduce elasticities of demand in open economies, and because lower demand elasticities make the predictions of HO theory more consistent with empirical evidence. The HOS model assumes that the demand for the goods produced by a small open economy is infinitely elastic. Most empirical applications of trade theory, however, assume demand elasticities to be finite because different national varieties of goods are imperfect substitutes for one another.[2] This

[2] Rauch and Trindade (2003) argue that elasticities of substitution in trade are reduced by information costs, too: even if varieties are actually identical, buyers are less sure that goods from foreign suppliers will be what they want.

insight of Armington (1969) motivates the form of the import functions of computable general equilibrium (CGE) models and is used also in gravity models (Anderson and van Wincoop, 2004). Armington elasticities are included in theoretical HO models by Robinson and Thierfelder (1996) and by Venables (2003). Krugman (1979) similarly used imperfect substitutability among varieties to explain intra-industry trade.

Finite demand elasticities bring HO theory closer to readily observable features of the real world. They help to explain why, in contrast to the textbook 2x2 one-cone HOS model, relative factor prices are not equalised and vary with endowments: factors earn less in countries where they are relatively more abundant, even though countries are open to trade (as noted by Davis and Weinstein, 2001).[3] Finite elasticities also help to explain why in reality most open economies are not highly specialised, contrary to the prediction of many-good HOS models that trade will cause each country to produce only a small subset of goods (no more than there are factors).[4] Less specialisation in production in turn helps to explain why, as pointed out by Trefler (1995), there is not nearly so much trade as HOS predicts.[5]

That local and foreign varieties of goods are generally imperfect substitutes is widely accepted, even for commodities such as oil and grain. More of a question is whether elasticities of substitution are low enough to make outcomes differ substantially from those of models which assume these elasticities to be infinite. CGE models achieve realistic results with substitution elasticities of 4 or less,[6] which, since these models are fairly aggregated, are consistent with the mean of 4.0 and median of 2.2 at the 3-digit SITC level estimated from US trade data by Broda and Weinstein (2006: 568). At the 10-digit level, however, Broda and Weinstein's mean rises to 12.6 (though the median is still only 3.1). Using worldwide data on bilateral trade flows and allowing for differences in product quality, Feenstra and Romalis (2011) estimate the median elasticity of substitution to be 9.4 even at the 4-digit level.

The gap between the low elasticities needed to reconcile HO models with empirical evidence and the higher elasticities estimated from disaggregated data can be bridged by recognising that price-ratio elasticities are pulled down by independent trade costs. Elasticities of substitution relate variation in sales to variation in purchaser prices. The demand elasticity that matters from the perspective of trade theory, however, is

---

[3] Factor prices and endowments are inversely related in multi-cone HOS models, but the relationship is discontinuous and requires unrealistic degrees of specialisation. They are also inversely related in HOS models with fewer goods than factors, but these seem implausible. In any sort of HOS model, any sort of trade cost tends to lower the earnings of abundant factors and raise those of scarce ones, but the sizes of the differences in factor prices among countries then depend on the height of trade costs, not on the sizes of the differences in endowments among countries.

[4] Sector-specific factors provide another possible theoretical reason for the lower-than-predicted degree of specialisation, though in the long run they are relevant mainly to primary sectors – in manufacturing and services, increasing returns should intensify specialisation. In the data, moreover, the true degree of specialisation is understated by aggregation of goods (Davis and Weinstein, 2001; Schott, 2003).

[5] Although, as Trefler shows, the main reason for 'missing trade' is 'missing production'. Calculations that assume US levels of total factor productivity in all countries predict unrealistically high quantities of output and thus of exports (and imports) in other countries, especially developing ones. Differences in average levels of total factor productivity also explain most of the big differences in absolute factor prices across countries, first emphasised by Leamer (1984).

[6] Of the elasticities in the 42 material-goods sectors in the widely-used GTAP model, 31 are below 4, with the mean being 3.5 and the median 3.3 (Dimaranan et al., 2011: table 20.2). Harrison et al. (1997) take 4 as their base elasticity in modelling the effects of the Uruguay Round.

the elasticity of a country's relative sales of different goods with respect to its relative production costs, which is the elasticity of substitution in consumption multiplied by the price-ratio elasticity. So if price-ratio elasticities are typically about one-half, the relevant demand elasticities are about half the level of substitution elasticities and in the same range as those used in calibrating CGE models.

This paper shows how trade costs determine price-ratio elasticities and illustrates the relevance of these elasticities by setting out a HO model that includes them, using the 'hat algebra' of Jones (1965). The model is labelled BHO, where B stands for 'better-behaved', and it nests HOS as a special case. In this model, the effect of variation in endowments on the composition of output in open economies is smaller than in HOS (and not necessarily magnified, as in the Rybczynski theorem), though larger than in a closed economy. Relative factor prices in open economies are correspondingly more sensitive to endowments than in HOS, though less so than in a closed economy.

In these respects, price-ratio elasticities in the BHO model simply reinforce the effects of less-than-infinite elasticities of substitution among national varieties, to a degree that depends on the average share of independent trade costs in purchaser prices. But price-ratio elasticities also have effects that could not be replicated by assuming lower elasticities of substitution. Most notably, low price-ratio elasticities tend to increase the impact on relative factor prices of changes in the prices of foreign goods caused by changes in world prices or in trade barriers. This is because price-ratio elasticities can work in both directions: they lessen the effect of changes in production costs on purchaser prices, but amplify the effect of changes in purchaser prices on producer prices (which are what drive factor prices).

The price-ratio elasticity, though apparently a new concept, is related to two sorts of earlier contributions to the literature. One is applications of HO theory to trade costs. Matsuyama (2007) emphasises that iceberg trade costs are inconsistent with HO logic and analyses the implications of assuming that trade is a skill-intensive activity for the effects of skill accumulation and technical progress in trade on the relative wages of skilled workers and the volume of trade. Marjit and Mandal (2009) undertake similar analysis, but assume that trade is relatively labour-intensive. Both papers treat goods as homogeneous, so the issue of reduction of elasticities does not arise.[7]

The other related earlier contribution is the effect of 'per-unit' trade costs on demand elasticities. This idea comes from industrial organisation theory, most famously in the 'shipping the good apples out' conjecture of Alchian and Allen (1964), who surmised that, since the cost of transport is the same for all qualities of a good, while the cost of production rises with quality, the relative price of better-quality varieties will be lower – and hence the relative demand for them will be higher – at the point of sale than at the point of production.

Hummels and Skiba (2004) confirm this conjecture empirically, using data on trade in varying qualities of the same goods. They find that transport costs are predominantly per-unit, rather than ad-valorem – *x* dollars per mile, rather than *x* percent of the value

[7] Marjit and Mandal use the HOS framework. Matsuyama for simplicity combines a HO treatment of trade costs with a Ricardian treatment of production costs (whereas the present paper does more or less the opposite, with a simplified treatment of trade costs and a HO treatment of production costs).

of the good per mile. Consistently, Bernard *et al*. (2007) and Baldwin and Harrigan (2011) find that the unit value of US exports rises with distance shipped. Irarrazabal *et al.* (2011) estimate per-unit international trade costs for Norwegian non-oil exports to be on average 33% of the fob price. Anderson and van Wincoop (2004) estimate the average share of all trade costs, international plus internal, in the purchaser prices of traded goods to be 63% for developed countries,[8] of which ad-valorem trade costs (discussed further in section 2.3) are likely to be a small minority. Their estimate is based partly on debatable inferences from gravity models (Chaney, 2008), but its high level fits with much casual evidence from the fair trade and value chain literatures of big international differences between producer prices and purchaser prices.

The price-ratio elasticity can be seen as a generalisation of Alchian-Allen. It extends their idea to differences across goods as well as across different qualities of one good. It lets trade costs as well as production costs differ across goods, rather than assuming the cost of trade to be the same for all goods. And rather than focusing on a fixed set of differences in production costs among different goods from one supplier, it focuses on the effect on the relative demand for different goods of differences across suppliers or changes over time in the relative production costs of these goods.

Low price-ratio elasticities, however, require not just that per-unit trade costs be large, relative to production costs and ad-valorem trade costs, but also that variation among countries and over time in the relative per-unit costs of trading goods be independent of variation in the relative costs of producing them. This independence is implied by HO theory, but can be illustrated in common-sense ways. A good that is heavier than another good, for example, is heavier and thus costs more to transport in all countries, regardless of how the relative production costs of the goods vary among countries.

As in this paper, Aldaz-Carroll (2003) and Deardorff (2006) show how HO theory can be made better-behaved by including non-ad-valorem trade costs. They both assume, though, that per-unit trade costs rise with the volume of trade, which seems unrealistic (as Deardorff notes) and differs from the price-ratio elasticity approach. The present approach differs also from that of the large body of work stimulated by Melitz (2003): fixed costs of entering particular markets are not required in the BHO model.

This paper's focus on how trade costs affect the relative prices of different goods sold by particular countries is not the same as that of most previous analysis of trade costs, which is on how such costs cause prices to differ between markets. In any sort of HO model, protection of home markets by any sort of trade costs has familiar effects: less trade, less output in abundant-factor-intensive sectors, and better-off scarce factors.[9] A complete analysis of trade costs thus has to cover both their effects on price-ratio elasticities and their protective effects, some of which are similar and others different, but until section 5 this paper abstracts from the protective effects by assuming that all sales and purchases are in a single world market.

[8] Anderson and van Wincoop (2004: 692) express their estimate of trade costs as a ratio of production costs: 170%, with international trade costs being 74% and internal trade costs adding a further 55%.
[9] Illuminatingly analysed, for instance, by Markusen and Venables (2007) in a three-good HOS model with ad-valorem trade costs.

Section 2 formalises the price-ratio elasticity and uses it to specify the demand side of a BHO model. Section 3 compares the properties of a two-good two-factor version of this model with those of a corresponding HOS model. Section 4 extends the analysis to cover more goods and factors, and section 5 explores the implications of the home market being protected by international trade costs. Section 6 concludes.

2. Price-ratio elasticities: form and function

The main purpose of this section is to explain how price-ratio elasticities depend on trade costs and hence how independent trade costs lower the elasticity of a country's relative sales of different goods with respect to its relative costs of producing them. But first it is necessary to specify an appropriate consumer demand system.

2.1 Response of sales to purchaser prices

Goods are indexed 1, .., *j*, *k* .., *n*, where *n* is large. Each country produces one variety of each good (indexed 1, .., *z*, .., *Z*, where *Z* is also large), and is assumed until section 5 to sell all its output in a unified world market. Country *z*'s output of good *j* is q_j^z, and the relative sales (and hence relative output) of its varieties of any two goods, *j* and *k*, are assumed to be describable by a demand function of the form

$$\frac{q_j^z}{q_k^z} = \alpha_{jk}^z \left(\frac{p_j^z}{p_k^z} \right)^{-\tilde{\varepsilon}_{jk}^z} \tag{2.1}$$

where *p* refers to purchaser prices, α_{jk}^z controls for other influences on relative sales (the prices of other varieties and goods, tastes and incomes), and the key parameter is the elasticity $\tilde{\varepsilon}_{jk}^z$.

To derive a specific such demand function, the behaviour of the world's consumers is assumed to be governed by a two-level CES utility function in which the varieties of goods produced by different countries are (to varying degrees) imperfect substitutes. Its upper (goods) level is

$$U = \left[\sum_{j=1}^{j=n} q_j^{(\gamma-1)/\gamma} \right]^{\gamma/(\gamma-1)} \tag{2.2}$$

and its lower (varieties) level is

$$q_j = \left[\sum_{z=1}^{z=Z} \left(q_j^z \right)^{(\beta_j - 1)/\beta_j} \right]^{\beta_j/(\beta_j - 1)} \tag{2.3}$$

where γ is the elasticity of substitution among different goods and β_j is the elasticity of substitution among different national varieties of good *j*.

This utility function assumes strong separability among different aspects of consumer choice. Consumers choose among varieties of each good on the basis of the relative prices of the varieties of that good alone. This assumption seems reasonable: choices between (say) French and Spanish cars are probably not affected by the relative prices of French and Spanish wines. Choices among goods depend on the relative prices of goods, each of which is a CES index of the prices of all varieties of that good.

To keep the algebra simple, this utility function also makes some other strong but less plausible assumptions, which could be relaxed without affecting the main argument of this paper. Its symmetry at both levels – all varieties of each good, and all goods, are equally desirable – has unrealistic implications.[10] The income elasticity of demand for varieties and goods is unity. The elasticities β and γ are constant, and they have no z superscripts (abstracting from variation in preferences among countries).

The purchaser-price elasticity of substitution, $\tilde{\varepsilon}_{j1}^{z}$, in (2.1) can be written for a two-level symmetrical CES function, following Sato (1967), as

$$\tilde{\varepsilon}_{jk}^{z} = \frac{\dfrac{1}{s_j^z} + \dfrac{1}{s_k^z}}{\dfrac{2}{\gamma} + \left(\dfrac{1}{s_j^z} - 1\right)\dfrac{1}{\beta_j} + \left(\dfrac{1}{s_k^z} - 1\right)\dfrac{1}{\beta_k}} \tag{2.4}$$

which is a weighted harmonic mean of the elasticities of substitution among varieties of the goods, β_j and β_k, and between the goods, γ. The weights depend on s_j^z and s_k^z, which are country z's shares of world sales of goods j and k. The smaller are these shares, the less are the overall prices of the goods affected by the prices of country z's varieties, and hence the smaller is the influence on $\tilde{\varepsilon}_{j1}^{z}$ of γ. Until section 5, it will be assumed that both s's are small, so γ hardly matters and $\tilde{\varepsilon}_{j1}^{z}$ is close to the unweighted harmonic mean of β_j and β_k, labelled β_{jk}.

The elasticity of substitution in (2.4) is a Hicks elasticity, which holds constant both total utility and the consumption of everything other than country z's varieties of j and k. Relaxing these strong assumptions would cause there to be no single symmetrical substitution elasticity: the effects of a given proportional change in the price of good j would differ from the effects of an equal and opposite change in the price of good k (Blackorby and Russell, 1989).[11] But using the Hicks elasticity makes it possible to illustrate the central argument of this paper in a simple model.

2.2 Trade costs and price-ratio elasticities

Having specified how relative sales depend on relative purchaser prices, the next step is to show how they are influenced by relative production costs and trade costs. The purchaser price of the variety of good j produced by country z can be written as

$$p_j^z = c_j^z + t_j^z \tag{2.5}$$

[10] The assumption of symmetry at the lower level implies that varieties produced by smaller countries command higher prices, which is not true, basically because bigger countries produce larger numbers of varieties than smaller countries, for reasons familiar from (and modellable on the basis of) Krugman (1979). The implication of (2.3) that every country, however small, produces its variety of every good is also unrealistic. Fixed costs of production and trade, as in Melitz (2003), and unwillingness to pay extremely high prices, put lower limits on the scale of production of individual varieties.

[11] Blackorby and Russell (1989) prove that the more accurate Morishima elasticity of substitution would be symmetrical with a CES function, but have a single-level function in mind. In the present two-level CES function, differences among the β_js would make Morishima elasticities asymmetric.

where c_j^z is the producer price (received at the factory gate or farm gate) and t_j^z is the trade cost per unit of output that country *z* incurs in supplying good *j* to the world market (including tariffs and other indirect taxes and subsidies).[12] In a competitive equilibrium, as is assumed throughout this paper (and in HOS), producer prices are equal to production costs. The demand function (2.1) can thus be rewritten as

$$\frac{q_j^z}{q_k^z} = \alpha_{jk}^z \left(\frac{c_j^z + t_j^z}{c_k^z + t_k^z} \right)^{-\tilde{\varepsilon}_{jk}^z} = \alpha_{jk}^z \left(\frac{c_j^z}{c_k^z} \right)^{-\varepsilon_{jk}^z} \qquad (2.6)$$

in which the exponent on the final term

$$\varepsilon_{jk}^z = \delta_{jk}^z \tilde{\varepsilon}_{jk}^z \qquad (2.7)$$

is the elasticity of relative sales with respect to relative producer prices (with no tilde), rather than with respect to relative purchaser prices (with the tilde). The link between ε_{jk}^z and $\tilde{\varepsilon}_{jk}^z$ is the price-ratio elasticity, δ_{jk}^z, which measures the responsiveness of relative purchaser prices to relative producer prices.

The value of δ_{jk}^z depends on the nature and size of trade costs. If, for each good, the cost of trade were proportional to its cost of production (not necessarily in the same proportion for both goods), t_j^z / t_k^z would vary in proportion to c_j^z / c_k^z, and hence so would p_j^z / p_k^z, making δ_{jk}^z unity and ε_{jk}^z the same as $\tilde{\varepsilon}_{jk}^z$. This would be the usual ad-valorem or iceberg assumption. But if t_j^z / t_k^z does not vary in proportion to c_j^z / c_k^z, then δ_{jk}^z is normally below unity. In other words, independent trade costs lower the producer-price demand elasticity, ε_{jk}^z.

To show more precisely how the price-ratio elasticity is determined, the identity

$$\frac{p_j^z}{p_k^z} = \frac{c_j^z + t_j^z}{c_k^z + t_k^z} \qquad (2.8)$$

can be rewritten (with subscripts and superscripts temporarily omitted) as

$$p = c\,\frac{1 + \tau\sqrt{t/c}}{1 + \tau\sqrt{c/t}} \qquad (2.9)$$

[12] The term 'purchaser price' means the same in this paper as in the UN System of National Accounts (SNA). The SNA makes a distinction between 'producer price' and 'basic price', which excludes more taxes, but the term 'producer price' is used here to make the meaning clearer.

where $p = p_j^z / p_k^z$ and $c = c_j^z / c_k^z$. Equation (2.9) includes two trade cost ratios: $t = t_j^z / t_k^z$ is the relative trade cost of the two goods, while $\tau = \tau_{jk}^z = \sqrt{\tau_j^z \tau_k^z}$, where $\tau_j^z = t_j^z / c_j^z$, is their average trade cost relative to their average production cost.

It is convenient initially to assume that both *t* and *τ* are given. On this basis, the price-ratio elasticity (the elasticity of *p* with respect to *c*) can be derived from (2.9) as

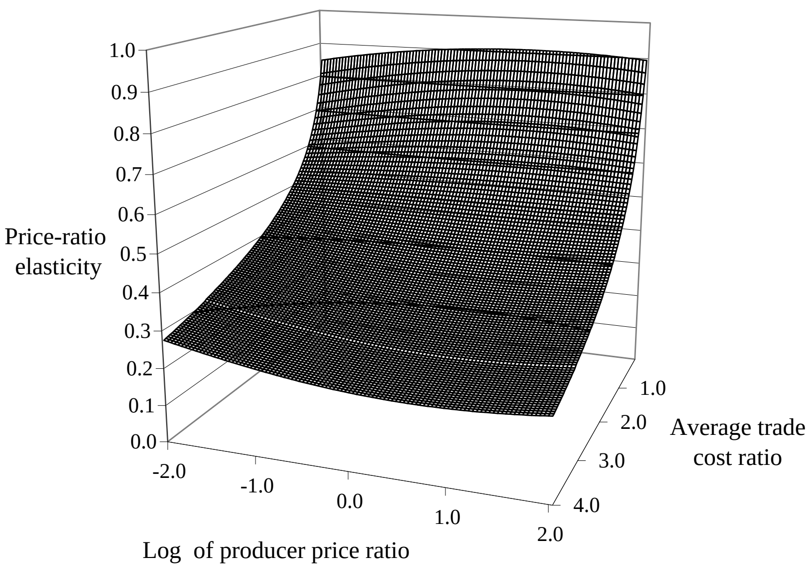
$$\delta = \frac{1 + 0.5\tau\left(\sqrt{c/t} + \sqrt{t/c}\right)}{1 + \tau\left(\sqrt{c/t} + \sqrt{t/c}\right) + \tau^2} \qquad (2.10)$$

in which the value of δ depends mainly on τ. When $c = t$, equation (2.10) reduces to

$$\delta = \frac{1}{1 + \tau} \qquad (2.11)$$

which is the extremum of a shallow curve over which, for any given τ, the size of δ varies with the ratio of *c* to *t*. Figure 1 plots equation (2.10) for wide ranges of τ (0.1 to 4.0) and of *c* (0.1 to 7.4, or in natural logs -2 to +2), with $t = 2$. The shape of the surface confirms that δ depends mainly on τ – a higher average trade cost ratio makes the price-ratio elasticity lower. The curvature of the function with respect to *c* is slight (it is flat when $\tau = 1$, U-shaped when $\tau > 1$ and an inverse U when $\tau < 1$).[13] Variation in *t* affects the position of the extremum in the curvature with respect to *c*, which in figure 1 is where $c = 2$ (and thus $\ln c = 0.69$).

Figure 1 Plot of equation (2.10)



[13] When $\tau = 1$, equation (2.10) implies that $\delta = 0.5$ for all values of *c* and *t*.

So, with given t and τ, the price-ratio elasticity is effectively determined, to a close approximation, by τ alone, at the value given by equation (2.11), and can conveniently be treated as constant with respect to c.[14] This approximation to δ must lie between zero and unity and, again conveniently, does not depend on the value of the relative trade cost ratio, t, provided that this ratio remains constant.

The intuition of equation (2.11) is simple. Consider a single good whose trade cost, t_j, is unaffected by its production cost, c_j. The expression $1/(1+\tau_j)$ is just another way of writing the share of production cost in the purchaser price, $c_j/(c_j + t_j)$. This share governs the size of the effect of a given proportional change in production cost on the purchaser price: for example, if c_j were half of p_j, a 10% rise in c_j, with no change in t_j, would raise p_j by 5%. Equation (2.11) can thus be interpreted as the average share of production costs in the purchaser prices of goods j and k, or as the complement of the average share of independent trade costs. This share is constant (by assumption) and thus determines the proportional change in p_j^z / p_k^z caused by a given proportional change in c_j^z / c_k^z if the trade cost ratio t_j^z / t_k^z remains constant.

The producer-price demand elasticity, $\varepsilon_{jk}^z = \delta_{jk}^z \tilde{\varepsilon}_{jk}^z$, which links the relative quantities of different goods sold (and therefore produced) by a country to its relative production costs, thus becomes, with the simplifying assumptions discussed above,

$$\varepsilon_{jk}^z = \frac{\beta_{jk}}{1+\tau_{jk}^z} \tag{2.12}$$

(derived from equation (2.7) by substituting β_{jk} for $\tilde{\varepsilon}_{jk}^z$, assuming that country z has only small shares of the world market for both goods, and by substituting (2.11), with subscripts and superscripts restored, for δ_{jk}^z). The value of ε_{jk}^z depends directly on the average degree of substitutability among national varieties of the two goods and inversely on their average independent trade cost ratio.[15]

2.3 Price-ratio elasticities further considered

It is important to consider the economic foundations of, and effects of relaxing, the assumptions of given t and τ used to derive (2.11). The more important parameter is t, whose invariance with respect to c has a clear HO basis, discussed more formally in appendix A, which is that both goods use the same type of trade services (in terms of the mix of factor inputs into these services), and hence that their relative trade costs would not change if their relative production costs were to change in response to changes in factor prices. This assumption could be relaxed considerably: so long as

---

[14] The approximation is better, the closer τ is to unity and (if τ is not unity) the closer t is to unity (since for any given range of c, divergence of t from unity increases the maximum difference – at one or other end of the range of c – between the extremum and the true value of δ).

[15] Producer-price elasticities of demand play no part in the HOS model, because national varieties are assumed to be perfect substitutes, so that β_{jk} is infinite – and thus so must be ε_{jk}^z, even if the price-ratio elasticity is well below unity.

relative trade costs vary less than in strict proportion to relative production costs, the message of equation (2.11) and the central argument of this paper remain valid.

The assumption of a constant τ has no comparably clear basis in HO theory. To keep the average ratio of the cost of trade services to production costs constant for a pair of goods in a HO model, the factor proportions of their trade services would need to be an average of the factor proportions of their production processes, which is unlikely to be true for any pair of goods, let alone for all pairs of goods, over all ranges of factor prices. But this assumption greatly simplifies the algebra, by keeping δ constant with respect to c, and its relaxation would not affect the central argument of the paper.

It is important to distinguish between two different ways in which t and τ, and thus δ, could vary with c, namely (i) that per-unit trade costs may be correlated with relative production costs and (ii) that some trade costs are ad-valorem. The following few paragraphs summarise a fuller discussion in appendices B and C.

Suppose initially that all trade costs consist of produced services such as transport. If different goods use different sorts of trade services, then how equation (2.11) needs to be modified depends on how the mixes of factor inputs to the trade services of the two goods relate to the mixes in the production of the goods. Define η_{jk} as the elasticity of t_j / t_k with respect to c_j / c_k. If goods j and k used trade services whose factor intensities differed in the same directions as those of their production processes, η_{jk} would be positive, and if the differences were in the opposite direction, η_{jk} would be negative. A positive η_{jk} would make δ_{jk} larger than equation (2.11) implies, since relative trade costs would move in the same direction as relative production costs (and $\eta_{jk} = 1$ would make $\delta_{jk} = 1$), while a negative η_{jk} would make δ_{jk} smaller than in equation (2.11).

Similarly, define ξ_{jk} as the elasticity of the average trade cost ratio, τ_{jk}, with respect to c_j / c_k. If average factor proportions in the production of trade services were closer to factor proportions in the production of good j than in the production of good k, then ξ_{jk} would be positive, and vice versa. Either way, δ_{jk} could not remain constant when c_j / c_k varied, though it would not vary by much unless the average factor proportions of trade services were close to those in the production of one of the goods. If $\xi_{jk} > 0$, τ_{jk} would rise with c_j / c_k, and so δ_{jk} would fall; and conversely if $\xi_{jk} < 0$.

Part of trade costs consists not of produced trade services but of charges that are paid in proportion to the value of the goods traded. Ad-valorem tariffs are an example, but there are also ad-valorem trade costs, some with economic foundations (insurance and trade finance) and some set on a percentage basis for convenience or by convention (fees and commissions). Moreover, as Hummels and Skiba (2004) point out, ocean liner cartels with monopoly power may charge higher freight rates for costlier items. To analyse how ad-valorem trade costs or policy barriers affect price-ratio elasticities, it is convenient to assume that these costs are based on the producer price, so that the purchaser price of any good j can be written as

$$p_j = \left(1 + \ddot{\tau}_j\right) c_j + \tilde{t}_j \qquad (2.13)$$

where $\ddot{\tau}_j$ is the rate of the ad-valorem trade costs and $\tilde{t}_j$ is the value of per-unit trade costs. Equation (2.11) can then be rewritten approximately as

$$\delta_{jk} = \frac{1}{1+\tau_{jk}} = \frac{1}{1+\tilde{\tau}_{jk}/\left(1+\ddot{\tau}_{jk}\right)} \quad (2.14)$$

where $\tilde{\tau}_{jk} = \sqrt{\left(\tilde{t}_j/c_j\right)\left(\tilde{t}_k/c_k\right)}$ and $\ddot{\tau}_{jk} = \sqrt{\ddot{\tau}_j \ddot{\tau}_k}$. The rhs of (2.14) is the complement of the average share of per-unit trade costs in the purchaser prices of the two goods, and so can be interpreted in the same way as equation (2.11). For given values of the $\tilde{t}$'s and *c*'s, higher $\ddot{\tau}$'s increase δ_{jk}, acting in the same direction as a positive η_{jk}, and if most trade costs were ad-valorem, most price-ratio elasticities would be close to unity. Equation (2.14) thus underlines that what matters for the size of price-ratio elasticities is not the height of trade costs in total but the extent of per-unit (or independent) trade costs relative to the sum of production costs and ad-valorem trade costs.

Price-ratio elasticities vary among countries, over time and among goods as a result of variation not only in relative production costs (as discussed above) but also in basic determinants of trade costs – distance from markets, infrastructure, trade policies and physical characteristics of goods. What matters most is variation in things that affect the average share of per-unit trade costs in purchaser prices, such as distance. Price-ratio elasticities are far less sensitive to differences in the relative trade costs of goods, *t* (as was explained in connection with figure 1).

3. Price-ratio elasticities in a 2x2 HO model

This section explores the effects of price-ratio elasticities in a BHO model that can be directly compared with the familiar 2x2 HOS model.

3.1 Shared elements of HOS and BHO

Two factors, *H* (human capital) and *L* (labour) are employed to produce two goods, *B* (biochemicals, which are *H*-intensive) and *G* (garments, which are *L*-intensive). In a competitive equilibrium, the prices of the goods, which in this context are producer prices, labelled *c*, are related to factor prices, labelled *w*, by

$$c_B = a_{HB}w_H + a_{LB}w_L \quad (3.1)$$

$$c_G = a_{HG}w_H + a_{LG}w_L \quad (3.2)$$

where the input coefficients, labelled *a*, depend on factor prices. Equilibrium in factor markets requires

$$v_H = a_{HB}q_B + a_{HG}q_G \quad (3.3)$$

$$v_L = a_{LB}q_B + a_{LG}q_G \quad (3.4)$$

where the supply of a factor is denoted by *v* and the output of a good by *q*.

These equations refer to one country, with the superscript *z* suppressed for simplicity. However, it will be assumed throughout this paper, following Trefler (1995), that the efficiency of the technology reflected in the *a*'s varies across countries only by an economy-wide scale factor dependent on, say, the quality of their institutions. Since the analysis will focus entirely on relative prices and quantities in a comparative-static framework, the results can thus be interpreted in terms either of movements over time in a country or of differences across countries.

Following Jones (1965), equations (3.1) to (3.4) can be rewritten in a more compact and illuminating way in terms of small proportional changes, denoted by hats. The price equations reduce to

$$\hat{c}_B - \hat{c}_G = (\theta_{HB} - \theta_{HG})(\hat{w}_H - \hat{w}_L) \tag{3.5}$$

where θ_{ij} is the share of factor *i* in the producer price or production cost of good *j*, and the equation relates changes in the relative prices of the two goods to changes in the relative prices of the two factors. If goods prices are assumed to be given by world prices and tariffs, equation (3.5) determines factor prices, as in the Stolper-Samuelson theorem of HOS, with the effects of changes in relative goods prices on relative factor prices being magnified because $\theta_{HB} - \theta_{HG}$ is less than unity.

The factor market equations reduce to

$$\hat{v}_H - \hat{v}_L = -\sigma_{BG}(\hat{w}_H - \hat{w}_L) + (\lambda_{HB} - \lambda_{LB})(\hat{q}_B - \hat{q}_G) \tag{3.6}$$

where λ_{ij} is the share of the total supply of factor *i* used by good *j*, and

$$\sigma_{BG} = \sum_{j=B,G} \left[\lambda_{Hj}(1-\theta_{Hj}) + \lambda_{Lj}\theta_{Hj}\right]\sigma_j \tag{3.7}$$

is a weighted average of the elasticities of substitution in production between *H* and *L* for the two goods, σ_B and σ_G. Equation (3.6) specifies that a rise (say) in the relative supply of *H* must be matched by a rise in the relative demand for *H*, which can be achieved by a fall in the relative price of *H* that induces a rise in the *H*-intensity of the techniques used in producing both goods (the first term), and/or by a shift in the composition of output towards the *H*-intensive good *B* (the second term).

If factor prices are assumed to be determined by world prices and do not change, (3.6) can be rewritten as

$$\hat{q}_B - \hat{q}_G = \frac{1}{\lambda_{HB} - \lambda_{LB}}(\hat{v}_H - \hat{v}_L) \tag{3.8}$$

in which changes in the relative outputs of the goods are determined by changes in the relative endowments of the factors, as in the Rybczynski theorem of the HOS model. Again, the effect is magnified, because $\lambda_{HB} - \lambda_{LB}$ is less than unity.

3.2 Effects of changes in endowments

In the BHO model, the HOS assumption that goods prices are exogenously given by world prices is replaced with a demand function that connects the relative quantities of goods sold by a country in the world market to its relative costs of producing them. Using the simplified form of the producer-price demand elasticity for a small country in (2.12), the demand function can be written in changes as

$$\hat{q}_B - \hat{q}_G = -\frac{\beta_{BG}}{1+\tau_{BG}}\left(\hat{c}_B - \hat{c}_G\right) \tag{3.9}$$

where β_{BG} is the average elasticity of substitution among varieties of goods *B* and *G*, and $1/(1 + \tau_{BG}) = \delta_{BG}$ is the price-ratio elasticity.

In principle, the use of factors to provide trade services should be explicitly modelled. But for simplicity and easier comparability with a 2x2 HOS model, it will be assumed that the mixture of *H* and *L* used in trade services is an average of the mixtures used in sectors *B* and *G* (which also ensures that τ_{BG} remains constant), so that trade services do not alter the relative supply of *H* and *L* to sectors *B* and *G*. Thus although part of the endowment of *H* and *L* is used in trade services, $\hat{v}_H - \hat{v}_L$ will be interpreted both as the change in the country's relative endowments of *H* and *L* and as the change in the relative supply of *H* and *L* to its *B* and *G* sectors.

Substituting (3.5) into (3.9) shows how changes in the relative sales and outputs of the two goods are affected by changes in relative factor prices

$$\hat{q}_B - \hat{q}_G = -\frac{\beta_{BG}}{1+\tau_{BG}}\left(\theta_{HB} - \theta_{HG}\right)\left(\hat{w}_H - \hat{w}_L\right) \tag{3.10}$$

The direction of the effect depends on the relative factor intensities of the two goods. In this case, $(\theta_{HB} - \theta_{HG})$ is positive because *B* is more *H*-intensive, so that a rise (say) in the relative price of *H* would reduce sales of *B* relative to *G*, to an extent governed directly by the sizes of $(\theta_{HB} - \theta_{HG})$ and β_{BG} and inversely by the size of τ_{BG}.

To derive the effect of endowments on factor prices, (3.10) can be substituted into the factor-market equilibrium condition (3.6) to yield, after rearrangement,

$$\hat{w}_H - \hat{w}_L = -\frac{1}{\sigma_{BG} + (\lambda_{HB} - \lambda_{LB})\dfrac{\beta_{BG}}{1+\tau_{BG}}\left(\theta_{HB} - \theta_{HG}\right)}\left(\hat{v}_H - \hat{v}_L\right) \tag{3.11}$$

This equation involves both factor market-clearing mechanisms from (3.6). The first term in the denominator of the big ratio shows that changes in endowments have more effect on factor prices if factors are less substitutable in production. The second term shows how changes in factor prices alter goods prices (via $\theta_{HB} - \theta_{HG}$) and so shift the composition of output in a direction that helps to absorb the change in endowments, to an extent that depends on the producer-price elasticity of demand, $\beta_{BG}/(1 + \tau_{BG})$.[16]

In a one-cone HOS model, with varieties of *B* and *G* being perfect substitutes, so that β_{BG} was infinite, the big ratio in equation (3.11) would be zero: changes in the relative supply of *H* and *L* would be fully absorbed by the shift in output composition, with no need for any change in factor prices. By contrast, in a BHO model, finite β_{BG} coupled with a price-ratio elasticity lowered by independent trade costs reduces the scope for shifts in output composition. Part of the factor supply change thus has to be absorbed by changes in technique, to induce which requires changes in factor prices.[17]

To get a feel for magnitudes, suppose $\sigma_{BG} = 1$, $(\lambda_{HB} - \lambda_{LB}) = (\theta_{HB} - \theta_{HG}) = 0.3$, $\beta_{BG} = 10$ and $\tau_{BG} = 1$.[18] The open-economy elasticity of relative factor prices with respect to relative factor endowments in (3.11) would then be –0.69, rather than zero as in HOS. The difference arises largely because of the much lower value of β_{BG}: even if $\tau_{BG} = 0$, for example because there were only ad-valorem trade costs, and hence the price-ratio elasticity were unity, the elasticity in (3.11) would be –0.53. But the higher the ratio of per-unit trade costs to other costs, the greater is this elasticity: for instance, if τ_{BG} were 2, as in a country with unusually high trade barriers, it would be –0.77.

To compare these outcomes with those in a closed economy, equation (3.11) has to be modified in two ways. With only the local variety available, the relevant elasticity of substitution in consumption is that among goods, γ (derived from equation (2.4) with $s_B^z = s_G^z = 1$), which is likely to be far lower than β_{BG}, say 3 rather than 10. The price-ratio elasticity, however, is likely to be higher, because there are no international trade costs, only internal ones, say $\tau_{BG} = 0.5$ rather than 1. These two modifications pull in opposite directions on the producer-price demand elasticity, $\beta_{BG}/(1 + \tau_{BG})$, but the net effect is likely to be a reduction, in this example from 5 to 2. The elasticity of factor prices with respect to endowments in a closed economy is thus greater than in a BHO open economy, but not a lot greater (in this example –0.85 rather than –0.69).

For brevity denoting the absolute value of the elasticity in (3.11) by φ_{HL}, so that

$$\hat{w}_H - \hat{w}_L = -\varphi_{HL}\left(\hat{v}_H - \hat{v}_L\right) \tag{3.12}$$

the effect of relative endowments on relative outputs in the BHO model can be shown by substituting (3.12) into (3.10) to yield

$$\hat{q}_B - \hat{q}_G = \frac{\beta_{BG}}{1+\tau_{BG}}\left(\theta_{HB} - \theta_{HG}\right)\varphi_{HL}\left(\hat{v}_H - \hat{v}_L\right) \tag{3.13}$$

---

[16] The shift in output mix tends to offset the change in endowments because in a 2x2 model $\lambda_{HB} - \lambda_{LB}$ and $\theta_{HB} - \theta_{HG}$ have the same sign, so the second term in the denominator of (3.11) is always positive.

[17] The differences between the HOS and BHO open-economy models are similar to those between the open-economy and closed-economy models of Jones (1965). φ_{HL} is the inverse of his 'economy-wide elasticity of substitution between factors' (p. 564), and equations (3.14) and (3.13) are substantively the same as his equations (11) and (11′).

[18] $\lambda_{HB} - \lambda_{LB} = \theta_{HB} - \theta_{HG} = 0.3$ if, for example, $\lambda_{HB} = 0.7$, $\lambda_{LB} = 0.4$, $w_H/w_L = 1.3$ and $v_H/v_L = 1$.

This equation can be interpreted causally, from right to left, in a way which resonates with basic HO intuition (e.g. Ohlin, 1967: 63): relative endowments influence relative factor prices (via φ_{HL}), which influence relative producer prices (via $\theta_{HB} - \theta_{HG}$), which influence relative purchaser prices (via the price-ratio elasticity, $1/(1 + \tau_{BG})$), which in turn influence relative sales in world markets (via β_{BG}). The effect of endowments on outputs is simply the product of these four elasticities: its size depends on their sizes, while its direction depends on the sign of ($\theta_{HB} - \theta_{HG}$), which here is positive: a larger endowment of *H* relative to *L* increases the output of *B* relative to *G*.

These elasticities, however, are not all independent of one another: in particular, a low price-ratio elasticity has two opposed effects: it reduces the responsiveness of sales to production costs (and therefore to factor prices), but it increases the responsiveness of factor prices to endowments. In a 2x2 framework, a reduced-form expression for the effect of endowments on outputs can be derived by substituting equation (3.11), rather than (3.12), into (3.10) and simplifying to yield

$$\hat{q}_B - \hat{q}_G = \frac{1}{\dfrac{\sigma_{BG}(1+\tau_{BG})}{\beta_{BG}(\theta_{HB}-\theta_{HG})} + (\lambda_{HB}-\lambda_{LB})}\left(\hat{v}_H - \hat{v}_L\right) \qquad (3.14)$$

In a HOS model, with an infinite β_{BG}, equation (3.14) would become the Rybczynski relationship (3.8). In the BHO model, with finite β_{BG} and a low price-ratio elasticity, the effects of changes in relative endowments on relative outputs are smaller. This is because an increased (say) endowment of *H*, by raising the output of *B*, lowers the price of *B* and hence also the price of *H*, which induces *H*-using changes in technique that absorb part of the increased endowment.

It is clear from the equation that a lower price-ratio elasticity (that is, a higher value of τ_{BG}) tends to reduce the effect of endowments on outputs. This is because it amplifies the effect on relative producer prices (and thus on relative factor prices and changes in technique) of the change in purchaser prices caused by the initial change in output. A corollary of a higher price-ratio elasticity decreasing the effect of changes in producer prices on purchaser prices is that it increases the effect in the other direction.

Using the same numbers as before for the variables in (3.14), the HOS Rybczynski elasticity would be 3.33, with substantial magnification, but in a BHO open economy, the elasticity is only 1.03. Much of the difference is due to the lower value of β_{BG}: even with $\tau_{BG} = 0$ and thus $\delta_{BG} = 1$, the elasticity in (3.11) would be 1.58. But if τ_{BG} were 2 rather than 1, for example, the elasticity would be reduced to 0.77.

To compare these open-economy outcomes with those in a closed economy, the same modifications are needed to (3.14) as were made above to (3.11): replacing β_{BG} with γ, and reducing the size of τ_{BG} to allow for the absence of international trade costs. With the same numerical values as before ($\gamma = 3$ and $\tau_{BG} = 0.5$), the elasticity in (3.14) is 0.51, so that the responsiveness of outputs to endowments in a BHO open economy is twice as big as it would be in a closed economy, though only one third as large as in the HOS version of this model.

The effects of relative endowments on exports and imports are smaller in a 2x2 BHO open-economy model than in a 2x2 HOS model.[19] In both models, the composition of trade varies more with endowments than the composition of output, because exports and imports are residuals – the differences between output and domestic consumption, taking consumer preferences to be given. But the composition of trade varies less in the BHO model, mainly because output composition varies less, but also because the composition of consumption varies more (countries consume more of their abundant factors, since their prices and those of goods that use them intensively are lower).[20]

A corollary of the smaller variation in the composition of trade in BHO than in HOS is a smaller total volume of trade. The three-fold difference in the elasticity in (3.14) between the two sorts of model in the numerical example above gives an indication of the size of the difference in trade volume.[21] In the present 2x2 BHO model with sales only in a large world market, a higher τ_{BG} reduces both the effect of endowments on the composition of trade and the total volume of trade. In the BHO models with more goods and a protected home market in later sections, higher average trade costs may increase the sensitivity of the composition of trade to endowments, but in any model higher average trade costs must reduce the total amount of trade.

3.3 Effects of changes in own trade costs

To analyse the effects of small changes in a country's own trade costs – those that it incurs in selling its outputs in world markets – changes in its relative purchaser prices can be written as a weighted average of changes in its producer prices and trade costs

$$\hat{p}_B - \hat{p}_G = \frac{1}{1+\tau_{BG}}\left(\hat{c}_B - \hat{c}_G + \hat{\ddot{\tau}}_B - \hat{\ddot{\tau}}_G\right) + \frac{\tau_{BG}}{1+\tau_{BG}}\left(\hat{\tilde{t}}_B - \hat{\tilde{t}}_G\right) \qquad (3.15)$$

where $\hat{\ddot{\tau}}_B - \hat{\ddot{\tau}}_G$ is the change in relative ad-valorem trade costs (measured as ratios of producer prices) and $\hat{\tilde{t}}_B - \hat{\tilde{t}}_G$ is the change in relative per-unit trade costs. The change in ad-valorem trade costs, like that in producer prices, is weighted by the price-ratio elasticity, $1/(1 + \tau_{BG}) = \delta_{BG}$, which (as explained in connection with equation (2.14)) is the average share of producer prices plus ad-valorem trade costs in the purchaser prices of the two goods. The change in per-unit trade costs is weighted by the average share of per-unit trade costs in purchaser prices, $\tau_{BG}/(1 + \tau_{BG}) = 1 - \delta_{BG}$.

The effect on factor prices, holding endowments constant, can then be written as

[19] Though a direct algebraic comparison is difficult: in HOS, countries either export or import a good, whereas in BHO, they both export and import different varieties of the same goods.

[20] Though the effect on the composition of consumption is trivial if each national variety accounts for only a tiny share of the total supply of each good, as has so far been assumed.

[21] Suppose that at world average v_H/v_L, $q_B = q_G$ in both production and consumption. A 20% deviation in v_H/v_L from the world average would raise q_B/q_G from unity to 1.21 in the BHO model and 1.84 in the HOS model, thus increasing the share of *B* in output from 50% to 55% in BHO and 65% in HOS. With no change in the composition of consumption (*B*'s share staying at 50% in both models), net exports of *B* would be only one-third as large in BHO (5% of output) as in HOS (15% of output). Allowing for the rise in *B*'s share of consumption in the BHO model would make the difference larger.

$$\hat{w}_H - \hat{w}_L = -\varphi_{HL}(\lambda_{HB} - \lambda_{LB})\beta_{BG}\left[\frac{1}{1+\tau_{BG}}\left(\hat{\ddot{\tau}}_B - \hat{\ddot{\tau}}_G\right) + \frac{\tau_{BG}}{1+\tau_{BG}}\left(\hat{\tilde{t}}_B - \hat{\tilde{t}}_G\right)\right] \qquad (3.16)$$

The terms in the square brackets show the impact effect of changes in own trade costs on relative purchaser prices (for given producer prices). The terms before the square brackets show how this change in purchaser prices affects relative sales of the goods (β_{BG}) and thus relative factor demands ($\lambda_{HB} - \lambda_{LB}$) and ultimately relative factor prices (φ_{HL}). Substituting for φ_{HL} from (3.11) and rearranging yields

$$\hat{w}_H - \hat{w}_L = -\frac{1}{\dfrac{\sigma_{BG}(1+\tau_{BG})}{\beta_{BG}(\lambda_{HB} - \lambda_{LB})} + (\theta_{HB} - \theta_{HG})}\left[\hat{\ddot{\tau}}_B - \hat{\ddot{\tau}}_G + \tau_{BG}\left(\hat{\tilde{t}}_B - \hat{\tilde{t}}_G\right)\right] \qquad (3.17)$$

The negative sign shows that a rise (say) in relative own trade costs on good *B*, either ad-valorem or per-unit, reduces the relative price of *H*, the factor used intensively by *B*. The effect is essentially the same as that of a change in relative indirect taxes.[22]

To understand the economic mechanisms involved, it is helpful to consider separately the two main components of equation (3.17). Its big ratio shows the effect of changes in relative producer prices on relative factor prices. In a HOS model, with an infinite β_{BG}, this ratio would become $1/(\theta_{HB} - \theta_{HG})$, as in the Stolper-Samuelson interpretation of equation (3.5), with the effects of changes in goods prices on factor prices being magnified. In the BHO model, however, this effect is smaller because of the negative feedback shown in the first term of the denominator of this ratio.[23] Shocks to factor prices induce changes in technique, which cause both sectors (for example) to release some of the factor that has become more expensive, permitting increased output of the good that uses that factor intensively, which drives down the price of that good and so also the price of its intensive factor, which offsets part of the initial shock.

The big ratio in equation (3.17) closely resembles that in (3.14), which relates outputs to endowments (illustrating HO reciprocity), and the two are identical if $\lambda_{HB} - \lambda_{LB} = \theta_{HB} - \theta_{HG}$, as was assumed in the earlier numerical examples. With these illustrative numbers, the HOS elasticity of the big ratio in (3.17) would again be 3.33, the BHO elasticity 1.03, and the elasticity in a closed economy 0.51.

The other main component of equation (3.17) is the term in square brackets, to which the minus sign belongs and which can be written more fully as

$$\hat{c}_B - \hat{c}_G = -(1+\tau_{BG})\left[\frac{1}{1+\tau_{BG}}\left(\hat{\ddot{\tau}}_B - \hat{\ddot{\tau}}_G\right) + \frac{\tau_{BG}}{1+\tau_{BG}}\left(\hat{\tilde{t}}_B - \hat{\tilde{t}}_G\right)\right] \qquad (3.18)$$

showing how an increase in own trade costs on a good would lower its producer price if its purchaser price did not change.[24] The price-ratio elasticity plays a double role:

---

[22] Much as in equations (14) and (14')of Jones (1965). The effects of changes in own trade costs on the relative outputs of the two goods can be derived in a similar way, but are of less economic interest.

[23] But in the same direction, because in a 2x2 model both denominator terms must have the same sign.

[24] Equation (3.18) is a rearrangement of (3.15) with $\hat{p}_B - \hat{p}_G = 0$.

inside the square brackets, it weights the effects on purchaser prices of changes in the two sorts of own trade costs; but in front of the brackets, the inverse of δ_{BG} amplifies the effects on relative producer prices of changes in relative purchaser prices. These two roles cancel out for changes in ad-valorem trade costs, whose effect on producer prices is thus independent of τ_{BG}. But the second role dominates for changes in per-unit trade costs: their effect on producer prices is greater, the bigger is τ_{BG}.

Reverting to equation (3.17) as a whole, the conclusion is that the effects of changes in own trade costs on factor prices are in the same direction in BHO as in HOS. But for a given value of τ_{BG} the effects are smaller in BHO than in HOS, albeit larger than in a closed economy, and in BHO the size of the effect declines with that of β_{BG}, all of which corresponds with the standard result that lower demand elasticities cause more shifting of taxes.

The impact on the outcome of the initial level of τ_{BG}, however, depends on which type of trade cost changes. If the change is in relative ad-valorem trade costs, in a BHO model a higher τ_{BG}, like a lower β_{BG}, diminishes the effect on relative factor prices (by lowering the producer-price demand elasticity and hence strengthening the negative feedback), and in HOS a higher τ_{BG} would make no difference. But if the change is in relative per-unit trade costs, a higher τ_{BG} increases the effect on factor prices: in BHO, as can be shown by rearranging (3.17), the greater amplification of the conversion of purchaser-price changes into producer-price changes outweighs the stronger negative feedback; and in HOS only the amplification is relevant.

3.4 Changes in foreign prices and trade costs

So far, the prices of foreign suppliers have implicitly been held constant. In analysing the effects of changes in foreign prices, the endowments of the country concerned will be held constant. Separating out the effects of foreign prices on this country's sales in the shift parameter (α_{jk}) of the demand function in equation (2.1), replacing $\tilde{\varepsilon}_{jk}$ with β_{BG} and rewriting in small changes yields

$$\hat{q}_B - \hat{q}_G = -\beta_{BG}\left(\hat{p}_B - \hat{p}_G\right) + \left(\beta_{BG} - \gamma\right)\left(\hat{p}_B^* - \hat{p}_G^*\right) \tag{3.19}$$

where p_j^* is the average purchaser price of all foreign varieties of good j, and γ is the elasticity of substitution in consumption between goods B and G. A change in the foreign relative price of B and G has the opposite effect on the relative sales of this country to a change in its own relative price. The effect is smaller, too, because a rise in p_B^* (for instance) not only increases this country's share of the market for good B (to the same degree as a fall in p_B would) but also offsettingly shrinks the market for B relative to G, to a degree determined by γ.[25]

Substituting (3.19) into the factor-market clearing equation (3.6), an equation can be derived that shows how changes in the purchaser prices of foreign suppliers affect this country's relative factor prices

[25] The derivation of (3.19) assumes for simplicity that $\beta_B \hat{p}_B^* - \beta_G \hat{p}_G^* = \beta_{BG}\left(\hat{p}_B^* - \hat{p}_G^*\right)$, where the Hicks elasticity of substitution, β_{BG}, is the same as for the effect of changes in the country's own prices.

$$\hat{w}_H - \hat{w}_L = \varphi_{HL}(\lambda_{HB} - \lambda_{LB})(\beta_{BG} - \gamma)(\hat{p}_B^* - \hat{p}_G^*) \tag{3.20}$$

As in any sort of HO model, a rise (say) in the relative foreign price of good *B* raises the relative price of *H*, the factor used intensively by *B*. The mechanism, reading (3.20) from right to left, is a rise in the country's relative sales of good *B* (via $\beta_{BG} - \gamma$), boosting the relative demand for *H* (via $\lambda_{HB} - \lambda_{LB}$) and so pulling up its relative price (via φ_{HL}). Substituting into (3.20) for φ_{HL} from (3.11) and rearranging yields[26]

$$\hat{w}_H - \hat{w}_L = \frac{1}{\dfrac{\sigma_{BG}(1+\tau_{BG})}{\beta_{BG}(\lambda_{HB} - \lambda_{LB})} + (\theta_{HB} - \theta_{HG})}(1+\tau_{BG})(1-\gamma/\beta_{BG})(\hat{p}_B^* - \hat{p}_G^*) \tag{3.21}$$

whose big ratio is the same as that in (3.17) already discussed. The terms to its right translate changes in relative foreign purchaser prices into changes in this country's relative producer prices. The expression $1 - \gamma/\beta_{BG}$ converts the change in the foreign price ratio into the equivalent change (in terms of its effects on this country's sales of its varieties of *B* and *G*) in this country's own purchaser-price ratio.[27] It is less than unity because (and to the degree that) foreign varieties are less than perfect substitutes for this country's varieties. The expression $1 + \tau_{BG}$, as in equation (3.18), amplifies the change in relative purchaser prices into a change in relative producer prices.

In a HOS model, with an infinite β_{BG}, equation (3.21) would reduce to

$$\hat{w}_H - \hat{w}_L = \frac{1}{(\theta_{HB} - \theta_{HG})}(1+\tau_{BG})(\hat{p}_B^* - \hat{p}_G^*) \tag{3.22}$$

which is the magnified Stolper-Samuelson relationship, but with an additional degree of magnification imparted by the inverse of the price-ratio elasticity. In a BHO model with a finite value of β_{BG}, the effect of changes in foreign purchaser prices on factor prices is smaller than in HOS for two reasons: a smaller effect of changes in foreign prices on domestic producer prices (via the $1 - \gamma/\beta_{BG}$ term); and the negative feedback discussed earlier in the first term of the denominator of the big ratio in (3.21).

The likelihood of magnification in the BHO model is increased by the $1 + \tau_{BG}$ term. With the same illustrative numerical values as before, the elasticity of relative factor prices with respect to foreign purchaser prices in the BHO model would be 2.69, well above unity, though well below the HOS value of 6.67. For any given value of β_{BG}, moreover, this elasticity increases with the per-unit trade cost ratio, τ_{BG}, as discussed earlier. If τ_{BG} were 2, for example, the BHO elasticity would be 3.00.

The causes of changes in foreign purchaser prices can be decomposed into changes in foreign producer prices and in foreign trade costs (those incurred by foreign firms in

---

[26] Equations comparable to (3.20) and (3.21) that show the effects of foreign prices on the relative sales and output of the two goods can be derived in a similar way.

[27] For a single good, the horizontal (quantity) shift in its demand curve caused by a change in foreign purchaser prices is $(\beta - \gamma)$, as in equation (3.19). The vertical (price) shift is thus $(\beta - \gamma)/\beta = 1 - \gamma/\beta$.

selling their outputs in world markets) by substituting for $\hat{p}_B^* - \hat{p}_G^*$ in equations (3.19) to (3.22) an identity similar to that in equation (3.15)

$$\hat{p}_B^* - \hat{p}_G^* = \frac{1}{1+\tau_{BG}^*}\left(\hat{c}_B^* - \hat{c}_G^* + \hat{\ddot{\tau}}_B^* - \hat{\ddot{\tau}}_G^*\right) + \frac{\tau_{BG}^*}{1+\tau_{BG}^*}\left(\hat{\tilde{t}}_B^* - \hat{\tilde{t}}_G^*\right) \qquad (3.23)$$

That the foreign price-ratio elasticity is less than unity (because $\tau_{BG}^* > 0$) reduces the effect of changes in foreign producer prices and ad-valorem trade costs. Higher levels of τ_{BG}^*, however, amplify the effect of changes in foreign per-unit trade costs.[28]

Another difference between the HOS and BHO models concerns the effects on factor prices of biased technical change. In a one-cone HOS model, as Jones (1965) shows, relative factor prices respond to technical change that is uneven across sectors, whose impact is like that of a change in relative own trade costs, but not to technical change that is uneven across factors, which acts like a change in relative endowments and so affects only the composition of output. In the BHO model, however, in which factor prices vary with endowments, both sorts of bias would alter relative factor prices. For example, technical change that displaced *L* in both the *B* and *G* sectors would reduce w_L / w_H, because the rise in the relative output of *B* caused by the increased availability of *L* would lower the relative price of *G*. This difference between the models matters for disentangling the effects of trade and technical change on wages in reality.

4. Increasing the numbers of goods and factors

Since in reality there are more than two goods and two factors, it is important to know whether and how the conclusions of the 2x2 analysis can be generalised.

4.1 A higher-dimensional BHO model

Following Smith and Wood (2009), the price and factor-market-clearing conditions of the 2x2 model – equations (3.1) to (3.4) – become, in higher dimensions,

$$c_j = \sum_{i=1}^{m} a_{ij} w_i \qquad j \text{ (or } k) = 1, \ldots., n \qquad (4.1)$$

$$v_i = \sum_{j=1}^{n} a_{ij} q_j \qquad i \text{ (or } h) = 1, \ldots., m \qquad (4.2)$$

with *n* goods and *m* factors. As in section 3, these equations refer to a single country, with the *z* superscript suppressed for simplicity. The higher-dimensional counterparts of the Jones 2x2 equations in small proportional changes are *n* – 1 equations that link changes in relative producer prices of goods to changes in relative factor prices

[28] These propositions about the effects of variation in τ_{BG}^* on the size of the impact of changes in trade costs on foreign purchaser prices remain true even after allowing for feedback to producer prices of the sort analysed in section 3.3 (which makes the impact smaller than equation (3.23) implies.)

$$\hat{c}_j - \hat{c}_1 = \sum_{h=2}^{m} (\theta_{hj} - \theta_{h1})(\hat{w}_h - \hat{w}_1) \tag{4.3}$$

and $m - 1$ factor market equilibrium conditions

$$\hat{v}_i - \hat{v}_1 = -\sum_{h=2}^{m} \left[\sum_{j=1}^{n} (\lambda_{ij}\sigma_{ijh} - \lambda_{1j}\sigma_{1jh}) \right] (\hat{w}_h - \hat{w}_1) + \sum_{j=2}^{n} (\lambda_{ij} - \lambda_{1j})(\hat{q}_j - \hat{q}_1) \tag{4.4}$$

which are basically similar to equations (3.5) and (3.6) of the 2x2 model, but require a bit more explanation.[29] The last term of (4.4) differs from (3.6) only in summing the effects on factor demands of changes in outputs over $n - 1$ pairs of goods, rather than one pair. The first term on the right-hand side, however, involves a redefinition. The σ terms are no longer elasticities of substitution: instead

$$\sigma_{ijh} = -\frac{\partial a_{ij}}{\partial w_h} \frac{w_h}{a_{ij}} \tag{4.5}$$

is a cross-price elasticity (of the factor i input coefficient for good j with respect to the price of factor h), with its sign reversed for easier comparability with the substitution elasticities in the 2x2 model: if $\partial a_{ij} / \partial w_h$ is negative, σ_{ijh} is positive. These elasticities are weighted in (4.4) by the shares of the endowment of i used in good j, and summed over all goods and factors. The first right-hand side term of (4.4) thus shows how changes in all relative factor prices affect the relative demands for factors i and 1.

In a higher-dimensional HOS model, equations (4.1) to (4.4) would be interpreted as referring to an economy facing given goods prices, c_j. In the higher-dimensional BHO model, there are instead assumed to be $n - 1$ demand equations of the same form as (3.9) in the 2x2 BHO model, namely

$$\hat{q}_j - \hat{q}_1 = -\frac{\beta_{j1}}{1 + \tau_{j1}} \left(\hat{c}_j - \hat{c}_1 \right) \tag{4.6}$$

each linking the world market sales of this country's variety of a good j, relative to its sales of the numeraire good 1, to its relative producer prices of these goods. These links depend on the (harmonic mean of the) elasticities of substitution among varieties of the two goods concerned and on price-ratio elasticities – which vary among pairs of goods depending on their average per-unit trade cost ratios.

The demand system in (4.6) is highly simplified. As explained in section 2.1, each of the β_{j1}s is formally a Hicks elasticity of substitution, derived on the assumption of no change in the consumption of anything other than these national varieties of these two goods. It is inaccurate to use the β_{j1}s in a context where the prices and quantities of many goods are changing.[30] The justification for doing so is that it permits a simple

---

[29] As in the 2x2 analysis, the use of factors to provide trade services is not explicitly modelled here.
[30] In a more general demand system, the rhs of equation (4.6) would show country z's prices of goods *j* and 1 separately (not just their ratio), as well as the prices of other varieties and goods (although, given

illustration of the forces at work in any higher-dimensional HO model with imperfect substitutability among national varieties and per-unit trade costs.

Equations (4.3) and (4.6) can be combined to yield *n* – 1 equations of the form

$$\hat{q}_j - \hat{q}_1 = -\frac{\beta_{j1}}{1+\tau_{j1}} \sum_{h=2}^{m} (\theta_{hj} - \theta_{h1})(\hat{w}_h - \hat{w}_1) \qquad (4.7)$$

which in conjunction with the *m* – 1 factor market-clearing equations (4.4) describe how changes in all relative factor prices and relative outputs depend on changes in the exogenous relative factor endowments. Changes in the relative producer prices of all goods are determined by equations (4.3). Extension of (4.6) along the same lines as in sections 3.3. and 3.4 would allow analysis of the effects on factor prices and outputs of changes in trade costs and foreign prices.

Comparing BHO and HOS models in higher dimensions is less straightforward than in the 2x2 case, because the properties of HOS models vary with their dimensions. In a HOS model with more factors than goods, for example, factor prices are affected by endowments, which they would not be in a one-cone model with the same numbers of factors and goods. With more goods than factors, the structure of output in a HOS models is indeterminate or highly specialised: changes in endowments cause switches between subsets of goods as well as changes in the relative outputs of the same goods. By contrast, a BHO model behaves in much the same way with any numbers of goods and factors (which is useful, because in practice the numbers of goods and factors can be varied arbitrarily by altering the level of aggregation of the data).

However, the general conclusions of section 3 about the comparisons between BHO and HOS are the same in higher dimensions. Factor prices tend to be more sensitive to endowments in BHO than in HOS, though less sensitive than in a closed economy, and vice versa for the sensitivity of relative outputs to endowments. The effects on factor prices of changes in own trade costs are smallest in a closed economy, larger in BHO and largest in HOS, and changes in foreign prices affect factor prices more in HOS than in BHO. These rankings may differ for particular goods or factors, but the general pattern is simply the result of producer-price elasticities of demand for goods being lower in BHO than in HOS.

4.2 Differences in price-ratio elasticities

Of particular interest are the effects on outcomes in higher dimensions of differences among goods and countries in price-ratio elasticities. These effects can be analysed quite satisfactorily in a simplified model with many goods but only two factors. (An alternative way of simplifying the higher-dimensional model, by omitting some of the inter-relationships among a larger number of factors, is outlined in Appendix D.)

With *n* goods and two factors, which as in section 3 are labelled *H* and *L*, equations (4.7) become

---

the separability and symmetry assumed in (2.2) and (2.3) plus small market shares, country *z*’s relative sales of goods *j* and 1 are unrelated to the prices of its varieties of goods other than *j* and 1).

$$\hat{q}_j - \hat{q}_1 = -\frac{\beta_{j1}}{1+\tau_{j1}}(\theta_{Hj} - \theta_{H1})(\hat{w}_H - \hat{w}_L) \qquad (4.8)$$

and the factor-market clearing equations (4.4) reduce to the single equation

$$\hat{v}_H - \hat{v}_L = -\sum_{j=1}^{n}(\lambda_{Hj}\sigma_{HjH} - \lambda_{Lj}\sigma_{LjH})(\hat{w}_H - \hat{w}_L) + \sum_{j=2}^{n}(\lambda_{Hj} - \lambda_{Lj})(\hat{q}_j - \hat{q}_1) \qquad (4.9)$$

in which the triple subscript on the σ's is a reminder that these are cross-elasticities. Substituting (4.8) into (4.9) and rearranging yields

$$\varphi_{HL} = -\frac{\hat{w}_H - \hat{w}_L}{\hat{v}_H - \hat{v}_L} = \frac{1}{\sum_{j=1}^{n}(\lambda_{Hj}\sigma_{HjH} - \lambda_{Lj}\sigma_{LjH}) + \sum_{j=2}^{n}(\lambda_{Hj} - \lambda_{Lj})\frac{\beta_{j1}}{1+\tau_{j1}}(\theta_{Hj} - \theta_{H1})} \qquad (4.10)$$

where φ_{HL}, as in the 2x2 BHO model, is the elasticity of the relative prices of the two factors with respect to their relative endowments. Equation (4.10) resembles (3.11),[31] but involves summations over many goods, which underlines that this is an economy-wide relationship. As in the 2x2 model, endowments affect relative factor prices, and by more, the lower are price-ratio elasticities. More precisely, φ_{HL} increases with the weighted average of all the trade cost ratios ($\bar{\tau}$, for short), where the weights on the individual τ_{j1}s are defined by the second term in the denominator of (4.10).

The effect of endowments on relative outputs can be shown by using equation (4.10) to extend equations (4.8) into

$$\hat{q}_j - \hat{q}_1 = \frac{\beta_{j1}}{1+\tau_{j1}}\left(\theta_{Hj} - \theta_{H1}\right)\varphi_{HL}\left(\hat{v}_H - \hat{v}_L\right) \qquad (4.11)$$

which are of the same form as (3.13), but there are now n – 1 of them. As in (3.13), price-ratio elasticities pull in two directions. The higher is the trade cost ratio, τ_{j1}, the smaller are the effects of changes in the relative producer prices of any pair of goods j and 1 on the relative outputs of those goods. But higher trade cost ratios also amplify the effects of changes in endowments on factor prices and hence on producer prices, by enlarging φ_{HL}. The amplification depends not on τ_{j1}, though, but on $\bar{\tau}$.

Given a country's $\bar{\tau}$ (and thus its φ_{HL}), the effect of the trade cost ratio for one good, τ_{j1}, being higher than for another good, τ_{k1}, is to make the output of j less responsive to endowments than the output of k, other things being equal. In a country whose τ_{j1}s were uniformly higher than those of some other country, the overall composition of output would likewise be less sensitive to endowments: the lesser responsiveness of sales to producer prices would outweigh the greater responsiveness of factor prices to

[31] In the second term of the denominator of (4.10), the elements that are summed need not be positive for each of the goods j, since with more than two goods the λ and θ difference terms need not have the same signs, but its sum over all goods, and hence (4.10) as a whole, will usually be positive.

endowments.[32] Comparing countries with different $\bar{\tau}$ s but uneven differences in τ_{j1}s, however, the outputs of goods with the same τ_{j1} in both countries would respond more to endowments in the country with the higher $\bar{\tau}$. For example, the output of (and thus trade in) goods with low trade costs could vary more with endowments in countries with mainly high trade costs than in countries with mainly low trade costs.

The effects on factor prices of changes in own trade costs can be shown by modifying equation (3.16) from the 2x2 model into

$$\hat{w}_H - \hat{w}_L = -\varphi_{HL} \sum_{j=2}^{n} (\lambda_{Hj} - \lambda_{Lj}) \beta_{j1} \left[\frac{1}{1+\tau_{j1}} \left(\hat{\hat{\tau}}_j - \hat{\hat{\tau}}_1 \right) + \frac{\tau_{j1}}{1+\tau_{j1}} \left(\hat{\hat{t}}_j - \hat{\hat{t}}_1 \right) \right] \qquad (4.12)$$

Each of the $n - 1$ terms covered by the summation sign shows the effect of changes in own trade costs for a particular good (relative to the numeraire good) on the relative demand for the two factors, which depends also on its purchaser-price elasticity (β_{j1}) and its shares of economy-wide use of the factors ($\lambda_{Hj} - \lambda_{Lj}$). The summation gives the total effect of the trade cost changes on relative factor demand, which φ_{HL} converts into the change in relative factor prices. In the simplest case, in which the numeraire good 1 is the least factor-*H* intensive and relative trade costs increase for other goods, the outcome, as usual, is a reduction in the relative price of *H*.

In a country with a higher $\bar{\tau}$, changes in relative ad-valorem trade costs would have smaller effects on factor prices, as explained in section 3.3 (and despite the apparent ambiguity of equation (4.12)): the impact of changes in ad-valorem trade costs on relative producer prices does not depend on τ_{j1}s, so the only result of generally higher τ_{j1}s would be stronger negative feedback from the initial factor price shock. Changes in relative per-unit trade costs, however, would have a larger effect on factor prices in a country with uniformly higher τ_{j1}s: the impact on purchaser prices would be greater, via the $\tau_{j1}/(1 + \tau_{j1})$ term in the square brackets of (4.12); and a higher $\bar{\tau}$ would raise φ_{HL}. With τ_{j1}s higher on average, but not uniformly, the effects of changes in per-unit trade costs on factor prices could be increased, decreased or reversed, depending on how the differences in τ_{j1}s matched up with the changes in per-unit trade costs.

The effects on factor prices of changes in foreign goods prices can be described by

$$\hat{w}_H - \hat{w}_L = \varphi_{HL} \sum_{j=2}^{n} (\lambda_{Hj} - \lambda_{Lj}) \left(\beta_{j1} - \gamma \right) \left(\hat{p}_j^* - \hat{p}_1^* \right) \qquad (4.13)$$

which is similar to equation (3.20), except that the terms covered by the summation sign reflect the effects on the relative demand for factors of changes in the prices of many (rather than two) foreign goods. Again, in the simple case where the numeraire

[32] For the same reasons as in the 2x2 model, which were explained in connection with equation (3.14). An increased (say) endowment of *H*, by raising the output of *H*-intensive goods, lowers their purchaser prices, to a degree that depends on the β_{j1}s, and also their producer prices, to a degree that increases with the τ_{j1}s, which in turn lowers the relative factor price of *H*. Higher τ_{j1}s thus amplify the fall in the relative price of *H*, which causes more of the increased endowment of *H* to be absorbed by *H*-using changes in technique and less to be absorbed by shifts in the composition of output.

good 1 is the least factor-*H* intensive and relative foreign prices rise for other goods, the outcome, as usual, is an increase in the relative price of *H*.

In a country with uniformly higher τ_{j1}s, the effect of a given set of changes in foreign prices on factor prices would be larger, as suggested by equation (4.13), in which φ_{HL} would be increased by a higher $\bar{\tau}$. However, the underlying mechanisms, discussed in section 3.4, are not visible in (4.13). Higher τ_{j1}s increase the impact of changes in foreign purchaser prices by amplifying the related changes in producer prices (which are what matter for factor prices), as can be seen by rewriting (4.13) as

$$\hat{w}_H - \hat{w}_L = \varphi_{HL} \sum_{j=2}^{n} (\lambda_{Hj} - \lambda_{Lj}) \frac{\beta_{j1}}{1+\tau_{j1}} \left[(1+\tau_{j1}) \left(1 - \frac{\gamma}{\beta_{j1}} \right) (\hat{p}_j^* - \hat{p}_1^*) \right] \qquad (4.14)$$

whose square-bracketed term shows, as in equation (3.21), the effect of foreign prices on producer prices, which rises with τ_{j1}. A neat reduced form for the terms to the left of the square bracket cannot be derived in higher dimensions, but as in the 2x2 model higher τ_{j1}s strengthen the negative feedback from the initial factor price shock, though not by enough to reduce the net effect of higher τ_{j1}s. If the higher level of τ_{j1}s were not uniform, the outcome would again depend on how the pattern of differences in the τ_{j1}s matched up with the pattern of changes in foreign prices.

The preceding analysis of the consequences of differences among goods and countries in price-ratio elasticities involved only two factors. Extension to many factors would be complicated, especially with uneven differences in τ_{j1}s across countries, and would create more possible exceptions to any generalisations, but would be unlikely to alter the basic conclusions. In countries with generally higher per-unit trade cost ratios, the composition of output would still tend to be less affected by variation in endowments (though the composition of trade might be more affected), and factor prices would still tend to respond more to variation in per-unit trade costs and foreign prices (but to respond less to variation in ad-valorem trade costs).

5. Protection of the home market

The effects of price-ratio elasticities in HO models were analysed in sections 3 and 4 on the assumption that each country sells in a single world market of which for any good it has only a tiny share. These sections thus abstracted from the usual focus of attention in the analysis of trade costs, which is the differences that such costs cause between prices in different markets and particularly between the world market and the home market, where trade costs protect a country's firms from foreign competition and thus discourage both importing and exporting.

The purpose of this section is to explore how the usual protective effects of trade costs combine and interact with those of independent trade costs by asking three questions. First, how does a protected home market alter the conclusions of earlier sections about the effects of low price-ratio elasticities (short answer: hardly at all). Second, how in the BHO model do the effects of protection compare with the effects of low price-ratio elasticities (short answer: similar in some ways, but different in other ways). Third, how do the effects of protection in the BHO model compare with those in the HOS model (short answer: broadly similar, but with some significant differences).

The degree of protection of the home market and the size of price-ratio elasticities are determined by different but overlapping aspects of trade costs. Protection depends on trade costs, both per-unit and ad-valorem, that fall only on foreign suppliers (such as international transport costs and tariffs) and not on internal trade costs that are paid by both home and foreign suppliers. Price-ratio elasticities, by contrast, depend on both international and internal trade costs, but only on per-unit costs. Greater protection may thus be associated with either lower or higher price-ratio elasticities, depending on the composition as well as the general level of trade costs.

In extending the BHO model to include protection, countries of origin are indexed as before by a superscript and markets (countries of destination) by a second superscript, and * refers to the rest of the world. Country *z*'s exports of good *j* to all destinations are thus q_j^{z*}, its home sales are q_j^{zz}, and its imports from all origins are q_j^{*z}.

5.1 Modified producer-price demand elasticities

International trade costs increase the share of domestic firms in home market sales by making their varieties relatively cheaper, which also increases the effect of the prices of a country's own varieties on the average prices of goods in its home market. The purchaser-price demand elasticities for domestic firms in home markets thus depend on substitutability between goods (γ) as well as among varieties (βs). Where they lie between the βs and γ, and how this depends on the sizes of market shares, is shown by the weighted harmonic mean in equation (2.4), whose spirit can be conveyed for the present purpose by a simpler weighted arithmetic mean,

$$\tilde{\varepsilon}_{j1}^{zz} = s_{j1}^{zz}\gamma + \left(1 - s_{j1}^{zz}\right)\beta_{j1} \qquad (5.1)$$

where β_{j1} is (as before) an average across the two goods and s_{j1}^{zz} is country *z*'s average home market share for these goods.[33] Since usually $\beta_{j1} > \gamma$, a higher s_{j1}^{zz} reduces $\tilde{\varepsilon}_{j1}^{zz}$, and if $s_{j1}^{zz} = 1$, as in a closed economy, $\tilde{\varepsilon}_{j1}^{zz} = \gamma$.

The producer-price demand elasticity in the home market

$$\varepsilon_{j1}^{zz} = \frac{\beta_{j1} - s_{j1}^{zz}\left(\beta_{j1} - \gamma\right)}{1 + \tau_{j1}^{zz}} \qquad (5.2)$$

(whose numerator is just a rearrangement of 5.1), can thus be compared with that in the export market for the same pair of goods, which because for most countries most export market shares are small is

[33] Equation (5.1) is highly simplified. It does not show explicitly how market shares depend on the βs and on trade costs. Nor does it convey that the individual values of the market shares matter, as well as their average: in the harmonic mean of equation (2.4), the elasticity is more strongly influenced by the β of the good with the lower market share. In consequence, if equation (2.4) were applied to many pairs of goods, the elasticities would depend on which good was chosen as numeraire, illustrating the limitations mentioned earlier of the Hicks elasticity of substitution.

$$\varepsilon_{j1}^{z*} = \frac{\beta_{j1}}{1+\tau_{j1}^{z*}} \tag{5.3}$$

as in earlier sections (apart from the addition of * superscripts). The lower purchaser-price elasticity tends to make (5.2) smaller than (5.3), but the price-ratio elasticity in the home market is also likely to be lower, because τ_{j1}^{zz} depends only on internal trade costs, while τ_{j1}^{z*} also includes international trade costs. The difference between ε_{j1}^{zz} and ε_{j1}^{z*} could thus in principle be in either direction.

It is also convenient to write an expression that shows how country *z*'s relative sales of goods *j* and 1 in all markets, and thus its relative total outputs of these goods, vary with its relative producer prices,

$$\varepsilon_{j1}^{z} = \frac{\beta_{j1} - s_{j1}^{z}\left(\beta_{j1} - \gamma\right)}{1+\tau_{j1}^{z}} \tag{5.4}$$

in which s_{j1}^{z} and τ_{j1}^{z} are averages across export and home markets, weighted by sales, and ε_{j1}^{z} lies between ε_{j1}^{z*} and ε_{j1}^{zz}. For a given pair of goods, the total output elasticity ε_{j1}^{z} need not be lower than the export market elasticity ε_{j1}^{z*} (since ε_{j1}^{zz} could be greater than ε_{j1}^{z*}). However, the average output elasticity of a protected economy is reduced by the existence of many non-traded goods: the output elasticities for these goods are just their home market elasticities, which are low both because the numerator of (5.2) is reduced by the high market share and because for non-traded goods even internal trade costs are likely to be high, raising the denominator of (5.2).

5.2 Effects of variation in endowments

In the BHO model, the presence of a non-traded sector increases the effect of variation in endowments on factor prices. Lower producer-price demand elasticities mean less scope for absorbing changes in endowments by altering the composition of output. More of the change in endowments thus needs to be absorbed by changes in technique within sectors, to induce which requires larger changes in relative factor prices. This can be illustrated by modifying equation (4.10) into

$$\varphi_{HL}^{z} = \frac{\hat{w}_H^z - \hat{w}_L^z}{\hat{v}_H^z - \hat{v}_L^z} = -\frac{1}{\sum_{j=1}^{n}(\lambda_{Hj}\sigma_{HjH} - \lambda_{Lj}\sigma_{LjH}) + \sum_{j=2}^{n}(\lambda_{Hj} - \lambda_{Lj})\varepsilon_{j1}^{z}(\theta_{Hj} - \theta_{H1})} \tag{5.5}$$

in which the export market elasticities (5.3) are replaced by output elasticities (5.4), which on average are smaller.

Higher overall trade costs, say in one country than another, tend to amplify the effect of variation in endowments on factor prices, regardless of whether the higher trade costs act through greater protection (reducing purchaser-price elasticities by raising home market shares) or through lowering price-ratio elasticities, since by either route producer-price demand elasticities are reduced. But the two routes are independent of one another: given the sizes of market shares, higher per-unit trade cost ratios increase the effect of variation in endowments on factor prices, much as in earlier sections.

The effect of protection on relative factor prices is basically the same in BHO as in HOS: scarce factors gain, and by more, the higher the degree of protection. Given the negative sign of (5.5), a lower average ε_{j1}^{z} causes the relative price of a factor to rise faster with its relative scarcity. In a two-factor HOS model, the relative price of the scarce factor also rises monotonically with trade costs, with either two goods or many goods (Markusen and Venables, 2007). In a one-cone HOS model, the existence of a non-traded sector does not affect factor prices, unlike the BHO model, but in a multi-cone HOS model a non-traded sector makes it more likely that factor prices will vary among countries with their endowments (Courant and Deardorff, 1990).

In the BHO model, as in the HOS model, the effects of a protected home market on the relationship between endowments and the composition of output are the opposite of those on the relationship between endowments and factor prices: factor prices vary more and the composition of output varies less. In particular, there is less output of goods that use a country's abundant factors intensively, and vice versa, and hence less trade. Higher overall trade costs reduce the responsiveness of output composition to endowments through both greater protection and lower price-ratio elasticities.

However, the reduced effect of endowments on the composition of output caused by greater protection is concentrated on the non-traded sector and the home market. The outputs of goods whose home markets are less protected become more sensitive to endowments, as can be seen by rewriting equations (4.11) as

$$\hat{q}_j^z - \hat{q}_1^z = \frac{\beta_{j1} - s_{j1}^z\left(\beta_{j1} - \gamma\right)}{1 + \tau_{j1}^z}\left(\theta_{Hj} - \theta_{H1}\right)\varphi_{HL}^z\left(\hat{v}_H^z - \hat{v}_L^z\right) \tag{5.6}$$

Protection of home markets raises both s_{j1}^z and φ_{HL}^z. For lightly protected goods, the rise in s_{j1}^z and the consequent fall in the responsiveness of relative outputs to relative producer prices is outweighed by the rise in φ_{HL}^z. Even for more protected goods, the responsiveness of exports to endowments may be as high as for less protected goods, despite the lower responsiveness of their outputs. Comparative advantage is therefore revealed more clearly in the composition of exports than of output, as would be true also in a HOS model if non-traded goods were of average factor intensity.[34]

Protection of home markets also increases the effect of endowments on the sectoral composition of imports (though the responsiveness of imports to endowments in the BHO model is usually lower than that of exports, as shown in appendix E). Larger home market shares cause the average prices of goods to depend more on the prices of a country's own varieties. People therefore tend to consume more of goods that are intensive in their country's endowments and hence cheaper, and vice versa, reducing

[34] As explained in Wood and Berge (1997: note 5) and Davis and Weinstein (2001: 1426-7, 1442). The idea of comparative advantage being revealed in the composition of exports is due to Balassa (1965).

the volume of trade, as is also the case in the HOS model, in which trade costs raise the home prices of imports and lower the home prices of exportables.

Substantial home market shares, moreover, increase the effect on the composition of output of differences among countries in consumer preferences (including those due to differing incomes), since if home market shares were tiny, greater consumption of a particular good would result mainly in increased imports of that good. In the BHO model, such differences in output composition would affect factor prices – a stronger taste for skill-intensive goods in a country, for instance, would raise the wages of its skilled workers. In a one-cone HOS model, there would be no such effect on factor prices, just a reallocation of factors between sectors, although in a multi-cone model factor prices could be affected through a change in the pattern of specialisation.

5.3 Variation in trade costs and foreign prices

In the BHO model, protection of home markets reduces the effect on factor prices of changes in own trade costs, because more of these costs are shifted on to purchasers than in export markets with higher demand elasticities. This difference is related to that in HOS between a closed and an open economy in the incidence of taxes. It can be illustrated by modifying equation (4.12) into

$$\hat{w}_H^z - \hat{w}_L^z = -\varphi_{HL}^z \sum_{j=2}^{n} (\lambda_{Hj} - \lambda_{Lj})\left(\beta_{j1} - s_{j1}^z\left(\beta_{j1} - \gamma\right)\right)\left[\frac{1}{1+\tau_{j1}^z}\left(\hat{\hat{\tau}}_j^z - \hat{\hat{\tau}}_1^z\right) + \frac{\tau_{j1}^z}{1+\tau_{j1}^z}\left(\hat{\hat{t}}_j^z - \hat{\hat{t}}_1^z\right)\right] \tag{5.7}$$

where each purchaser-price elasticity, formerly β_{j1}, is replaced by $\beta_{j1} - s_{j1}^z\left(\beta_{j1} - \gamma\right)$, and the own trade costs are averages of those in home and export markets. Changes in relative own trade costs cause smaller shifts in relative factor demand, only partly offset by the increase in φ_{HL}^z, and thus smaller changes in relative factor prices.

In this case, the effects of higher trade costs through greater protection differ from the effects through lower price-ratio elasticities, which are the same as in earlier sections. As can be seen in equation (5.7), both a higher s_{j1}^z and a higher per-unit trade cost ratio τ_{j1}^z would reduce the effect on factor prices of a change in ad-valorem own trade costs. However, the effect on factor prices of a change in per-unit own trade costs, given s_{j1}^z, would be amplified by a higher τ_{j1}^z (which would give per-unit trade costs more influence on purchaser prices), whereas a higher s_{j1}^z would lessen the effect.

In the BHO model, the impact of changes in foreign prices on factor prices is reduced by protection of the home market, as can be illustrated by revising (4.13) into

$$\hat{w}_H^z - \hat{w}_L^z = \varphi_{HL}^z \sum_{j=2}^{n} (\lambda_{Hj} - \lambda_{Lj})\left(1 - s_{j1}^z\right)\left(\beta_{j1} - \gamma\right)\left(\hat{p}_j^* - \hat{p}_1^*\right) \tag{5.8}$$

to which the $1-s_{j1}^{z}$ terms have been added, and in which s_{j1}^{z}, p_j^* and p_1^* are weighted averages across markets. With a larger s_{j1}^{z}, a rise (say) in the relative foreign price of good *j* causes less of an increase in the relative sales of domestic producers because it causes less substitution towards domestic varieties (the β effect), which dominates the smaller rise in the overall price of good *j* and thus smaller fall in overall sales of good *j* (the γ effect).[35] The effect of this reduction in the impact on relative factor demand is only partly offset by the increase in φ_{HL}^{z} caused by greater protection.

The effects of higher overall trade costs through greater protection again differ from the effects through lower price-ratio elasticities, which are the same as were explained with regard to equation (4.14). For a given set of s_{j1}^{z}s, generally higher τ_{j1}^{z}s would increase the effect of changes in foreign prices on factor prices, the mechanism (not visible in 5.8) being the amplified effect of changes in purchaser prices on producer prices (which could increase the degree of magnification in a HOS model, too, as was shown in equation 3.22). By contrast, for a given τ_{j1}^{z}s, generally higher s_{j1}^{z}s reduce the effect of foreign prices on factor prices.

In both the BHO and the HOS models, protection of the home market tends to reduce the effect of foreign prices on factor prices, but the details of the relationship differ significantly between the models. In neither model can foreign prices directly affect the domestic prices of non-traded goods, and in both models foreign prices have big effects on the prices of goods with small home market shares. Between these market share extremes, however, the effect of foreign prices on the domestic prices of goods and factors remains high in the HOS model, whereas in the BHO model it declines as home market shares rise. In the BHO model, in other words, it is not just the level of foreign prices but also the volume of foreign trade that matters for the determination of factor prices (which makes factor content of trade calculations relevant).[36]

6. Conclusion

HO logic implies that relative trade costs are independent of relative production costs. Allowing for this independence enables the insights of Heckscher and Ohlin to be conveyed more realistically than in the standard HOS model with iceberg trade costs. The key contribution of independent trade costs, embodied in the concept of the price-ratio elasticity, is to amplify the effects of imperfect substitutability among different national varieties of similar goods, but independent trade costs have other effects, too, most notably to increase the effects on factor prices of changes in foreign prices.

The practical relevance of the analysis in this paper obviously depends on the size of independent trade costs compared to other components of purchaser prices, including

---

[35] This convenient formulation assumes β_j to be the elasticity of q_j^z / q_j with respect to p_j^z / p_j, and so is not strictly consistent with the utility function in equation (2.3), which implies that β_j is the elasticity of q_j^z / q_j^* with respect to p_j^z / p_j^*. The elasticity of q_j^z / q_j with respect to p_j^z / p_j^* does decline as s_j^z rises, but not so neatly as is implied by $(1-s_j^z)\beta_j$.

[36] Factor content of trade calculations were debated in JIE (2000). Even in a BHO model, the biggest practical challenge for such calculations is non-competing imports (Wood, 1994: 72-4).

ad-valorem trade costs. On the basis of the limited evidence available, it appears that independent trade costs – international plus internal – typically account for about half of purchaser prices (though with wide variation among goods and countries), and thus have a big effect on how the world works. Further research might show the true share to be less than a half, implying that the effects of independent trade costs on outcomes are smaller, but the share and the effects are surely not trivial.

Econometric applications of HO theory need to allow for independent trade costs. To estimate how endowments affect the structure of output and trade, the BHO model, as outlined in appendix D, suggests a specification that is similar to those of Keesing and Sherk (1971) and Wood and Berge (1997) and rather different from those of Trefler (1995) and Davis and Weinstein (2001), who start from the Vanek (1968) version of HOS. In estimating the effects of trade on factor prices, the BHO model implies that the amount of trade matters (not just world prices), and that it is necessary to control for changes in endowments and in domestic demand. In disentangling the effects of trade and technology on factor prices, the BHO model, unlike HOS, requires attention to the factor bias as well as the sector bias of technical change.

This paper has used one particular analytical framework and addressed only a limited set of issues. Other frameworks, including hybrids of HO and other elements, could shed more light on the issues analysed in this paper and extend the range of issues that could be addressed. The incidence of independent trade costs could be investigated more fully, as could their welfare effects (as in Irarrazabal *et al.*, 2011), with possible policy implications. The reduction by independent trade costs of the economic impact of differences between places in relative production costs seems relevant also to other models of trade and economic geography, including gravity models, and perhaps even to international macroeconomics (Obstfeld and Rogoff, 2000).

Appendix A: HO foundations of elasticity reduction

That HO theory implies that price-ratio elasticities are normally less than unity can be illustrated formally in the small-changes framework of sections 3 and 4. Define $\tilde{\theta}_{ij}$ as the share of factor *i* in the purchaser price of good *j*, including the factor content of its trade costs, all of which are assumed in this appendix to be per-unit costs arising from the use of produced trade services (omitting ad-valorem trade costs, whose effects are analysed in appendix C). This share term can be written as a weighted average

$$\tilde{\theta}_{ij} = (1-\theta_{tj})\theta_{ij} + \theta_{tj}\theta_{it}^{j} \qquad \text{(A.1)}$$

of θ_{ij}, which as before is the share of *i* in c_j, the production cost or producer price of good *j*, and of θ_{it}^{j}, which is the share of *i* in the cost of the trade services used by good *j*, with the weights determined by the share, θ_{tj}, of trade costs in the purchaser price, p_j. The equation showing the effect of changes in relative factor prices on relative purchaser prices, which if written in the notation of sections 3 or 4 would be

$$\hat{p}_j - \hat{p}_1 = \frac{1}{1+\tau_{j1}}(\theta_{ij} - \theta_{i1})(\hat{w}_i - \hat{w}_1) \qquad \text{(A.2)}$$

where the price-ratio elasticity $\delta_{j1} = 1/(1+\tau_{j1})$, can thus be written instead as

$$\hat{p}_j - \hat{p}_1 = \left(\tilde{\theta}_{ij} - \tilde{\theta}_{i1}\right)\left(\hat{w}_i - \hat{w}_1\right) \quad \text{(A.3)}$$

implying $\delta_{j1} = \left(\tilde{\theta}_{ij} - \tilde{\theta}_{i1}\right)/\left(\theta_{ij} - \theta_{i1}\right)$. Using (A.1), equation (A.3) can be expanded to

$$\hat{p}_j - \hat{p}_1 = \left[\left((1-\theta_{tj})\theta_{ij} + \theta_{tj}\theta_{it}^{j}\right) - \left((1-\theta_{t1})\theta_{i1} + \theta_{t1}\theta_{it}^{1}\right)\right]\left(\hat{w}_i - \hat{w}_1\right) \quad \text{(A.4)}$$

which shows the effects of trade costs on the price-ratio elasticity.

The first question is whether trade costs are likely to affect the outcome by causing the price-ratio elasticity to differ from unity. Equation (A.4) shows the answer to be a clear 'yes'. Except in special cases, the necessary and sufficient conditions for $\delta_{j1} = 1$ are $\theta_{ij} = \theta_{it}^{j}$ and $\theta_{i1} = \theta_{it}^{1}$ (which reduce the square-bracketed term to $\theta_{ij} - \theta_{i1}$). These conditions require (a) each good to use its own sort of trade services and (b) the factor *i* intensity of the trade services used by each good to equal the factor *i* intensity of the production of that good. This is surely not even approximately true in reality.

The next question is the likely direction and size of the effect of trade costs. Suppose for simplicity that both goods use the same sort of trade services (with the same factor input proportions), though not necessarily equal amounts of trade services, so that θ_{it} loses its good-specific superscript. Equation (A.4) can then be rewritten as

$$\hat{p}_j - \hat{p}_1 = \left[(1-\theta_{t1})(\theta_{ij} - \theta_{i1}) - (\theta_{tj} - \theta_{t1})(\theta_{ij} - \theta_{it})\right]\left(\hat{w}_i - \hat{w}_1\right) \quad \text{(A.5)}$$

This equation implies that in most cases $\delta_{j1} < 1$, as is clearest if trade cost shares are assumed to be the same for both goods ($\theta_{tj} = \theta_{t1} = \theta_t$), reducing it to

$$\hat{p}_j - \hat{p}_1 = (1-\theta_t)(\theta_{ij} - \theta_{i1})\left(\hat{w}_i - \hat{w}_1\right) \quad \text{(A.6)}$$

Equation (A.6) neatly illustrates the basic HO reason for price-ratio elasticities being less than unity: changes in relative factor prices alter the relative production costs of the two goods, because the factor intensities of their production technologies differ, but do not alter their relative trade costs, because they use trade services of similar factor intensity. (What the factor intensity of trade services is matters little, as witness the absence of θ_{it} from A.6.) Relative purchaser prices – the sum of production costs and trade costs – thus change by proportionally less than relative production costs.

Equation (A.6) is identical in substance to equation (A.2), since $(1-\theta_t)$ and $1/(1 + \tau_{j1})$ are two ways of writing the average share of production costs in the purchaser prices of the goods. Both equations show that how far below unity the price-ratio elasticity lies depends on the size of (per-unit) trade costs, relative to production costs.

The second term in the square brackets of equation (A.5) complicates the analysis. Unless one of its components is zero (equal trade cost shares for both goods, or equal factor *i* intensity of trade costs and good *j* production costs), it modifies the simple reducing effect of the first term. The direction and size of the modification depend on the signs and sizes of ($\theta_{tj} - \theta_{t1}$) and ($\theta_{ij} - \theta_{it}$), compared to the sign of ($\theta_{ij} - \theta_{i1}$), but there is no reason to suppose that it will generally be in any particular direction, so the presumption remains that price-ratio elasticities are normally less than unity.[37]

Equation (A.5) also shows that if the second term in square brackets were big enough, it could take the price-ratio elasticity out of the zero-unity range. One such possibility is 'amplification', where $\tilde{\theta}_{ij} - \tilde{\theta}_{i1}$ is larger in absolute size than (and of the same sign as) $\theta_{ij} - \theta_{i1}$, so that $\delta_{j1} > 1$ and p_j/p_1 changes by proportionally more than c_j/c_1. The other possibility is 'reversal', where $\tilde{\theta}_{ij} - \tilde{\theta}_{i1}$ and $\theta_{ij} - \theta_{i1}$ have different signs, so that $\delta_{j1} < 0$ and p_j/p_1 and c_j/c_1 move in opposite directions. Both outcomes are likely to be rare, but cannot be ruled out (Appendix B shows how they might arise).

Appendix B: Effects of variation of per-unit trade costs with production costs

This appendix provides support for the analysis in section 2.3 of the effects on price-ratio elasticities of changes in the relative trade costs and average trade cost ratios of pairs of goods that are correlated with changes in their relative production costs. It assumes that all trade costs are per-unit and arise from the use of produced services (the effects of ad-valorem trade costs are analysed in Appendix C).

Equation (2.9) can be rewritten as

$$p = c\,\frac{1+\tau(c)\sqrt{t(c)/c}}{1+\tau(c)\sqrt{c/t(c)}} \tag{B.1}$$

in which both trade cost ratios are functions of c. In the simple case where $t(c) = \breve{t}\,c^{\eta}$ and $\tau(c) = \breve{\tau}\,c^{\xi}$, the price-ratio elasticity can then be derived as

$$\delta = \frac{1+\breve{\tau}\left[\breve{t}^{-0.5}\left(0.5(1+\eta)-\xi\right)c^{(0.5(1-\eta)+\xi)} + \breve{t}^{0.5}\left(0.5(1+\eta)+\xi\right)c^{(-0.5(1-\eta)+\xi)}\right] + \eta\breve{\tau}^{2}c^{2\xi}}{1+\breve{\tau}\left(\breve{t}^{-0.5}c^{(0.5(1-\eta)+\xi)} + \breve{t}^{0.5}c^{(-0.5(1-\eta)+\xi)}\right) + \breve{\tau}^{2}c^{2\xi}} \tag{B.2}$$

The meaning of equation (B.2) is clearest when $t = c^{1-\eta}$, which reduces it to

$$\delta = \frac{1+\eta\breve{\tau}\,c^{\xi}}{1+\breve{\tau}\,c^{\xi}} \tag{B.3}$$

The effect on δ of relative trade costs, t, being influenced by relative producer prices, c, can be shown by assuming also $\xi = 0$, which makes (B.3), with subscripts restored,

$$\delta_{jk} = \frac{1+\eta_{jk}\breve{\tau}_{jk}}{1+\breve{\tau}_{jk}} \tag{B.4}$$

---

[37] Nor would this presumption be affected by restoring the goods superscripts on θ_{it} (different goods using different sorts as well as different amounts of trade services).

where η_{jk} is the elasticity of t_j/t_k with respect to c_j/c_k. A positive η increases δ_{jk}, and if $\eta = 1$, the price-ratio elasticity is unity, with $\eta > 1$ yielding amplification ($\delta_{jk} > 1$). A negative η reduces δ_{jk}, and if the absolute value of η were to exceed $1/\tau_{jk}$, there would be reversal ($\delta_{jk} < 0$). Like equation (2.11), to which it reduces if $\eta_{jk} = 0$, equation (B.4) is the extremum of a shallow curve with respect to c_j/c_k and thus in effect a constant elasticity, so long as η_{jk} and τ_{jk} do not change.[38]

The effect on δ of the average trade cost ratio, τ, being influenced by relative producer prices can be shown by assuming $\eta_{jk} = 0$, which makes (B.3), with subscripts restored,

$$\delta_{jk} = \frac{1}{1 + \breve{\tau}_{jk}\left(c_j/c_k\right)^{\xi_{jk}}} \tag{B.5}$$

In this case, the function $\delta(c)$ is not U-shaped, but slopes up or down, depending on whether ξ_{jk} is negative or positive. If average trade costs rise with c_j/c_k, the price-ratio elasticity falls, and vice versa. If ξ were big, moreover, the value of δ in (B.2) over some ranges of c_j/c_k could lie above unity or below zero.

If δ varied a lot with respect to c, because ξ was far from zero, it would be necessary to recognise that all the equations above refer to point elasticities. For example, in assessing the effect of a big difference in endowments on the output structures of two countries, the relevant price-ratio elasticity would be between the elasticities relevant to small endowment changes in each of the countries individually, a complication that can be ignored if the elasticity is more or less constant.

Appendix C: Effect of ad-valorem trade costs on price-ratio elasticities

This appendix provides support for the analysis in section 2.3. Assuming ad-valorem. trade costs to be based on the producer price, the purchaser price of any good *j* is

$$p_j = \left(1 + \ddot{\tau}_j\right)c_j + \tilde{t}_j \tag{C.1}$$

where $\ddot{\tau}_j$ is the rate of the ad-valorem trade costs and $\tilde{t}_j$ is the value of per-unit trade costs. Assuming for simplicity that $\eta_{jk} = \xi_{jk} = 0$ (see Appendix B), the price-ratio elasticity for goods *j* and *k* becomes

$$\delta = \frac{1 + 0.5\tilde{\tau}\left(\sqrt{c/\tilde{t}} + \sqrt{\tilde{t}/c}\right) + \ddot{\tau}_j\left(1 + 0.5\tilde{\tau}\sqrt{c/\tilde{t}}\right) + \ddot{\tau}_1\left(1 + 0.5\tilde{\tau}\sqrt{\tilde{t}/c}\right) + \ddot{\tau}_j\ddot{\tau}_1}{1 + \tilde{\tau}\left(\sqrt{c/\tilde{t}} + \sqrt{\tilde{t}/c}\right) + \ddot{\tau}_j\left(1 + \tilde{\tau}\sqrt{c/\tilde{t}}\right) + \ddot{\tau}_1\left(1 + \tilde{\tau}\sqrt{\tilde{t}/c}\right) + \ddot{\tau}_j\ddot{\tau}_1 + \tilde{\tau}^2} \tag{C.2}$$

where $\tilde{t} = \tilde{t}_j/\tilde{t}_k$ and $\tilde{\tau} = \sqrt{\left(\tilde{t}_j/c_j\right)\left(\tilde{t}_k/c_k\right)}$. The meaning of (C.2) is clearest when $c = t$ and $\ddot{\tau}_j = \ddot{\tau}_1 = \ddot{\tau}$, which makes it (with subscripts restored)

---

[38] A positive value of η_{jk} makes δ_{jk} vary less with τ_{jk} than when $\eta_{jk} = 0$ (provided $\eta_{jk} < 2$). A negative η_{jk} amplifies the effect on δ_{jk} of variation in τ_{jk}.

$$\delta_{jk} = \frac{1}{1+\tilde{\tau}_{jk}/(1+\ddot{\tau})} \qquad \text{(C.3)}$$

Given $\tilde{\tau}_{jk}$, a higher $\ddot{\tau}$ (or, more usefully but more approximately, as in section 2.3, a higher average of the two ad-valorem trade cost rates, $\ddot{\tau}_{jk}$) raises δ_{jk}. If all trade costs were ad-valorem and $\tilde{\tau}_{jk} = 0$, the price-ratio elasticity would be unity. Ad-valorem trade costs thus act in the same direction as a positive value of η (the elasticity of t with respect to c). However, extreme values of η could take δ out of the range between zero and unity, while no values of $\ddot{\tau}_j$ and $\ddot{\tau}_k$ could do so.

In reality, ad-valorem trade costs are not paid just at the factory gate (that is, based on c's, as assumed above), but also at later stages of the trading process, of which there may be several – for example, from factory to port of embarkation, from there to port of destination, and thence to the shops. A full analysis of the effects of ad-valorem trade costs would need to treat δ_{jk} as the product of a series of sub-elasticities, one for each stage in the trading process (but ignoring ad-valorem taxes at the final stage, on purchaser prices, which have no effect on the overall price-ratio elasticity).[39]

Appendix D: A simplified BHO model with many goods and many factors

Higher-dimensional HO models are notoriously complicated, and clear results require restrictions on the parameters and/or simplifying assumptions (Bliss, 2007: 128). The complexity is most challenging in the factor dimension because, with more than two factors, a change in the endowment of one factor could in principle alter its own price and the prices of other factors in almost any way. Section 4.2 simplifies by assuming that there are only two factors, but an alternative approach to simplification, retaining many factors but omitting some of the relationships among them, may be more useful for some purposes.

Equations (4.7) can be substituted into equations (4.4) to yield

$$\hat{v}_i - \hat{v}_1 = -\sum_{h=2}^{m}\left[\sum_{j=1}^{n}(\lambda_{ij}\sigma_{ijh} - \lambda_{1j}\sigma_{1jh})\right](\hat{w}_h - \hat{w}_1) - \sum_{j=2}^{n}(\lambda_{ij} - \lambda_{1j})\frac{\beta_{j1}}{1+\tau_{j1}}\sum_{h=2}^{m}(\theta_{hj} - \theta_{h1})(\hat{w}_h - \hat{w}_1) \qquad \text{(D.1)}$$

a set of $m - 1$ equations which implicitly describes the relationship between all factor endowments and all factor prices. The simplification suggested here is to reduce each of these equations to a relationship between the relative prices and endowments of a single pair of factors, i and 1. It involves suppressing the two summations over h: in the first rhs term omitting the effects on the choice of technique as between factors i and 1 of the prices of all factors other than i and 1; and in the second term omitting the effects on the relative prices and hence the relative outputs of goods j and 1 of the relative prices of all factors other than i and 1.

[39] Ad-valorem sales taxes at different rates on goods affect the relative prices paid by consumers, but proportional changes in net-of-tax price ratios cause equal proportional changes in gross-of-tax price ratios, so the sub-price-ratio elasticity at this stage is unity and does not affect the overall elasticity, which is the product of all the sub-elasticities.

With these simplifications, (D.1) can be rearranged to yield m – 1 elasticities, φ_{i1}, that show how the relative price of each factor pair depends on the corresponding relative endowment pair

$$\varphi_{i1} = -\frac{\hat{w}_i - \hat{w}_1}{\hat{v}_i - \hat{v}_1} = \frac{1}{\sum_{j=1}^{n} (\lambda_{ij}\sigma_{iji} - \lambda_{1j}\sigma_{1ji}) + \sum_{j=2}^{n} (\lambda_{ij} - \lambda_{1j}) \frac{\beta_{j1}}{1+\tau_{j1}} (\theta_{ij} - \theta_{i1})} \qquad \text{(D.2)}$$

each equation being of the same form as the single two-factor equation (4.10).

Equations (4.11) describing how a single pair of factors influences the relative outputs of all pairs of goods then become

$$\hat{q}_j - \hat{q}_1 = \frac{\beta_{j1}}{1+\tau_{j1}} \sum_{i=2}^{m} (\theta_{ij} - \theta_{i1}) \varphi_{i1} (\hat{v}_i - \hat{v}_1) \qquad \text{(D.3)}$$

a set of n – 1 equations showing how the relative outputs of all pairs of goods depend on the relative endowments of all pairs of factors. The meaning of (D.3) can be made clearer by writing the equation for (say) good 2 as a series of m – 1 terms, each of which is a change in one factor endowment ratio multiplied by a coefficient

$$\hat{q}_2 - \hat{q}_1 = \frac{\beta_{21}}{1+\tau_{21}} (\theta_{22} - \theta_{21}) \varphi_{21} (\hat{v}_2 - \hat{v}_1) + ... + \frac{\beta_{21}}{1+\tau_{21}} (\theta_{m2} - \theta_{m1}) \varphi_{m1} (\hat{v}_m - \hat{v}_1) \qquad \text{(D.4)}$$

The negative signs on β_{21} and the φ_{i1}'s cancel, so the sign of each coefficient depends on that of its ($\theta_{i2} - \theta_{i1}$) term: if good 2 uses factor i more intensively than good 1, the coefficient is positive, and if less intensively it is negative. The ($\theta_{i2} - \theta_{i1}$) term affects also the size of the coefficient: the bigger the difference between goods 1 and 2 in the intensity of their use of factor i, the larger is the coefficient (and if the two goods were of equal factor intensity, the term would vanish). The size of each coefficient depends also on the size of the relevant φ_{i1}, which can vary among pairs of factors, depending on the parameters of equation (D.2). All the coefficients in (D.4) depend in the same way on $\beta_{21}/(1+\tau_{21})$: the less substitutable are different varieties of goods 1 and 2 and the higher the independent trade cost ratios of these goods, the smaller are the effects of all pairs of factor endowments on the relative outputs of goods 1 and 2.

Equation (D.3), with its hatted differences replaced by log ratios, is the specification in Wood and Berge (1997) and subsequent papers reviewed in Wood (2003: 168-78). It yields sensible coefficients and a good fit, albeit with only a few goods and factors, confirming that in practice the effects of endowments of many factors on the structure of output and trade can be estimated (as was shown earlier by Leamer, 1984), despite the potential theoretical difficulties of doing so.

Equations (4.12) and (4.13), showing the effects of own trade costs and foreign prices on the single pair of factor prices, become sets of m – 1 equations of similar form that show the effects on all pairs of factor prices

$$\hat{w}_i - \hat{w}_1 = -\varphi_{i1}\sum_{j=2}^{n}(\lambda_{ij} - \lambda_{1j})\beta_{j1}\left[\frac{1}{1+\tau_{j1}}\left(\hat{\hat{\tau}}_j - \hat{\hat{\tau}}_1\right) + \frac{\tau_{j1}}{1+\tau_{j1}}\left(\hat{\hat{t}}_j - \hat{\hat{t}}_1\right)\right] \quad \text{(D.5)}$$

$$\hat{w}_i - \hat{w}_1 = \varphi_{i1}\sum_{j=2}^{n}(\lambda_{ij} - \lambda_{1j})(\beta_{j1} - \gamma)(\hat{p}_j^* - \hat{p}_1^*) \quad \text{(D.6)}$$

The simplification that is buried in the φ_{i1}'s is that the change in the relative demand for each pair of factors resulting from the sum of all the changes in own trade costs or foreign prices maps simply into a change in the relative price of this particular pair of factors, with no spillover effects either to or from the prices of other factors.

Equations (D.2)–(D.6) show, for any numbers of goods and factors, how the structure of a country's output and its relative factor prices are influenced by its endowments, trade costs and foreign prices far more clearly than in the general higher-dimensional BHO model of section 4.1. This clarity was achieved, however, by cutting out parts of the factor market-clearing equations (D.1), making the simplified equations in this appendix less accurate. The inaccuracy can be explained by considering, say, the first term in equation (D.4)

$$\hat{q}_2 - \hat{q}_1 = \frac{\beta_{21}}{1+\tau_{21}}(\theta_{22} - \theta_{21})\varphi_{21}(\hat{v}_2 - \hat{v}_1) + \ldots$$

in the case of an increase in the endowment of factor 2. The expression for φ_{21} in (D.2), and more specifically the second term in its denominator, is inaccurate because a fall in the relative price of factor 2 as a result of its increased supply would also alter the relative prices of goods other than 1 and 2, so the fall in the relative price of factor 2 could be larger or smaller than (D.2) implies. Moreover, increasing the endowment of factor 2 would lower the prices of other factors for which it was a substitute (and raise the prices of factors for which it was a complement), which would affect the relative production costs of goods 1 and 2, and thus their relative purchaser prices and relative outputs. Equation (D.4) should thus be expanded to something like

$$\hat{q}_2 - \hat{q}_1 = \frac{\beta_{21}}{1+\tau_{21}}\left[(\theta_{22} - \theta_{21})\varphi_{21} + \sum_{i=3}^{m}(\theta_{i2} - \theta_{i1})\varphi_{i2}\right](\hat{v}_2 - \hat{v}_1) + \ldots \quad \text{(D.7)}$$

in which the added summation could be of either sign, since the φ_{i2}s could be of either sign (though most would probably be positive, since substitutability is more common than complementarity) and so could the ($\theta_{i2} - \theta_{i1}$)s.

For these reasons, the effect of a change in the relative endowments of any given pair of factors on the relative outputs of any given pair of goods is not exactly as specified in equations (D.2) and (D.3): it could be either larger or smaller. Much the same is true of the relationships in equations (D.6) between changes in relative foreign prices and in relative factor prices, because of the inaccuracy of the φ_{i1} terms. Shifts in the relative demand for factors 1 and 2 caused by changes in relative foreign prices could have larger or smaller effects on the prices of factors 1 and 2 than equations (D.2) and

(D.6) suggest. This is because changes in the prices of factors 1 and 2 will affect the prices of other factors for which they are substitutes or complements and thus alter the relative demands for, and prices of, these two.

The costs of these inaccuracies, relative to the benefits of this simplified version of the higher-dimensional model, depend on the purpose for which the model is to be used. The inaccuracies of the simplified many-factor model should not be forgotten, but for some practical purposes it may be more useful than either the general version in section 4.1 or the simplified version in section 4.2 with only two factors.

Appendix E: Export and import structure with a protected home market

A protected home market makes the sectoral structure of exports more sensitive to variation in endowments than in the small-market-share models of sections 3 and 4. The effect of endowments on exports is described by equations similar to (4.11), but with superscripts added:

$$\hat{q}_j^{z*} - \hat{q}_1^{z*} = \frac{\beta_{j1}}{1+\tau_{j1}^{z*}}\left(\theta_{Hj} - \theta_{H1}\right)\varphi_{HL}^z\left(\hat{v}_H^z - \hat{v}_L^z\right) \tag{E.1}$$

The producer-price demand elasticity for exports in each sector is unaltered by the existence of a protected home market, but the economy-wide responsiveness of factor prices (and producer prices) to endowments is increased (a greater φ_{HL}^z, from equation (5.5)), and hence so is the responsiveness of relative exports to endowments.

A protected home market also increases the effect of endowments on import structure, which is described by equations of the form[40]

$$\hat{q}_j^{*z} - \hat{q}_1^{*z} = -\frac{s_{j1}^{zz}\left(\beta_{j1} - \gamma\right)}{1+\tau_{j1}^{zz}}\left(\theta_{Hj} - \theta_{H1}\right)\varphi_{HL}^z\left(\hat{v}_H^z - \hat{v}_L^z\right) \tag{E.2}$$

With $\theta_{Hj} - \theta_{H1}$ positive, the sign is negative: as in any HO model, a larger endowment of a factor reduces imports of goods in whose production it is used intensively. The size of the effect of endowments increases with country *z*'s home market share for the goods concerned, and would be negligible if this share were close to zero.

The effect of endowments on the structure of imports in (E.2) is likely to be smaller than on the structure of exports in (E.1), since the fact that usually $\tau_{j1}^{zz} < \tau_{j1}^{z*}$ is in most cases substantially outweighed by the fact that $s_{j1}^{zz}\left(\beta_{j1} - \gamma\right) < \beta_{j1}$.

[40] To derive the purchaser-price demand elasticity (the numerator of the first term) in (E.2), rewrite equation (5.2) as the relationship in country *z*'s home market between foreign sales and foreign prices, denoting the (foreign) market share by *s**. Convert this into a relationship between foreign sales and the prices of country *z*'s varieties by subtracting *γ*, as explained in connection with equation (3.19), then rewrite *s** as (1 – *s*), simplify, and add a negative sign to define it like a substitution elasticity. The denominator of the first term in (E.2) is the same as in (5.2), since what matters for imports is how changes in the country's producer prices affect its purchaser prices in its home market.

References

Alchian, Armen, and William Allen (1964), *University Economics*, Belmont, California: Wadsworth.

Aldaz-Carroll, Enrique (2003), 'Getting things in proportion: essays on the development and application of Heckscher-Ohlin trade theory', D.Phil. thesis, University of Sussex.

Anderson, James, and Eric van Wincoop (2004), 'Trade costs', *Journal of Economic Literature*, 42/3: 691-751.

Armington, Paul (1969), 'A theory of demand for products distinguished by place of production', *IMF Staff Papers*, 16/1: 159-78.

Balassa, Bela (1965), 'Trade liberalisation and "revealed" comparative advantage', *The Manchester School*, 33/2: 99-123

Baldwin, Richard, and James Harrigan (2011), 'Zeros, quality and space: trade theory and trade evidence', *American Economic Journal: Microeconomics*, 3/2: 60-88.

Bernard, Andrew, Bradford Jensen, Stephen Redding and Peter Schott (2007), 'Firms in international trade', *Journal of Economic Perspectives*, 21/3: 105-30.

Blackorby, Charles, and Robert Russell (1989), 'Will the real elasticity of substitution please stand up? (A comparison of the Allen/Uzawa and Morishima elasticities), *American Economic Review*, 79/4: 882-8.

Bliss, Christopher (2007), *Trade, Growth, and Inequality*, Oxford: Oxford University Press.

Broda, Christian, and David Weinstein (2006), 'Globalisation and the gains from variety', *Quarterly Journal of Economics*, 121/2: 541-85.

Chaney, Thomas (2008), 'Distorted gravity: the intensive and extensive margins of international trade', *American Economic Review*, 98/4: 1707-21.

Courant, Paul, and Alan Deardorff (1990), 'On the likelihood of factor price equalisation with non-traded goods', *International Economic Review*, 31/3: 589-96.

Davis, Donald, and David Weinstein (2001), 'An account of global factor trade', *American Economic Review*, 91/5: 1423-53.

Deardorff, Alan (2006), 'Needs and means for a better workhorse trade model', slides for Graham Lecture, Princeton University (available on AD's website).

Dimaranan, Betina, Robert McDougall and Thomas Hertel (2011), 'Behavioral parameters', GTAP Data Base Documentation, ch. 20, downloaded 2 January 2012 from https://www.gtap.agecon.purdue.edu/databases/v6/v6_doco.asp

Feenstra, Robert, and John Romalis (2011), 'International prices and endogenous quality', unpublished paper, November 2011 version.

Harrison, Glen, Thomas Rutherford and David Tarr (1997), 'Quantifying the Uruguay round', *Economic Journal*, 107 (September): 1405-30.

Hummels, David, and Alexandre Skiba (2004), 'Shipping the good apples out? An empirical confirmation of the Alchian-Allen conjecture', *Journal of Political Economy*, 112/6: 1384-1402.

Irarrazabal, Alfonso, Andreas Moxnes and Luca David Opromolla (2010), 'The tip of the iceberg: a quantitative framework for estimating trade costs', unpublished paper, downloaded 4 January 2012 from http://dl.dropbox.com/u/3719891/tc_paper32.pdf

JIE (2000), Symposium by Deardorff, Krugman, Leamer and Panagariya on the factor content of trade, *Journal of International Economics*, 50/1: 17-116.

Jones, Ronald (1965), 'The structure of simple general equilibrium models', *Journal of Political Economy*, LXXIII/6: 557-72.

Keesing, Donald, and Donald Sherk (1971), 'Population density in patterns of trade and development', *American Economic Review*, 61/5: 956-61.

Krugman, Paul (1979), 'Increasing returns, monopolistic competition, and international trade, *Journal of International Economics*, 9/4: 469-79.

Leamer, Edward (1984), *Sources of International Comparative Advantage*, Cambridge: MIT Press.

Marjit, Sugata, and Biswajit Mandal (2009), 'General trading costs in pure theory of international trade', MPRA Paper 19290, University Library of Munich, Germany.

Markusen, James, and Anthony Venables (2007), 'Interacting factor endowments and trade costs: a multi-country, multi-good approach to trade theory', *Journal of International Economics*, 73/2: 333-54.

Matsuyama, Kiminori (2007), 'Beyond icebergs: towards a theory of biased globalisation', *Review of Economic Studies*, 74/1: 237-53.

Melitz, Marc (2003), 'The impact of trade on intra-industry reallocations and aggregate industry productivity', *Econometrica*, 71/6: 1695-1725.

Obstfeld, Maurice, and Kenneth Rogoff (2000), 'The six major puzzles in international macroeconomics: is there a common cause?', *NBER Macroeconomics Annual*, 15: 339-90.

Ohlin, Bertil (1967), *Interregional and International Trade*, Cambridge MA: Harvard University Press (revised edition).

Rauch, James, and Vitor Trindade (2003), 'Information, international substitutability, and globalisation', *American Economic Review*, 93/3: 775-91.

Robinson, Sherman, and Karen Thierfelder (1996), 'The trade-wage debate in a model with non-traded goods: making room for labor economists in trade theory', *TMD Discussion Paper 9*, Trade and Macroeconomics Division, Washington DC: IFPRI.

Romalis, John (2004), 'Factor proportions and the structure of commodity trade', *American Economic Review*, 94/1: 67-90.

Sato, Kazuo (1967), 'A two-level constant-elasticity-of-substitution production function', *Review of Economic Studies*, 34/2: 201-18.

Schott, Peter (2003), 'One size fits all? Heckscher-Ohlin specialisation in global production', *American Economic Review*, 93/2: 686-708.

Smith, Alasdair, and Adrian Wood (2009), 'Factor intensity similarity and the dimensionality of Heckscher-Ohlin models', unpublished paper.

Trefler, Daniel (1995), 'The case of the missing trade and other mysteries', *American Economic Review*, 85/5: 1029-46.

Vanek, Jaroslav (1968), 'The factor proportions theory: the *n*-factor case', *Kyklos*, 21/4: 749-56.

Venables, Anthony (2003), 'Winners and losers from regional integration agreements', *Economic Journal*, 113: 747-61.

Wood, Adrian (1994), *North-South Trade, Employment and Inequality: Changing Fortunes in a Skill-Driven World*, Oxford: Clarendon Press.

Wood, Adrian (2003), 'Could Africa be like America?', in Boris Pleskovic and Nicholas Stern (eds), *Annual Bank Conference on Development Economics 2003*, World Bank and Oxford University Press.

Wood, Adrian, and Kersti Berge (1997), 'Exporting manufactures: human resources, natural resources, and trade policy', *Journal of Development Studies* 34/1: 35-59.