Working Paper Number 137

Looking ahead optimally in allocating aid

Adrian Wood[1]*

The Collier-Dollar approach to aid allocation among countries has been less than fully embraced by donors – even those focused on poverty reduction – partly because it conflicts with the approach to aid allocation implied by the Millennium Development Goals. These two approaches are shown to be special cases of a more general model of aid allocation, in which donors care about future as well as current poverty. This model is illustratively applied to data for developing regions. Adding a poverty decline adjustment to the allocation formulae used by donors could resolve the conflict between the two approaches.

October 2006 (revised July 2007)

* Queen Elizabeth House, University of Oxford.

1. Introduction

In a pair of brilliant and influential articles, Collier and Dollar (2001, 2002) proposed and illustrated empirically a formal model for allocating total world aid among recipient countries in such a way as to maximise its impact on the reduction of world poverty. Their model has become widely known in the aid donor community, and has influenced the actions of important development agencies.

The actual allocation of aid, however, continues to diverge widely from that suggested by the Collier-Dollar (C-D) model, even among donors whose main objective is the reduction of world poverty. One consistent feature of this divergence is the allocation of more aid to Africa than the C-D model recommends, and correspondingly less aid to other poor countries, particularly those in South Asia. This emphasis was reinforced by the Commission for Africa (2005) and Millennium Project (2005), and by the 2005 G8 and UN Millennium Summits, which all called for more aid to Africa because of its slow progress towards the Millennium Development Goals (MDGs).

The reason for this striking divergence, it will be argued in this paper, is that the C-D model does not fully capture the preferences of donors (and their tax-payers) about reduction of world poverty. C-D omit time, so that the relative needs for aid of different countries depend only on their current levels of poverty, whereas donors and people care – for intellectually and morally defensible reasons – about both current and future levels of poverty. Specifically, donors are attracted by the MDGs and allocate more aid to Africa and less aid to Asia than would be justified by the current relative poverty of these regions (a) because what they want to reduce is some combination of current and future poverty and (b) because they believe that in the absence of aid poverty would decline more slowly in Africa than in Asia.

At present, however, donors are struggling to apply their preferences and beliefs to aid allocation in a consistent way. In addition to the time-related inconsistency of principle between the MDGs and the C-D-based allocation formulae used by donors, there is a practical inconsistency between the target-based method of the MDGs and the optimising method of the C-D model. A key purpose of this paper is to help donors to act more consistently by suggesting a way of combining the C-D and MDG approaches into a single approach that retains the strong points of each of them.

The essence of the combined approach is conveyed by Figure 1. The vertical axis measures the amount of poverty in a country (poverty rate multiplied by population), and the horizontal axis measures time, from the present to a planning horizon, T. The aspect that matters for aid allocation in the C-D model is the initial height, H_0: the greater is poverty today, the more aid should the country get. The aspect that matters for MDG-based aid allocation is the slope of H_0H_T: the more slowly would poverty otherwise decline in the future, the more aid should the country get. The combined approach proposed in this paper is to base allocations on the area $0H_0H_TT$, which depends both on a country's initial level of poverty and on its projected rate of decline in poverty: the larger is this area, the more aid should the country get.

The rest of the paper develops this simple idea. Section 2 recapitulates the C-D model, and section 3 reviews its influence – and the limits of its influence – on donors. Section 4 develops a forward-looking allocation model, of which it is shown

in section 5 that the C-D and MDG-based approaches are special cases. Section 6 applies this model illustratively to data for developing regions, and compares its allocations with those of the C-D and MDG approaches. Section 7 shows how donors could modify their existing allocation formulae (which are formally compared to the C-D model in an appendix) to make them forward-looking. Section 8 concludes.

2. The Collier-Dollar model

Collier and Dollar conceptualised the donor problem as

$$\text{Max poverty reduction } \sum_i G^i \alpha^i h^i N^i \tag{1}$$

$$\text{subject to } \sum_i A^i y^i N^i = \overline{A}, \quad A^i \geq 0 \tag{2}$$

where the superscript i indexes countries and h^i is a measure of poverty relative to each country's population, N^i, so that $h^i N^i$ is the total amount of poverty in each country. G^i is the growth rate of per capita income (usually non-negative) and α^i is the (negative) elasticity of h^i with respect to per capita income, so that the objective, summed over all countries, is to maximise the reduction in the total amount of poverty in the world. If h^i were a poverty headcount, for example, the objective would be to achieve the largest possible cut in the global number of poor people.

The constraints (2) are that the sum of aid to all countries equals the global total of available aid, $\overline{A}$, and that no country can receive negative aid. Aid to each country, A^i, is measured as a ratio of its GDP, which is multiplied by its per capita income, y^i, and population to yield its absolute receipts of aid. The objective is connected to the constraints by a functional relationship between growth and aid for each country, which C-D specify and estimate as

$$G^i = c + b_1 X^i + b_2 P^i + b_3 A^i + b_4 A^{i2} + b_5 A^i P^i \tag{3}$$

where P^i measures the quality of each country's policies and X^i is a vector of exogenous influences. They derive from the solution to the constrained maximisation problem an expression for the optimal level of aid to each recipient country, which can be written as

$$a^i = -\frac{N^i y^i}{2b_4}\left\{\left(b_3 + b_5 P^i\right) - \frac{\lambda}{\alpha^i}\frac{y^i}{h^i}\right\} \tag{4}$$

where λ is the global shadow value of aid and $a^i = A^i y^i N^i$ is the absolute amount of aid (the initial $N^i y^i$ just converts the whole expression from A^i to a^i). The initial negative sign is offset by b_4 also being negative (it measures the speed with which returns to aid diminish), so that the whole expression is positive for countries that are allocated some aid (for those which are not, it is zero or negative).

Inside the chain bracket, the two terms in the round bracket show that countries with better policies are allocated more aid (b_3 and b_5 must be considered jointly – C-D,

2002: 1478). This is because the specification of equation (3) makes the effectiveness of aid in increasing growth and hence in reducing poverty depend on the quality of policies (and only on the quality of policies). The presence of α^i in the denominator of a term with a negative sign shows that more aid is also allocated to countries with larger poverty elasticities (where growth is more effective in reducing poverty).

The final ratio, y^i/h^i, which is also part of a term with a negative sign, shows that more aid is allocated to countries with higher poverty rates and that less aid tends to be allocated to countries with higher per capita incomes (though the y^i in front of the chain bracket pulls the other way). The ratio between h^i and y^i matters because C-D explicitly assume, and equation (1) implies, that aid can reduce poverty only by raising national income, which makes it more efficient to give aid to countries with more poverty per dollar of national income (H^i/y^iN^i, which is equivalent to h^i/y^i).

N^i in the first term of equation (4) implies that the allocation of aid to a country varies in proportion to its population (other things being equal). Both C-D papers recognise that in reality the allocation of aid is biased against large countries. In C-D (2001), N^i is raised to a power between zero and one (the estimated value was 0.32, which means that a ten-fold increase in population only roughly doubles the amount of aid). In C-D (2002), aid to India was capped. Both these adjustments were purely pragmatic: size bias contradicts the logic of the C-D model and will therefore be largely neglected in this paper until the discussion of donor allocation formulae in section 7.

3. Use and non-use of the C-D model

The C-D framework has powerful attractions to donors who want to reduce world poverty. Its logic is simple and compelling: give more aid to countries where there is more poverty and where aid will have more impact on poverty (because the policy environment for growth is better or because growth reduces poverty more). Its form gives precision to this logic by showing exactly how much more aid should go to countries which have more poverty and use aid better, and exactly how these two criteria should be combined and traded off against one another.

These attractions have led to widespread discussion of the C-D model by donor agencies and to improvements in aid allocation practice (Beynon, 2002; McGillivray, 2003). Most multilateral agencies – for example, the World Bank, African and Asian Development Banks, and European Development Fund – use related models for aid allocation (Keith, 2002), though some of them long pre-date C-D.[2] A few bilateral agencies, for example in the UK and the Netherlands, also use allocation models informed by C-D principles, and even in agencies which do not use formal models allocation decisions are influenced by these principles (Jones *et al.*, 2004)

There are also many divergences of donor practice from the C-D model. Most donors have motives other than poverty reduction for giving aid (Alesina and Dollar, 2000), which explain, for example, why middle-income countries receive far more aid than in a 'poverty-efficient' C-D allocation. Even where poverty reduction is the focus, no agency uses the C-D model in the exact form described in section 2: it refers to the

---

[2] The World Bank's model for allocating IDA funds on the basis of per capita income, population size and performance has been in use for three decades (Isenman *et al.*, 1977). It has proved more durable than the project appraisal methods initiated at about the same time (Little and Mirrlees, 1990).

whole developing world, while most donors deal with limited sets of countries (even the World Bank is restricted to IDA-eligible ones). Donors prefer simple formulae to optimising models (with implications to be discussed in section 7 and the appendix). Their formulae include causes of aid effectiveness other than the quality of policies, and they deviate from formula allocations where they have more information than in the formula about the effectiveness of aid to specific countries.

None of these considerations, however, can explain why donors whose professed aim is to reduce world poverty allocate more aid to Africa and less aid to Asia than a C-D model would recommend. The IDA allocation formula, for example, is subject to the constraint (imposed by contributing donors) that at least 50% of the total should go to Africa. The 'European Consensus' on development policy earmarks for Africa half of its large planned increase in aid. The DFID model used in allocating aid among low-income countries is likewise applied separately to Africa, whose overall allocation is a prior political decision. The relatively low allocations to Asia partly reflect size bias against India and China, but there is clearly a strong bias towards Africa, too.

C-D aid allocation principles are also in conflict with the now-dominant approach to world poverty reduction of the MDGs, particularly as interpreted in the report of the Millennium Project (2005) directed by Jeffrey Sachs. Associated with the MDGs are targets, mostly proportional reductions in poverty indicators – for example, halving the income poverty headcount rate and a two-thirds fall in the under-five mortality rate. These targets were originally set for the world as a whole, but Sachs applied them to individual countries (e.g. that each country should halve its headcount rate between 1990 and 2015), and argued that the basic principle of aid allocation should be to enable every developing country to achieve all its targets.[3]

Comparisons between the Sachs and C-D approaches must recognise that the MDGs are multi-dimensional, while the C-D model has a single-valued measure of poverty. The two approaches also make entirely different assumptions about total aid, which in the C-D model is predetermined but for Sachs is an end-result of the calculations (the sum needed for every country to achieve its targets). But they are clearly inconsistent, as is illustrated by Anderson and Waddington (2007). The poverty reduction targets of Sachs are set without regard to initial poverty levels (e.g. halving the headcount rate, regardless of whether it started at 50% or 5%). As a result, the Sachs aid total could be reallocated on C-D principles to achieve a larger reduction in world poverty – but with many countries falling short of their MDG targets. Each approach yields a sub-optimal allocation from the point of view of the other approach.

4. A forward-looking allocation model

To understand these discrepancies between C-D model allocations and how donors actually allocate aid – to Africa and for achieving MDG targets – it is necessary to bring time into the formal analysis. When policy-makers write about 'eliminating world poverty' (as in the titles of all DFID's White Papers) or 'dream of a world free of poverty' (as inscribed in the lobby of the World Bank's headquarters), they are looking forward. They have in mind the future, as well as the present – unavoidably,

---

[3] In one of their two articles, Collier and Dollar (2001) applied their model to the MDGs to develop a scenario of improved aid allocation and policy reform that would efficiently achieve the target of halving poverty in Africa as a whole, as well as globally (but not in each individual country).

since although the global total of abject poverty can and eventually will be reduced to almost zero, this is bound to take a long time.

Analytically, this aspiration of donors is best thought of as being to minimise future world poverty. Future world poverty in turn can be thought of as world poverty in each future year, added up in some way over some appropriate time horizon, T. The simplest possible way to express this objective formally would thus be:

$$\text{Min} \quad \sum_i H_0^i + \sum_i H_1^i + \ldots + \sum_i H_T^i = \sum_i \sum_{t=0}^{t=T} H_t^i \qquad (5)$$

where the superscript i as before indexes countries and the subscript t (= 0, 1, ..., T) indexes years. The amount of poverty in each country and year is $H_t^i = h_t^i N_t^i$, using whatever measure of poverty donors prefer and allowing for population size as well as the poverty rate. Policy-makers may be systematically more concerned about poverty in the nearer-term future than in the more distant future, which can be formalised by discounting future poverty at a rate, r, so that the objective becomes:

$$\text{Min} \quad \sum_i \sum_{t=0}^{t=T} \frac{H_t^i}{(1+r)^t} \qquad (6)$$

This objective can more conveniently be rewritten in continuous time as

$$\text{Min} \quad \sum_i H_0^i \int_0^T e^{-\left(r+\Delta_H^i\right)t} dt \qquad (7)$$

where $\Delta_H^i = \Delta_h^i - n^i$ is the proportional decline per period in the level of country i's poverty, allowing for both the reduction in its poverty rate, Δ_h^i, and the growth of its population, n^i (which slows the decline in its absolute amount of poverty). These rates of change and the discount rate are assumed for simplicity to be constant up to period T. Replacing the integral by its solution yields

$$\text{Min} \quad \sum_i H_0^i \frac{1 - e^{-\left(r+\Delta_H^i\right)T}}{r + \Delta_H^i} \qquad (8)$$

The longer the time horizon, the closer to unity is the numerator of the ratio, so that if T were infinite the expression would be simply

$$\text{Min} \quad \sum_i \frac{H_0^i}{r + \Delta_H^i} \qquad (9)$$

The 'present value' of each country's future poverty thus depends on its current poverty, H_0^i, but is reduced by a faster future rate of decline in poverty and by a higher discount rate, to an extent dependent on the length of the time horizon. It is the

sum across all countries of these present values that donors are assumed to be seeking to minimise through the allocation of their aid.

As in C-D (2002), it is also assumed that what needs to be allocated is just the current period's aid total, $\overline{A}_0$. The future rate of decline of poverty will be affected by aid in future periods, for which allowance needs to be made in projecting Δ^i_H, together with all other influences on each country's future poverty except the current period's aid. An ideal forward-looking model would allocate aid in all future periods, too, since the impact on world poverty reduction of a given total amount of aid could be increased by distributing it optimally among periods as well as among countries. But in reality, agencies allocate only the aid budgets for their current planning periods (for example, each three-year IDA replenishment) and cannot shift money between periods.[4]

To complete the allocation model, it is necessary to specify how current aid affects future poverty, which it could do in two possible ways. It could cause a step change, reducing the level of poverty in all future periods: thus $H^i_0(a^i_0)$ relates the amount of poverty in country i at the end of period zero to the aid it receives during that period, a^i_0. The current period's aid could also speed the future rate of decline of poverty, the relationship being $\Delta^i_H(a^i_0)$. The Lagrangian based on equation (8) is thus

$$L = \sum_i H^i_0(a^i_0) \frac{1 - e^{-(r + \Delta^i_H(a^i_0))T}}{r + \Delta^i_H(a^i_0)} + \lambda \left(\overline{A}_0 - \sum_i a^i_0 \right) \qquad (10)$$

and the first-order conditions for an optimum are

$$\left[H^i_a + \Delta^i_a H^i_0(a^i_0) \left(\frac{T}{e^{(r + \Delta^i_H(a^i_0))T} - 1} - \frac{1}{r + \Delta^i_H(a^i_0)} \right) \right] \frac{1 - e^{-(r + \Delta^i_H(a^i_0))T}}{r + \Delta^i_H(a^i_0)} = \lambda \qquad (11)$$

where H^i_a (< 0) and Δ^i_a (> 0) are the first derivatives of $H^i_0(a^i_0)$ and $\Delta^i_H(a^i_0)$.

A convenient simplification of the model is to assume that $\Delta^i_a = 0$, which reduces the first-order conditions to[5]

$$H^i_a \frac{1 - e^{-(r + \Delta^i_H)T}}{r + \Delta^i_H} = \lambda \qquad (12)$$

This assumption – that current aid reduces the future level of poverty, but not its future rate of decline – is convenient both because it eases the exposition of the model and because equation (12) is simple enough to be applied in practice (as is explained in section 7). It is of course not always accurate: current aid could affect the future rate of decline of poverty, for example by removing a bottleneck to growth through

---

[4] The International Finance Facility for Immunisation is a partial exception: it allows future aid to be brought forward by borrowing, though only for this particular purpose. Aid is allocated in all periods up to the time horizon in Collier and Dollar (2001) and Cogneau and Naudet (2007).
[5] I am indebted to Tony Venables for suggesting this simplification.

construction of a port. But it is unlikely that the $\Delta_H^i(a_0^i)$ function could in practice be quantified or included in a model, because its form varies so widely among countries (though ad hoc adjustments could be made to the allocations for countries where the effects of current aid on future poverty decline were believed to be large).

Depending on the form of H_a^i, it may be possible to rearrange (12) into an explicit expression for each country's optimal aid. The relationship between aid and poverty proposed by C-D yields such an expression, but before turning to this specific form, it is worth considering further the general form of the model. With the assumption that $\Delta_a^i = 0$, the model just modifies a current-period optimal allocation by including an exogenous and multiplicative 'poverty decline adjustment', defined as

$$D_{rT}^i = \frac{1 - e^{-(r + \Delta_H^i)T}}{r + \Delta_H^i} \tag{13}$$

Thus, for example, the first-order conditions (12) can be rewritten as

$$H_a^i D_{rT}^i = \lambda \tag{14}$$

and if the time horizon of policy-makers did not extend beyond the current period (as in C-D) would reduce to $H_a^i = \lambda$, with no need for a poverty decline adjustment.

Multiplication by D_{rT}^i affects the allocation of aid because the size of the adjustment varies, being larger for countries where poverty is falling slower (or rising) than for countries where poverty is falling faster. How much difference this makes depends also on the discount rate and time horizon, as is illustrated in table 1, which shows, for a wide range of values of r and T, the ratio D_{rT}^i / D_{rT}^j for a country i where poverty is declining at 1% per year and a country j where it is declining at 7% per year.

The benefit of the adjustment to the slow-poverty-decline country i obviously rises as the discount rate falls and the time horizon becomes longer, both of which signify increased donor concern about future poverty relative to current poverty. With a high discount rate or short time horizon, its adjustment is close to that of the rapid-poverty-decline country j. With a low r or long T, however, the slow-poverty-decline country benefits more, and with a zero discount rate and an infinite horizon, its adjustment is seven times that of the rapid-poverty-decline country.

Cogneau and Naudet (2007) also propose a model in which the optimal allocation of aid depends on future rates of poverty decline. Their objective is not to minimise total world poverty, but rather, on Rawlsian principles of fairness, to reduce inequality of poverty rates among countries. More precisely, their model uses aid to reduce high values of h_T^i (poverty rates at the end of the planning period) due to forces beyond the control of the countries concerned. As in the present model, more aid thus tends to be allocated to countries where Δ_H^i is low (and to countries where h_0^i is high). Unlike the present model, however, countries with high total poverty (H_0^i) as a result of large populations rather than high poverty rates are not entitled to more aid.

5. Collier and Dollar as a special case

The specific form of $H_0^i(a_0^i)$ used by Collier and Dollar is $(1 + G^i(A_0^i)\alpha^i)H_0^i$, where H_0^i is a measure of income poverty, α^i is the poverty elasticity and $G^i(A_0^i)$ is the relationship between growth of per capita income and the ratio of aid to GDP in equation (3) above. The specific form for its derivative is thus

$$H_A^i = (b_3 + 2b_4 A_0^i + b_5 P^i)\alpha^i H_0^i \tag{15}$$

and the optimal allocation of aid to each country is

$$a_0^i = -\frac{N_0^i y_0^i}{2b_4}\left\{(b_3 + b_5 P^i) - \frac{\lambda}{\alpha^i}\frac{y_0^i}{h_0^i}\left(\frac{r + \Delta_H^i}{1 - e^{-(r + \Delta_H^i)T}}\right)\right\} \tag{16}$$

assuming as in the previous section that the future rate of decline in poverty, Δ_H^i, is unaffected by this period's aid.

Apart from the appearance of time (zero) subscripts on some of the variables, this is the same as the equation (4) derived above from the C-D model, with the exception of the last term in the chain bracket. This term is now multiplied by a round-bracketed expression, which is simply the reciprocal of the poverty decline adjustment, D_{rT}^i. As in equation (4), aid to a country rises with its current poverty rate, h_0^i (which makes the negatively-signed second term in the chain bracket smaller). In equation (16), however, aid is increased also by a slower future rate of decline in poverty, Δ_H^i (which also makes this second term smaller).

The relative impact of present and future poverty on aid allocations depends on the discount rate, r, which reflects the degree of concern of policy makers about more distant as compared to nearer-term poverty. For any given degree of variation across countries in Δ_H^i, a larger r, since it is the same for all countries, reduces variation in $(r + \Delta_H^i)$, so that variation among countries in their future poverty reduction rates has less effect on the allocation of aid among them, relative to variation in their current poverty rates, h_0^i. A shorter time horizon, T, likewise tilts the balance of influence on allocations more towards present, relative to future, poverty.[6]

The C-D allocation model can thus be seen as a special case of the present more general model in which aid allocations are based only on current levels of poverty, without regard to future rates of change in poverty. Equation (16) would generate much the same allocation of aid as equation (4) if r were set high and/or T low, implying concern entirely about the present, since this would make the poverty

---

[6] Reducing T decreases D for all countries, but by proportionally more for those with slower rates of decline in poverty, which lessens the influence on allocations of differences in rates of decline, as can be seen from the numbers in table 1.

decline adjustment (the only substantive difference between the two equations) almost identical in value for all countries.

Comparing the present model with MDG-based aid allocations is less easy, because the MDG approach is based on numerical targets for multiple poverty indicators to be attained by specific dates, rather than optimisation. But if the single-valued poverty rate, h^i, is interpreted as a suitable average of these indicators, the spirit of the MDG-based approach is captured reasonably well as a special case of equation (16) in which r is zero and T is high. For any given degree of cross-country variation in Δ^i_H, these parameter values would maximise variation in D^i_{rT} and thus maximise the influence on allocations of differences in poverty reduction prospects, broadly as was advocated in Millennium Project (2005).

The present more general model is attractive because it combines the strong points of the C-D and MDG-based special cases while avoiding their shortcomings. It retains the crucial C-D principle of allocating aid efficiently – an extra dollar of aid achieves the same reduction of poverty in every recipient country – while avoiding the myopia of the formal C-D model. It also retains the attention rightly paid by the MDG-based approach to cross-country differences in poverty reduction prospects, while avoiding the inefficiency of its proposed allocations (in which an extra dollar of aid would reduce poverty by different amounts in different recipient countries).

The C-D model is a special case in a second important sense. Not only is its objective myopic, but it is also based on one specific form of the function $H^i_0(a^i_0)$ that links aid to poverty reduction. To operationalise any allocation model, it is clearly necessary to choose a specific form, but the choice matters greatly for the results. The allocations generated by a model depend on what it assumes about the impact of aid on poverty, as well as on how its poverty objective is specified. Moreover, there is much disagreement about the effects of aid on poverty reduction: how big, how rapid, and conditional on what characteristics of recipient countries, aid instruments and donor behaviour?[7] Disagreement about facts could cause disagreement about allocations, even among people with identical views on objectives.

Collier and Dollar (2001, 2002) allow for some disagreement about $H^i_0(a^i_0)$ by trying alternative values for their estimated parameters b_3, b_4 and b_5.[8] But the effectiveness of aid in increasing a country's growth clearly depends on more than the quality of its policies.[9] So the $b_5 A^i P^i$ term in equation (3) should be modified to make P^i into a vector of relevant country characteristics (including some of the exogenous influences on growth, X^i). Even more fundamental is the C-D assumption that aid can reduce poverty only through growth – raising a whole country's income – and that it cannot be targeted on poor people within countries. Conclusions about optimal aid allocation based on the C-D specification of $H^i_0(a^i_0)$ must thus be interpreted with caution.

---

[7] The most extreme disagreement is between Sachs (2005) and Easterly (2006). Milder but still strong disagreement is evident in, for example, Clemens *et al.* (2004) and Rajan and Subramanian (2005).
[8] Beynon (2003) and Anderson and Waddington (2007) survey the debate about these parameters.
[9] E.g. Guillaumont and Chauvet (2001), Collier and Hoeffler (2004), Dalgaard *et al.* (2004).

6. Illustration with regional data

Although models are used to inform decisions about allocating aid among countries, the implications of sections 4 and 5 can be illustrated with data for the much smaller number of developing regions. Table 2 adopts the World Bank regional classification, but separates out India and China to yield eight geographical units (all referred to as 'regions' below). For each region, the table shows the values of the variables in equation (16) which determine its aid allocation ($h_0^i, N_0^i, y^i, P^i, \Delta_H^i, \alpha^i$), with 2004 as the base year. The poverty data – the initial headcount ratio, the projected rate of decline in the number of poor people (from 2004 to 2030), and the income elasticity (2004-15) – are from the World Bank.[10] The proxies for P^i are population-weighted regional average CPIA scores.[11]

Table 3 presents one illustrative optimal allocation of aid. Its first column shows the initial number of poor people in each region, based on the same World Bank data as in table 2.[12] The second column shows the value of the poverty decline adjustment (equation 13), with a zero discount rate and a time horizon of 25 years (corresponding to the 1990-2015 time horizon of the MDGs, and to the 2004-30 poverty projections in table 2). Policy makers are thus assumed to attach equal weight to poverty in each year up to their time horizon and not to care at all about poverty beyond this horizon. The third column, obtained by multiplying the first and second columns, shows the 'present value' of future poverty (equation 8) prior to this period's aid.

The allocation of aid among regions which minimises the world total of the present value of future poverty (the last row of column 3) is shown in column 4. The percentages of aid are also absolute amounts, since the world aid total is set at $100 billion, to correspond roughly with the commitments made by donors in 2005. The allocation is based on the C-D form of $H_0^i(a_0^i)$, and uses the values of b_3, b_4 and b_5 in C-D (2002) Variant I (-0.54, -0.02, and 0.31 respectively).

The middle-income regions in the bottom half of the table (East Asia, Latin America, Europe and Central Asia, and the Middle East and North Africa) are allocated no aid – not even China, which starts with more than 100 million poor people. This outcome is similar to that of C-D (2001, 2002), though it is exaggerated by the use of regional aggregate data, which conceal the low-income countries in middle-income regions. It arises because, with the C-D form of $H_0^i(a_0^i)$, aid can reduce poverty only by raising national income, which makes it more efficient to give aid to regions with many poor people per dollar of national income (H^i/Y^i or h^i/y^i) – an indicator on which, as is shown in column 5 of table 3, the middle-income regions are far below Africa and South Asia, both because of their lower h^i and because of their higher y^i.

---

[10] These data were kindly supplied by Dominique van der Mensbrugghe. The poverty levels and trends are similar to those in Table 2.3 of World Bank (2007a), revised and updated for World Bank (2007b), including a change of base year from 2003 to 2004. The elasticities, which are not published, are explained in note 37 of chapter 2 of World Bank (2007a).

[11] CPIA (Country Policy and Institutional Assessment) scores, calculated by the World Bank, are also used by C-D (2001, 2002). These regional averages are based on unofficial estimates of the scores for individual countries (which are not published by the Bank for most countries and years).

[12] These numbers are not exactly consistent with those that could be obtained by multiplying together the first two columns of table 2, and differ considerably for Other South Asia and Other East Asia.

The allocations among the low-income regions are at first sight surprising. Africa gets only one-third of the aid, despite the forward-looking poverty reduction objective and a slower projected decline of poverty than in South Asia. India, moreover, gets substantially less aid than Other South Asia, despite having five times as much poverty initially and a slower projected decline of poverty. The explanations for both outcomes are differences among these regions in the effectiveness of aid, which offset the differences in their needs for aid. The combination of a lower CPIA score and a lower poverty elasticity causes aid to reduce poverty by substantially less in Africa than in South Asia. In India, likewise, a far lower poverty elasticity than in Other South Asia outweighs a somewhat higher CPIA score, making it efficient to give more aid to Other South Asia.

This illustrative optimal allocation of aid differs greatly from its actual allocation, shown for 2004 in the last column of table 3. The middle-income regions got half of all aid, in contrast to their zero optimal shares, confirming that much aid is given for reasons other than poverty reduction (Alesina and Dollar, 2000; C-D, 2002). The allocation of the other half among the low-income regions is also different: India's actual 3% share of the low-income subtotal is far lower than its optimal 26% share, reflecting size bias; and Africa actually gets four times as much aid as Other South Asia, rather than somewhat less aid, as in the optimal allocation (though its share of total world aid is by coincidence fairly similar in columns 4 and 6 of table 3).

Table 4, which addresses the central point of this paper, shows how the illustrative allocations among low-income regions vary with the choice of discount rate and time horizon. The middle row is the same as in table 3, showing the optimal shares for a zero discount rate and a time horizon of 25 years. The two rows above it show allocations with less future-oriented objectives: the C-D special case of a one-year time horizon is in the first row, and a 25-year horizon combined with a 10% discount rate in the second row. The last two rows of table 4 are more future-oriented: as in table 3, a zero discount rate is assumed in both cases, but the time horizon is extended from 25 years to 50 years in the fourth row, and to 100 years in the final row (the 'MDG' special case, with maximum weight on poverty reduction prospects).[13]

The first column shows the share of Africa in the low-income subtotal (which in all cases is also the world total, since in no case does any middle-income region get any aid). In the first row, in which the aim is to minimise near-term world poverty, Africa gets no aid at all, despite its 40% share of all the poor people in low-income regions. Given the CPIA scores and poverty elasticities in table 2, and the assumed values of the parameters of the C-D aid-growth function, the effectiveness of aid to Africa is too low to justify any allocation. (This outcome is different from C-D (2001, 2002), in which, even with no future orientation, African countries are allocated substantial amounts of aid, mainly because the C-D poverty elasticities are more favourable to Africa than those in table 2.)

However, moving down the first column of table 4, it can be seen that, despite the low effectiveness of aid to Africa in these calculations, its allocation of aid increases rapidly as the poverty objective becomes more future-oriented. Assuming $r = 0$, a 25-

---

[13] Over all these time horizons, poverty is assumed to decline at the 2004-30 rates shown in table 2.

year horizon raises its share of the low-income subtotal from zero to one-third, a 50-year horizon to two-thirds, and with a 100-year horizon Africa gets all the aid. The share of India in the low-income subtotal (column 2) correspondingly declines, but there is not much change in the relative shares of India and Other South Asia (column 3), and the share of Africa relative to Other South Asia rises steeply (column 4).

The numbers in table 4 illustrate the effects of variation in donor objectives. Those in table 5, in which *r* and *T* are held constant, show how allocations vary with the values of the parameters of the growth function (equation 3) and of the poverty elasticities. The base case, in the first row, is again that of table 3. The next two rows alter b_3 and b_5, increasing and decreasing respectively the sensitivity of allocations to policy scores (these are Variants III and IV of C-D, 2002). The following two rows alter b_4, increasing and decreasing respectively the rate at which returns to aid diminish.[14] The final row, which reverts to the base case growth parameters, replaces the poverty elasticities in table 2, which vary widely among regions, with a uniform elasticity of -2 in all regions, as assumed in C-D (2002).[15]

The last four columns of table 5 show the effects of these parameter variations on the four aspects of the allocation of aid among low-income regions explored in table 4 (none of these variations gives any aid to to middle-income regions). For example, less sensitivity to policy scores substantially increases Africa's share of aid and reduces India's share to almost zero, so that Other South Asia also gains slightly. By contrast, faster diminution of returns to aid doubles India's share, at the expense of both Africa and (somewhat more) other South Asia. Removing the variation among regions in poverty elasticities has almost no effect on India, but the share of Other South Asia, which had by far the most favourable elasticity, drops to zero, and the share of Africa rises from one-third to three-quarters.

The calculations that Cogneau and Naudet (2007, table 5) make to illustrate their forward-looking allocation model also show the sensitivity of the results to poverty elasticities. When they assume, as in the last row of the present table 5, the C-D form of $H_0^i(a_0^i)$ and uniform elasticities of -2, Africa gets almost all of the aid to low-income regions (though, unlike the present calculations, one-third of world aid goes to countries in middle-income regions). When Cogneau and Naudet let poverty elasticities vary among countries, however, Africa gets only about half of the aid to low-income regions (in the present calculations, using the base case parameters and with elasticities varying among regions, Africa's share would be about half if *r* were zero and *T* 35 years).

In summary, the numerical illustrations in table 4 confirm that the optimal allocation of aid among regions depends heavily on the degree of future orientation of the world poverty reduction objective. With a near-term view, it makes sense to allocate Africa

---

[14] The coefficient is raised in the 'faster' case to the value in Variant II of C-D (2002) and is lowered in the 'slower' case by one standard deviation (from regression 1 of table 1 in C-D, 2002).

[15] C-D (2002) use this uniform elasticity for their calculations based on the headcount poverty measure, but use varying elasticities for other poverty measures. See also Beynon (2003: 13-14). In a forward-looking model, variation in poverty elasticities among countries can have offsetting effects: for a given growth rate of income, a higher elasticity accelerates future poverty reduction and so tends to reduce a country's aid allocation (by lowering the present value of its future poverty); but a higher elasticity also increases the effectiveness of aid and hence tends to increase the country's allocation.

no aid, because the effectiveness of aid in reducing poverty in Africa is low. With an extremely long-term view, by contrast, it makes sense to allocate Africa all the aid. However, the numbers in table 5 are a reminder that disagreements about the optimal allocation of aid are as likely to arise from disagreements about its effects on poverty as from disagreements about the correct specification of the objective of aid.

7. Application in practice

The central idea of this paper, summarised initially in figure 1, is that any poverty-efficient allocation of aid must recognise that donors care about both present and future poverty. Section 4 developed this idea algebraically, and sections 5 and 6 showed that it has a large effect on allocations. The issue for this section is whether and how this idea could be implemented by donor agencies.

It would be unrealistic to try to specify a single model which all donors should adopt. Each agency has its allocation practices, which will not be swept away and replaced by something new. Moreover, practices necessarily vary among agencies, depending on their circumstances, both political (e.g. the weight attached to poverty reduction as an objective) and administrative (e.g. a multilateral or a bilateral, and, if a bilateral, whether one or several agencies are involved). The most that a paper such as this can offer is suggestions that might be used to improve current practices.

Donors use simple allocation formulae rather than explicit optimising models (though, as is shown in the appendix, there are implicit and unrecognised models behind these formulae). The typical formula allocates aid among countries in proportion to scores, S^i, based on the combination of some measure of each country's current poverty with some measure of the quality of its policies

$$S^i = h_0^i N_0^i P^{i\gamma} \tag{17}$$

where $h_0^i N_0^i = H_0^i$ is poverty (rate multiplied by population size), P^i is policy, and γ is the importance attached to policy.[16] Some donor formulae include other variables that affect how much aid reduces poverty, in which case $P^{i\gamma}$ can be interpreted simply as some composite indicator of aid effectiveness.[17]

The formula in (17), like the C-D model, focuses only on current levels of poverty, and thus tends to allocate a smaller share of aid to Africa than poverty-oriented donors want to provide, so that it is not used (or modified by some other rule) in this aspect of their decisions. Similarly, the prescriptions of the formula conflict with those of the MDG-based approach. The solution to these problems suggested by the analysis in section 4 is to adjust the measure of poverty to allow for its future trajectory as well as its current level.[18] The formula thus becomes

---

[16] The formulae used by donors are surveyed by Keith (2002) and Jones *et al*. (2004). From 2003 to 2004, the DFID formula was systematically biased against larger countries by attaching an exponent of 0.6 to population size, following Collier and Dollar (2001).

[17] Because each donor allocates only part of the world's aid, usually among only a subset of countries, some formulae also adjust for the amounts of aid that countries are likely to get from other donors (Beynon and Landymore, 2002; Jones *et al*., 2004).

[18] DFID's allocation formula has already moved in this direction. As a result of the recommendations in Dyer *et al*. (2003), it was modified to give (slightly) more aid to countries that were making slower

$$S^i = h_0^i D_{rT}^i N_0^i P^{i\gamma} \qquad\qquad (18)$$

where

$$D_{rT}^i = \frac{1 - e^{-(r+\Delta_H^i)T}}{r + \Delta_H^i} \qquad\qquad (19)$$

is the poverty decline adjustment for the country concerned (copied from equation 13 earlier). As before, Δ_H^i is the projected annual rate of decline in poverty over the next T years, and r is the rate at which, up to this time horizon, policy-makers discount future poverty relative to current poverty.

The modified formula (18) and adjustment (19) can be applied to whatever measure of poverty a donor regards as best, subject to the availability of data. The poverty rate, h_0^i, can be any index (for example, the headcount or the squared poverty gap) and based on any number of dimensions of poverty, provided that these are combined in some appropriate way into a single number (which has yet to be done for the MDG indicators). The rate of decline in poverty, $\Delta_H^i = \Delta_h^i - n^i$, then has to be projected for the chosen measure of the poverty rate, adjusted by population growth.

One obvious practical issue in using the modified formula is how to project the decline in poverty for each country. It would be unreasonable to expect each donor to make its own projections – and undesirable in relation to the objective of improving aid coordination. A better approach would be for all donors to use projections made by one international agency such as the World Bank. Relevant projections are already published, both for income headcount poverty (e.g. World Bank, 2007a, table 2.3) and for progress towards non-income MDG targets (e.g. World Bank, 2007b).

Expected future aid should be taken into account in projecting each country's rate of poverty decline, Δ_H^i. As was explained in section 4, however, the adjustment (19) is based on the simplifying assumption that this period's aid lowers the future level of poverty, but does not affect its rate of decline. Thus if a donor had reason to believe that the amount of aid it allocated to a country in the current period would raise Δ_H^i substantially, for example by removing a bottleneck, then it should make an ad hoc (upward) adjustment to the formula allocation for that country.[19]

Another practical issue in using the modified formula is the choice of values for r and T, which affects the relative sizes of the adjustment for different countries (given their projected rates of poverty decline). Politicians and other senior policy-makers are unlikely to be able or willing to specify particular values at the outset, but their preferences could be inferred by asking them to choose among alternative allocations based on different values of r and T. They might allow the values that emerged to be

---

progress towards MDG targets. Since 2006, the poverty measure has been the projected level of per capita income in 2010 rather than the latest available actual level.

[19] Donors should make other ad hoc adjustments to allow for information about specific countries that is not adequately captured by the formula or reflected in the available data.

used routinely in the allocation formula. Conceivably, all donor agencies might be able to agree, in the framework of the DAC, to use the same values of *r* and *T*.

An attractively simple option would be to set *r* at zero and *T* at 25 years. The 25 years has some basis in reality, since it matches the 1990-2015 timespan of the MDGs. The combination of a zero discount rate for the quarter-century ahead and (in effect) an infinite discount rate thereafter would also capture much of the greater concern of policy-makers about current than about future poverty. Even over the next 25 years, they probably care more about nearer-term than about more distant poverty, but there has been little in the debate about the MDGs to suggest that this is a major concern.

The use of the modified formula (18) can be illustrated by applying it to the data in the previous section. Table 6 reports the allocations prescribed by the formula for the same range of values of *T* as in table 4 (with a zero discount rate), assuming that $\gamma = 2$. Its first four columns are also the same as in earlier tables: they show how the formula allocates aid among low-income regions. The last column shows how much in total it allocates to middle-income regions, although in practice poverty-oriented donors limit aid to these regions, on the grounds that they can afford to pay for their own poverty reduction or have better access to international capital markets.[20]

In the first row of panel A, a low *T* yields a result similar to the unmodified formula (17), with only 30% of aid to low-income regions assigned to Africa and well over half to India. Allowing for differences in poverty reduction prospects by extending *T* to 25 years raises Africa's share by 10 percentage points and lowers India's. Raising *T* to 50 years makes Africa's share similar to that of India. Extending *T* to 100 years raises Africa's share to 62% and lowers that of India to 36%.

The general pattern of these results is the same as in table 4, but the variation is smaller, especially for Africa. Moreover, middle-income regions got no aid in table 4, but are allocated substantial amounts in table 6. Since the poverty reduction objective is identical in the corresponding rows of tables 4 and 6, the difference in allocations between the two tables reflects a difference of views about how aid reduces poverty or, to be more precise, about $H_0^i\left(a_0^i\right)$: table 4 is based on the C-D specification, while table 6 uses the specification implicit in the donor formula. The two specifications overlap, but are different, as is shown in the appendix. Not least, C-D assume that aid can reduce poverty only by making entire countries richer, while the donor formula assumes that aid can be targeted on poor people within countries.

Since in practice donors cap their allocations to India, panel B of table 6 shows what the other shares would become if India were limited to 3% of the world aid total. Africa's share of the low-income subtotal is substantially increased: for example, with *r* = 0 and *T* = 25, it rises from 39% to 84%. The India cap also raises the share of Other South Asia in total aid to South Asia (in this case up from 9% to 73%), but it does not alter the relative shares of Africa and Other South Asia.

Although the modified formula (18) is usable with any measure of poverty, the pattern of allocations implied by the formula will depend on which measure is chosen, since

---

[20] Thus, for example, DFID reserves 90% of its bilateral aid for low-income countries, and the World Bank allocates IDA funds only among a restricted set of low-income countries.

different measures will alter the relative poverty levels of different countries (or their relative projected rates of decline in poverty). To illustrate this point, panel C of table 6 shows what the shares would be if poverty were measured, as it is at present in most donor formulae, by the average per capita income of a country rather than by its income poverty headcount, as in panel A.[21] In all other respects, the calculations for panels A and C are identical (and in both the allocations to India are uncapped).

Comparing panels A and C shows that this change of poverty measure, which makes India seem relatively less poor than in terms of its poverty headcount, and so less deserving of aid, lowers its share of the low-income subtotal, and raises the shares of the other two low-income regions. The increase in Africa's share of the low-income subtotal is fairly small. It is larger for Other South Asia, which looks much poorer, relative to Africa, in terms of per capita income than of poverty headcount.

8. Conclusion

The puzzle that prompted this paper is that poverty-oriented donors allocate more aid to Africa than the C-D poverty-minimising allocation model suggests and, relatedly, that this model clashes with the poverty-focused MDGs. The solution to this puzzle, the paper argues, is that the poverty reduction objective of the C-D model is myopic, whereas donors (and the MDGs) are forward-looking – they care about future poverty as well as current poverty.

The paper shows how an improved model could include concern about future poverty in its objective while retaining the best feature of the C-D model, which is to allocate aid efficiently (meaning so as to get closest to the objective). The paper also suggests a specific modification to the aid allocation formulae used by donors, which is to add a future poverty decline adjustment. This modification could make donor decisions more consistent (e.g. avoiding the need for separate Africa quotas) and reduce conflict and confusion between the formulae and the MDG targets.

Improvements of these sorts could not solve all the problems of aid allocation among countries, because inefficient or inconsistent treatment of the balance between current and future poverty is only one reason for disagreement about the relative amounts of aid that different countries should receive. Even people who agree that the objective of aid is to minimise world poverty may disagree about allocations also because they have differing views about how poverty should be measured. In addition and more fundamentally, they may disagree about allocations because of factual disagreements about how aid contributes to the reduction of poverty.

This paper thus makes only a partial contribution to improving the allocation of aid among countries, which itself is only part of the overall problem of improving aid. It takes the global total of aid as given, and does not ask how to deliver aid to countries better. Nor, of course, would improving aid be sufficient to eliminate world poverty. What the paper offers is much more limited, but still worth having: a better grasp of the time dimension of the objective of poverty-oriented aid.

---

[21] For these calculations, using the data on y^i in table 2, $h_0^i = y^{i-1}$. The same values of Δ_H^i, also from table 2, are used as for the calculations in panels A and B.

Appendix: Comparison of typical donor formula with Collier-Dollar model

The allocation formulae used by donors are often labelled 'Collier-Dollar', and are based on similar principles – the aim of using aid to reduce poverty and the belief that the effectiveness of aid depends on the quality of a country's policies. However, their algebraic forms are clearly different (compare equations 4 and 17) and they generate different numbers (for given measures of poverty and policy: compare table 4 and panel A of table 6). The differences arise from different views about exactly how aid contributes to poverty reduction – different assumptions about the form of $H_0^i(a_0^i)$.

The typical donor formula, though not derived from an optimising model (as in C-D), does imply such a model, which can be made explicit. The formula in equation (17) allocates aid to each country in proportion to its poverty-policy score

$$a_0^i = \varphi h_0^i N_0^i P^{i\gamma} \qquad\qquad (A1)$$

where

$$\varphi = \frac{\overline{A}_0}{\sum_i h_0^i N_0^i P^{i\gamma}} \qquad\qquad (A2)$$

which divides up total available aid, $\overline{A}_0$. The factor of proportionality, φ, can be interpreted as the inverse, $1/\lambda$, of a Lagrangian multiplier (provided that all countries have positive scores, and hence all are allocated some aid by the formula). Equation (A1) can thus be rearranged as first-order conditions

$$\frac{h_0^i N_0^i P^{i\gamma}}{a_0^i} = \lambda \qquad\qquad (A3)$$

and integrated to infer the relevant parts of the implied Lagrangian

$$L = \sum_i \left(C^i - h_0^i N_0^i P^{i\gamma} \ln a_0^i \right) + \lambda \left(\overline{A}_0 - \sum_i a_0^i \right) \qquad (A4)$$

where C is the constant of integration (a sum of terms which affect the amount of poverty in a country but are unrelated to its level of aid). The specific form of the function $H_0^i(a_0^i)$ implicitly assumed by donors is thus

$$H_0^i(a_0^i) = C^i - h_0^i N_0^i P^{i\gamma} \ln a_0^i \qquad\qquad (A5)$$

in which the effect of aid on poverty depends on the initial level of poverty, multiplied by the natural log of the amount of aid and by the country's policy score to the power γ. The specific form assumed by C-D, in comparable notation, is

$$H_0^i\!\left(a_0^i\right)= C^i + \frac{h_0^i N_0^i}{Y^i}\,\alpha^i\!\left(b_3 a_0^i + b_4\,\frac{a_0^{i2}}{Y^i} + b_5 a_0^i P^i \right) \qquad \text{(A6)}$$

which includes two extra variables: the poverty elasticity, α^i, whose negative sign makes the whole non-constant term negative, as in (A5), and national income, Y^i.

Their inclusion reflects a basic divergence of views about how aid reduces poverty. The donor formula assumes that aid can be targeted within each country on people who are poor, so that there is a direct link (through unspecified channels) from the amount of aid to the amount of poverty reduction. By contrast, as emphasised by Beynon (2003), C-D assume that aid cannot be targeted within countries. In their model, the only channel through which aid can reduce poverty in a country is by accelerating its growth – thus raising its per capita income, which lowers poverty to an extent dependent on its poverty elasticity. Thus in (A6), unlike (A5), total initial poverty, $h_0^i N_0^i$, is divided by national income, Y^i (= $y^i N_0^i$), which reflects the fact that with the C-D form of $H_0^i\!\left(a_0^i\right)$ a given amount of aid reduces poverty more if it is allocated to a country with more poverty per dollar of national income.

This difference in assumptions explains why middle-income regions do worse in table 4 than in table 6. According to the donor equation (A5), to maximise the impact on world poverty reduction, aid should be allocated among countries in proportion to the absolute amounts of poverty in them. But the C-D equation (A6) allocates little or no aid to countries where poverty is large in absolute terms but small relative to national income (which depends on population size and per capita income). If the poor are a small share of the population (a low h^i), most aid (if it simply raised average incomes) would be 'wasted' on the non-poor majority. And because growth in the C-D model depends on the ratio of aid to national income, a higher per capita income (y^i) means that a given absolute amount of aid would add less to the growth of income.

There are other differences between (A5) and (A6) in their assumptions about the effects of aid on poverty, which also affect optimal allocations. One concerns how fast returns to aid diminish, which can be compared by calculating second derivatives, to obtain for the donor equation

$$\frac{d^2 H_0^i}{da_0^{i2}} = \frac{h_0^i N_0^i P^{i\gamma}}{a_0^{i2}} \qquad \text{(A7)}$$

and for the C-D equation

$$\frac{d^2 H_0^i}{da_0^{i2}} = \alpha^i\,\frac{2b_4}{N_0^i}\,\frac{h_0^i}{y^{i2}} \qquad \text{(A8)}$$

which like (A7) is positive (because both α^i and b_4 have negative signs), meaning that the poverty-reducing effect of aid declines as the level of aid increases. The relative speeds at which returns diminish in the two equations depends on the values of their other variables and parameters, but the log form of (A5) implies rapid diminution (e.g. each doubling of aid cuts poverty by only the same absolute amount).

Another important influence on optimal allocation is how much the effectiveness of aid in reducing poverty in a country is thought to depend on the quality of its policies. To compare this aspect of equations (A5) and (A6), they can be differentiated first with respect to a_0^i and then with respect to P^i, to obtain for the donor equation

$$\frac{\partial^2 H_0^i}{\partial a_0^i \partial P^i} = -\gamma \frac{h_0^i N_0^i P^{i(\gamma-1)}}{a_0^i} \qquad \text{(A9)}$$

and for the C-D equation

$$\frac{\partial^2 H_0^i}{\partial a_0^i \partial P^i} = \alpha^i b_5 \frac{h_0^i}{y^i} \qquad \text{(A10)}$$

which like (A9) is negative (because of the sign of α^i), meaning that aid reduces poverty by more as the policy score increases. The relative impact of policy in these two equations again depends on the values of their other variables and parameters.

References

Alesina, Alberto, and David Dollar (2000), 'Who gives foreign aid to whom, and why', *Journal of Economic Growth*, 5/1: 33-63.

Anderson, Edward, and Hugh Waddington (2007), 'Aid and the Millennium Development Goal poverty target: how much is required and how should it be allocated?', *Oxford Development Studies*, 35/1: 1-32.

Beynon, Jonathan (2002), 'Policy implications for aid allocations of recent research on aid effectiveness and selectivity', in B. M. Arvin (ed.), *New Perspectives on Foreign Aid and Development*, Westport CT: Praeger.

Beynon, Jonathan (2003), 'Poverty efficient aid allocations: Collier/Dollar revisited', ESAU Working Paper 2, London: Overseas Development Institute, available at http://www.odi.org.uk/esau/publications/working_papers/esau_wp2.pdf

Beynon, Jonathan, and Peter Landymore (2002), 'Bilateral programme: poverty efficient aid allocation by country', background paper for Dyer *et al*. (2003), available at http://www.dfid.gov.uk/pubs/files/bilateralworkstream.pdf

Clemens, Michael, Steven Radelet and Rikhil Bhavnani (2004), 'Counting chickens when they hatch: the short term effects of aid on growth', Washington DC: Center for Global Development Working Paper 44. Available from http://www.cgdev.org/files/2744_file_CountingChickensFINAL3.pdf

Cogneau, Denis, and Jean-David Naudet (2007), 'Who deserves aid? Equality of opportunity, international aid, and poverty reduction', *World Development*, 35/1: 104-20.

Collier, Paul, and David Dollar (2001), 'Can the world cut poverty in half? How policy reform and effective aid can meet international development goals,' *World Development*, 29/11: 1787-1802.

Collier, Paul, and David Dollar (2002), 'Aid allocation and poverty reduction', *European Economic Review*, 46/8: 1475-1500.

Collier, Paul, and Anke Hoeffler (2004), 'Aid, policy and growth in post-conflict societies', *European Economic Review*, 48/5: 1125-45.

Commission for Africa (2005), *Our Common Interest*, London. Available from www.commissionforafrica.org.

Dalgaard, Carl-Johan, Henrik Hansen and Finn Tarp (2004), 'On the empirics of foreign aid and growth', *Economic Journal*, 114 (June): F191-F216.

Dyer, Nick, with many others (2003), 'Strategic review of resource allocation priorities', DFID, available from http://www.dfid.gov.uk/pubs/files/resourceallocation.pdf

Easterly, William (2006), *The white man's burden: why the West's efforts to aid the rest have done so much ill and so little good*, New York: Penguin Press.

Guillaumont, Patrick, and Lisa Chauvet (2001), 'Aid and performance: a reassessment', *Journal of Development Studies*, 37/6: 66-92.

Isenman, Paul, Michel Devaux and John Edelman (1977), 'An analysis of the World Bank's lending program FY78-82: technical note', unpublished paper, Policy Planning and Program Review Department, Washington DC: World Bank.

Jones, Stephen, Roger Riddell and Katarina Kotoglou (2004), 'Aid allocation: managing for development results and difficult partnerships', unpublished paper, Oxford Policy Management.

Keith, Andrew (2002), 'Review of allocation mechanisms of IDA, AfDB, AsDB and EDF', unpublished background paper for Dyer *et al.* (2003).

Little, Ian, and James Mirrlees (1990), 'Project appraisal and planning twenty years on', in *Proceedings of the Annual Bank Conference on Development Economics 1990*, Washington DC: World Bank: 351-95.

McGillivray, Mark (2003), 'Aid effectiveness and selectivity: integrating multiple objectives into aid allocations', *DAC Journal* 4/3: 27-40.

Millennium Project (2005), *Investing in Development: A Practical Plan to Achieve the Millennium Development Goals*, New York: UNDP

Rajan, Raghuram, and Arvind Subramanian (2005), 'Aid and growth: what does the cross-country evidence really show?', *IMF Staff Working Paper* 05/127. Available from http://www.imf.org/external/pubs/ft/wp/2005/wp05127.pdf

Sachs, Jeffrey (2005), *The end of poverty: how we can make it happen in our lifetime*, London and New York: Penguin.

World Bank (2007a), *Global Economic Prospects*, Washington DC.

World Bank (2007b), *Global Monitoring Report*, Washington DC.

Figure 1: Combining current and future poverty

Poverty

H_0

H_T

0   T Time
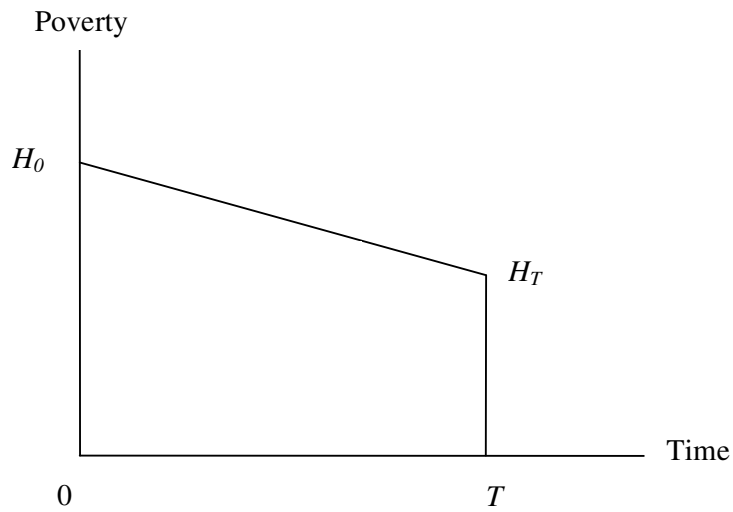
Table 1. *Effects of variation in r and T on poverty decline adjustment*
(D for country with Δ_H = 1% p.a., divided by D for country with Δ_H = 7% p.a.)

Values of			Values of T			
r (% p.a.)	1	5	10	25	100	∞
0	1.0	1.2	1.3	1.9	4.4	7.0
5	1.0	1.1	1.3	1.6	2.0	2.0
10	1.0	1.1	1.3	1.5	1.5	1.5
25	1.0	1.1	1.2	1.2	1.2	1.2
100	1.0	1.1	1.1	1.1	1.1	1.1

Source: Calculations using equation (13).

Table 2. *Variables influencing the allocation of aid*

	$1-a-day poverty headcount 2004 (%)	Population 2004 (millions)	Average per capita income 2004 (PPP$000)	Average CPIA score 2001	Projected decline in poverty 2004-30 (% p.a.)	Estimated income elasticity of poverty 2004-15
Sub-Saharan Africa	41.1	719	1.9	3.3	-0.3	-0.7
South Asia	32.0	1448	2.8	4.0	3.4	-1.4
India	35.8	1080	3.1	4.2	2.9	-1.1
Other South Asia	28.8	368	2.0	3.6	7.3	-3.8
East Asia	9.1	1870	5.1	3.9	9.6	-2.3
China	9.9	1297	5.5	4.1	9.7	-1.9
Other East Asia	8.8	574	4.0	3.6	9.2	-3.7
Latin America	8.6	541	7.7	4.0	0.8	-1.7
Europe and Central Asia	0.9	472	8.4	3.2	2.7	-1.7
Middle East and North Africa	1.5	294	5.8	4.0	1.8	-2.5
Developing regions	18.4	5344	4.6	3.8	2.1	-1.3

Notes: East Asia includes Pacific; Latin America includes Caribbean.
Sources: population and per capita income from World Development Indicators; for others, see text.

Table 3. *Illustrative aid allocation with poverty-decline-adjusted C-D model (r = 0, T = 25)*

	$1-a-day poverty headcount 2004 (millions)	Poverty decline adjust-ment	'Present value' of future poverty	Optimal shares of total aid (%)	Poverty over GNI (poor per $million) 2004	Actual shares of total aid in 2004 (%)
Sub-Saharan Africa	298	26	7785	32.5	222	37.1
South Asia	462	17	7825	67.5	113	10.4
India	386	18	6873	26.0	115	1.5
Other South Asia	76	12	873	41.6	141	8.8
East Asia	169	9	1603	0.0	18	10.7
China	128	9	1205	0.0	18	1.9
Other East Asia	41	10	398	0.0	22	8.9
Latin America	47	23	1060	0.0	11	10.1
Europe and Central Asia	4	18	81	0.0	1	16.9
Middle East and North Africa	4	20	89	0.0	3	14.8
Developing regions	985	20	18443	100.0	40	100.0

Sources: Actual aid from World Development Indicators (2004 total $70 billion); for others, see text.

Table 4. *Effects of alternative values of r and T on illustrative aid allocation*

| | Aid shares (%) | | | |
	Africa/ low-income	India/ low-income	OSA/ South Asia	Africa/ Africa+OSA
No weight on future ($r = 0$, $T = 1$)	0.0	34.7	65.3	0.0
Some weight on future ($r = 10\%$, $T = 25$)	14.0	33.1	61.5	20.9
More weight on future ($r = 0$, $T = 25$)	32.5	26.0	61.6	43.8
Greater weight on future ($r = 0$, $T = 50$)	67.2	12.2	63.0	76.5
Most weight on future ($r = 0$, $T = 100$)	100.0	0.0	n.a.	100.0

Notes: 'Low-income' is sub-Saharan Africa plus South Asia; 'OSA' is Other South Asia
Source: simulations explained in text, identical in all other respects to those in table 3.

Table 5. *Sensitivity of allocation to parameters of C-D aid-poverty reduction function*

| | Parameter values | | | Aid shares (%) | | | |
	b_3	b_4	b_5	Africa/ low-income	India/ low-income	OSA/ South Asia	Africa/ Africa+OSA
Base case	-0.54	-0.02	0.31	32.5	26.0	61.6	43.8
Sensitivity to quality of policy							
Greater	-0.93	-0.02	0.42	15.1	45.9	45.9	27.8
Lesser	-0.15	-0.02	0.21	52.4	1.1	97.7	53.0
Diminishing returns to aid							
Faster	-0.54	-0.04	0.31	24.2	50.8	33.0	49.1
Slower	-0.54	-0.01	0.31	33.6	0.0	100.0	33.6
Poverty elasticities in all regions = -2							
	-0.54	-0.02	0.31	74.9	25.1	0.0	100.0

Source: Simulations explained in text. 'Base case' is that in table 3, and in all variants only the indicated parameters are altered: $r = 0$ and $T = 25$ in all variants.

Table 6. *Illustrative aid allocations with poverty-decline-adjusted donor formula*

| | Aid shares (%) | | | | |
	Africa/ low-income	India/ low-income	OSA/ South Asia	Africa/ Africa+OSA	Middle- income
A. Headcount poverty measure					
No weight on future ($r = 0$, $T = 1$)	29.8	61.3	12.7	77.0	23.7
More weight on future ($r = 0$, $T = 25$)	39.0	55.6	8.7	88.0	16.9
Greater weight on future ($r = 0$, $T = 50$)	47.9	48.5	7.0	92.9	14.3
Most weight on future ($r = 0$, $T = 100$)	62.2	35.6	5.9	96.6	11.8
B. Headcount poverty measure, India capped					
No weight on future ($r = 0$, $T = 1$)	73.0	5.3	80.5	77.0	43.1
More weight on future ($r = 0$, $T = 25$)	84.2	4.3	72.7	88.0	30.5
Greater weight on future ($r = 0$, $T = 50$)	89.3	3.9	63.3	92.9	23.7
Most weight on future ($r = 0$, $T = 100$)	93.1	3.6	47.9	96.6	16.7
C. Per capita income poverty measure					
No weight on future ($r = 0$, $T = 1$)	33.7	48.0	27.7	64.7	38.7
More weight on future ($r = 0$, $T = 25$)	44.7	44.1	20.2	80.0	30.2
Greater weight on future ($r = 0$, $T = 50$)	54.4	38.1	16.5	87.8	25.9
Most weight on future ($r = 0$, $T = 100$)	68.5	27.1	14.1	93.9	20.9

Source: Calculations with equations (18 - 19), using the same data as in tables 2 and 3. In all cases, $\gamma = 2$. Equations modified for panel C, as explained in text.