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The comparative study of chaoticity and dynamical complexity of the low-latitude ionosphere, over Nigeria, during quiet and disturbed days

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Abstract. The deterministic chaotic behavior and dynamical complexity of the space plasma dynamical system over Nigeria are analyzed in this study and characterized. The study was carried out using GPS (Global Positioning System) TEC (Total Electron Content) time series, measured in the year 2011 at three GPS receiver stations within Nigeria, which lies within the equatorial ionization anomaly region. The TEC time series for the five quietest and five most disturbed days of each month of the year were selected for the study. The nonlinear aspect of the TEC time series was obtained by detrending the data. The detrended TEC time series were subjected to various analyses for phase space reconstruction and to obtain the values of chaotic quantifiers like Lyapunov exponents, correlation dimension and also Tsallis entropy for the measurement of dynamical complexity. The observations made show positive Lyapunov exponents (LE) for both quiet and disturbed days, which indicates chaoticity, and for different days the chaoticity of the ionosphere exhibits no definite pattern for either quiet or disturbed days. However, values of LE were lower for the storm period compared with its nearest relative quiet periods for all the stations. The monthly averages of LE and entropy also show no definite pattern for the month of the year. The values of the correlation dimension computed range from 2.8 to 3.5, with the lowest values recorded at the storm period of October 2011. The surrogate data test shows a significance of difference greater than 2 for all the quantifiers. The entropy values remain relatively close, with slight changes in these values during storm periods. The values of Tsallis entropy show similar variation patterns to those of Lyapunov exponents, with a lot of agreement in their comparison, with all computed values of Lyapunov exponents correlating with values of Tsallis entropy within the range of 0.79 to 0.81. These results show that both quantifiers can be used together as indices in the study of the variation of the dynamical complexity of the ionosphere. The results also show a strong play between determinism and stochasticity. The behavior of the ionosphere during these storm and quiet periods for the seasons of the year are discussed based on the results obtained from the chaotic quantifiers.

1 Introduction

In our real world, most natural systems are nonlinear. However, a natural system can be either deterministic or stochastic. There is usually no distinction between the two in our natural system since all systems in one way or the other interact with their surroundings. Although few natural systems have been found to be low-dimensional deterministic in the sense of the theory, the concept of low-dimensional chaos has been proven to be fruitful in the understanding of many complex phenomena (Hegger et al., 1999). This is also evident in the study on magnetospheric dynamics and the ionosphere in the past decade. The study of chaos in magnetospheric index time series such as the AE index and the AL index was initially conducted by Vassiliadis et al. (1990), Shan et al. (1991), Pavlos et al. (1992) and Unnikrishnan (2008). These efforts have led to further advances in the study of chaos in the upper atmosphere. The development of this framework is

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based on the advances in the study of chaos in the ionosphere, revealing its complexity.

Excellent studies have been carried out relating to this framework, by some investigators like Bhattacharyya (1990), who studied chaotic behavior of ionospheric density fluctuation using amplitude and phase scintillation data, and found the existence of low-dimension chaos. Also, Wernik and Yeh (1994) further revealed the chaotic behavior of the ionospheric turbulence using scintillation data and numerical modeling of scintillation at high latitudes. They showed that the ionospheric turbulence attractor (if it exists) cannot be reconstructed from amplitude scintillation data, and their measured phase scintillation data adequately reproduce the assumed chaotic structure in the ionosphere. Also, Kumar et al. (2004) reported the evidence of chaos in the ionosphere by showing the chaotic nature of the underlying dynamics of the fluctuations of the TEC power spectrum, indicating exponential decay and the calculated positive value of the Lyapunov exponent. This is also supported by the results of the comparison of the chaotic characteristics of the time series of variations of TEC with the pseudochaotic characteristic of the colored noise time series. Also, Unnikrishnan et al. (2006a, b) analyzed the deterministic chaos at mid-latitude. Also, Unnikrishnan (2010); Unnikrishnan and Ravindran (2010) analyzed some TEC data from some Indian low-latitude stations for quiet and major storm periods, and found in their results the presence of chaos, which was indicated by a positive Lyapunov exponent and also showed storm periods with lower values compared with quiet periods.

A good number of investigators have also worked on the dynamical complexity of magnetospheric processes and the ionosphere. Balasis et al. (2008) investigated the dynamical complexity of the magnetosphere by using Tsallis entropy as a dynamical complexity measure in D_{st} time series; Balasis et al. (2009) also investigated the dynamical complexity in D_{st} further by considering different entropy measures. Coco et al. (2011), using the information theory approach, studied the dynamical changes of the polar cap potential, which is characteristic of the polar region ionosphere, by considering three cases: (i) steady IMF $B_{\mathrm{z}} < 0$, (ii) steady IMF $B_{\mathrm{z}} > 0$ and (iii) a double rotation from negative to positive and then positive to negative B_{z}. They observed a neat dynamical topological transition when the IMF B_{z} turns from negative to positive and vice versa, pointing to the possible occurrence of an order–disorder phase transition, which is the counterpart of the large-scale convection rearrangement and of the increase in the global coherence. Further studies of the chaotic behavior are however needed to improve our understanding of the dynamical behavior of the ionosphere of the low-latitude ionosphere, especially over Africa during quiet and storm periods for different seasons of the year, so as to be able to characterize chaoticity for different seasons.

The Nigerian subcontinent of Africa is situated in the low-latitude region of the globe within the equatorial anomaly region where the magnetic field B is almost totally parallel to the Earth's surface. In a low-latitude region such as this, off the Equator map along the F region, the eastward electric field (E) of the E region interacts with the magnetic field B during the day. This results in the electrodynamic lifting of the F region plasma over the Equator, which is known as EXB drift. The uplifted plasma over the Equator moves along the magnetic field lines in response to gravity, diffusion and pressure gradients and hence, the fountain effect. The fountain effect being controlled by the EXB drift shows the dynamics of the diurnal variation equatorial anomaly (Abdu, 1997; Unnikrishnan, 2010). There is a reduction in the F region ionization density at the magnetic Equator and much enhanced ionization density at the two anomaly crests within $\pm 15°$ of the magnetic latitude north and south of the Equator (Rama Rao et al., 2006).

The equatorial ionization anomaly and other natural processes, which include various ionization processes and recombination, the influx of solar wind, and photoionization processes among many others have a great influence on the internal dynamics of the systems of the ionosphere, and form the natural internal dynamics of the ionosphere. However, as mentioned before, there is no system in the real world without interaction with its external environment, which leads us to further study of the influence of the Sun on the ionosphere. Therefore in this study the Lyapunov exponent and Tsallis entropy from the TEC data from the ionosphere were studied for storm and quiet periods throughout the months of the year for a better understanding of the deterministic chaotic behavior and dynamical complexity of the ionosphere.

The motivation for this work is based on the fact that there is a significant level of nonlinear variability in the ionosphere that needs more investigation. Characterizing the ionosphere is of utmost importance due to numerous complexities associated with the region (Rabiu et al., 2007). The concept of chaos applied to ionospheric and magnetospheric studies of the quiet and stormy conditions is limited. The review of the works mentioned earlier shows that a good number of investigations has been carried out on the chaoticity of the upper atmosphere, especially in the magnetosphere and the ionosphere, and a number of investigations have also been carried out on the dynamical complexity of the upper atmosphere, but mainly on the magnetosphere.

To the best of our knowledge no extensive study has been carried out for all geomagnetic conditions, including quiet days, disturbed days and stormy conditions. Most investigations have only been based on quiet and storm conditions, and for all studies carried out none of the investigators has carried out a study of these concepts using the geomagnetically quiet and disturbed day classification to enable the proper characterization of the ionosphere. Secondly, this concept has not been used to study the ionosphere over the low-latitude region of Africa, and finally the study of dynamical complexity using the Tsallis entropy of the ionosphere has not been carried out using the total electron content in low-latitude Africa or any other part of the world. However, for

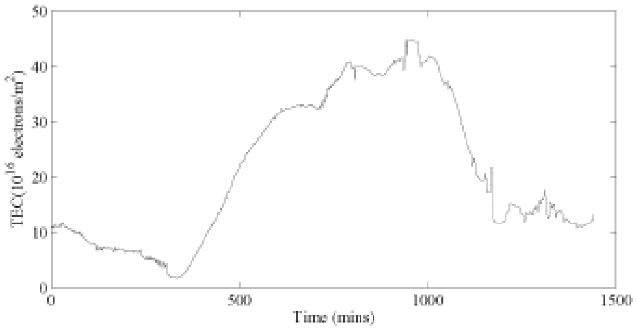

Fig. 1. A typical daily time series plot for TEC data measured from Lagos showing the typical pattern of variation.

the first time these issues mentioned are considered in this work.

2 Data and methodology

In Nigeria a number of GPS receivers are installed in different parts of the country, which falls within the equatorial ionization anomaly region. These receiver stations are called the NIGNET stations. The TEC values are gotten from these GPS receivers taking the measurement of slant TEC within a 1 m^2 columnar unit of the cross section along the ray path of the satellite and the receiver, which is given by

$$\mathrm{STEC} = \int_{\mathrm{Receiver}}^{\mathrm{Satellite}} N \mathrm{d}l. \tag{1}$$

The observation of the total number of free electrons along the ray path is derived from the frequencies L_1 (1572.42 MHz) and L_2 (1227.60 MHz) of the GPS that provide the relative ionospheric delay of electromagnetic waves travelling through the medium (Saito et al., 1998). The slant TEC is projected to vertical TEC using the thin shell model assuming a height of 350 km (Klobuchar, 1986).

$$\mathrm{VTEC} = \mathrm{STEC} \cdot \cos[\arcsin(R_\mathrm{e} \cos\Theta / R_\mathrm{e} + h_{\max})], \tag{2}$$

where $R_\mathrm{e} = 6378$ km (radius of the Earth), $h_{\max} = 350$ km (vertical height assumed in the model) and Θ = the elevation angle at the ground station.

In this study, three stations were considered: Birnin Kebbi (12°32′ N, 4°12′ E and 0.62° N geomagnetically), Enugu (6°26′ N, 7°30′ E and −3.21° N geomagnetically) and Lagos (6°27′ N, 3°23′ E and −3.07° N geomagnetically) within the low-latitude region. The TEC data obtained for January to December 2011 were considered for this study and the data are given at 1 min sampling time. The values of the five quietest days of each month were taken and the same was done for the five most disturbed days using the list of International Quiet Days (IQD) and International Disturbed Days (IDD) data provided by Geoscience Australia to compare the chaotic behavior of quiet days with that of disturbed days. The TEC data were subjected to various analyses that will be discussed in the next section.

3 Data analysis and results

3.1 Time series analysis

A given time series Sn can be defined as a sequence of scalar measurement of the same quantity taken as a series at different portions in time for a given time interval (δt). The time series describe the physical appearance of an entire system. However, it may not always describe the internal dynamics of that system. A system like the ionosphere possesses a dominant dynamics that can be seen as diurnal, so the data should be treated so as to be able to see its internal dynamics. The measured TEC time series were plotted to see the dynamics of the system. A typical plot of TEC usually has a dominant dynamics (see Fig. 1), which may be seen as the diurnal behavior. However, it can also be seen that there is a presence of fluctuations (which appear to be nonlinear) in the system as a result of the internal dynamics of the ionosphere and space plasma system due to different activities in the ionosphere. Therefore there is a need to minimize the influence of the diurnal variations, since we are more interested in the nonlinear internal dynamics of the system in this study. To do so the TEC time series was detrended by carrying out the analysis below, since it is known that there are 1440 data points in a day for daily data of 1 min sampling time, if we let a_i and t_i, $i = 1, 2, 3, \ldots, 1440$ represent the actual and observed values, respectively, of the time series of TEC. The diurnal variation reduced time is given by

$$T_i = t_i - a_j, \tag{3}$$

where $j = \mathrm{mod}\ (i, 1440)$ if $\mathrm{mod}\ (j, 1440) \neq 0$, and $j = 1440$ if $d\ (j, 1440) = 0$.

This method will give the detrended time series from the original TEC data as shown in Fig. 2. This method is similar to that used by Unnikrishnan et al. (2006a, b); Unnikrishnan (2010). Further explanations of the dynamical results can be found in Kumar et al. (2004).

3.1.1 Phase space reconstruction and nonlinear time series analysis

The chaoticity and dynamical complexity of a system are nonlinear phenomena that can describe the state of some dynamical systems. Such systems may be seen as nonlinear complex systems. The magnetosphere and the ionosphere are good examples of such systems. To be able to study such phenomena, some nonlinear time series analyses can be carried out on the time series data describing such a system. The detrended time series of TEC measurement is subjected to some

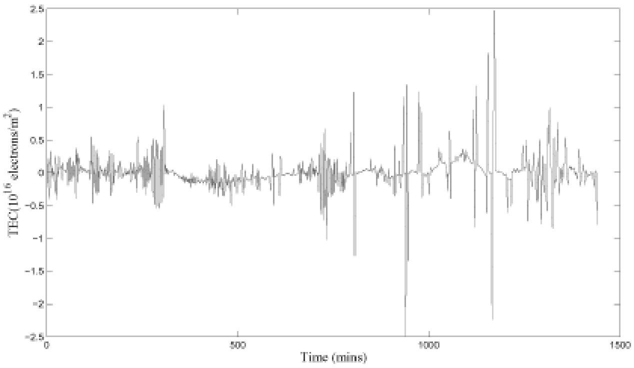

Fig. 2. A typical daily time series plot for the detrended TEC data measured from Lagos showing the typical pattern of variation.

nonlinear time series data analyses to obtain the mutual information and false nearest neighbors, embedding dimension and delay coordinates for the phase space reconstruction, and the evaluation of other chaotic quantifiers: Lyapunov exponents, entropy. The phase space reconstruction helps to reveal the multidirectional aspect of the system. The phase space reconstruction is based on the embedding theorem, such that the phase space is reconstructed to show the multidimensional nature as follows:

$$\boldsymbol{Y}_n = (s_{n-(m-1)\tau}, s_{n-(m-2)\tau}, \ldots, s_n), \tag{4}$$

where $\boldsymbol{Y}_{\mathrm{n}}$ are vectors in phase space. The proper choice of embedding dimension (m) and delay time (τ) is essential for phase space reconstruction (Fraser and Swinney, 1986; Kennel et al., 1992). If a plot showing the time-delayed mutual information shows a marked minimum, that value can be considered as a responsible time delay. Figure 3 shows the mutual information plotted against time delay. Likewise, the minimal embedding dimension, which corresponds to the minimum number of the nearest false neighbors as shown in Fig. 4, can be treated as the optimum value of the embedding dimension in Unnikrishnan et al. (2006a, b) and Unnikrishnan (2010). It was observed that for all the daily detrended TEC time series the choice of $\tau \geq 25$ and $m = 3$ values of delay and embedding dimension above these values are suitable for analysis of data for all stations. The choice of $\tau = 30$ and $m = 5$ was also used for further analysis for most of the stations. The reconstructed phase space trajectory is shown in Fig. 5.

3.1.2 Lyapunov exponents

The Lyapunov exponent is an important chaotic quantifier. It indicates divergence of trajectory in one dimension, or alternatively an expansion of volume, which can also be said to indicate repulsion, or attraction from a fixed point. A positive Lyapunov exponent indicates that there is evidence of chaos in a dissipative deterministic system, where the positive Lyapunov exponent indicates divergence of trajectory in one direction or expansion of values, and a negative value shows convergence of trajectory or contraction of volume along another direction. The largest Lyapunov exponent (λ_1) can be used to determine the rate of divergence as indicated by Wolf et al. (1985) where

$$\lambda_1 = \lim_{r \to \inf} \frac{1}{t} \log \frac{\Delta x(t)}{x(0)} = \lim_{r \to \inf} \frac{1}{t} \sum_{i=1}^{t} \log \frac{\Delta x(t_i)}{\Delta x(t_{i-1})}. \tag{5}$$

The Lyapunov exponent was computed for the TEC values measured from different stations. The evolution in state space was scanned with $\tau = 30$, $m = 5$, as shown in Fig. 6. The values of the Lyapunov exponent were computed for all stations for the five quietest and five most disturbed days of every month of the year according to the International Quiet Days (IQD) and International Disturbed Days (IDD) classification by Geoscience Australia. The difference in values was studied for these situations and compared. This was done using the implementation introduced by Rosenstein et al. (1993), and Hegger et al. (1999), both algorithms using very similar methods. The values of the Lyapunov exponent plotted for the five quietest and five most disturbed days are shown in Fig. 8a–d and show the plot values of the computed Lyapunov exponent from January to April 2011 for Birnin Kebbi station. The monthly averages of Lyapunov exponents for quiet and disturbed periods are shown in Fig. 8e.

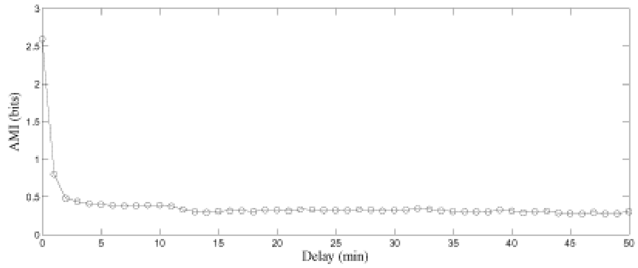

Fig. 3. Plot of mutual information against time delay obtained from the detrended TEC data measured from Lagos.

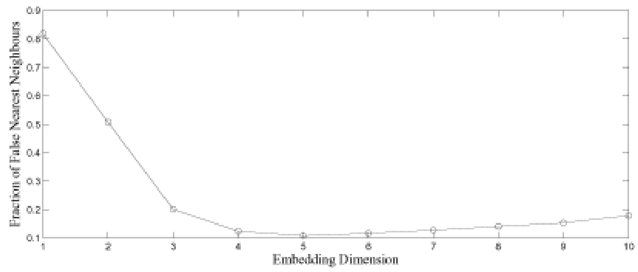

Fig. 4. Plot of fraction of false nearest neighbors against embedding dimension obtained from the detrended TEC data measured from Lagos.

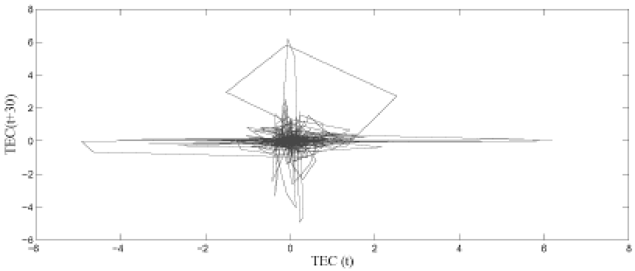

Fig. 5. Delay representation from the phase space reconstruction of the detrended time series.

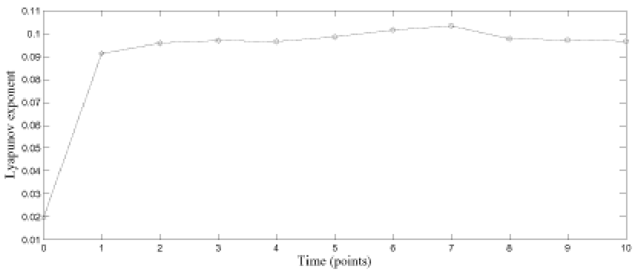

Fig. 6. Lyapunov exponent and its evolution, computed as the state space trajectory with $\tau = 30$, and $m = 5$ for detrended time series measured at Lagos with the largest Lyapunov exponent equal to 0.1035.

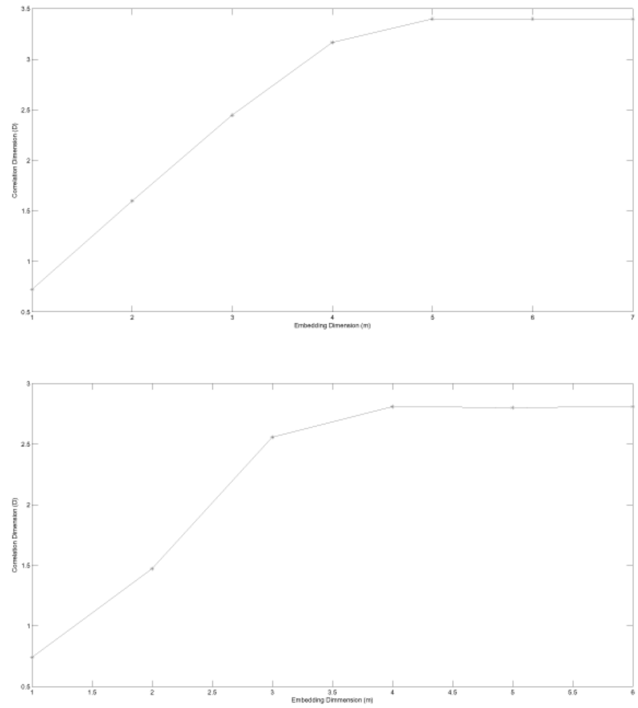

Fig. 7. Correlation dimension for (upper panel) the quietest day of October 2011 for TEC measured at Lagos that saturates at $\tau = 30$, and $m \geq 5$ (lower panel) the most disturbed day of October 2011 for TEC measured at Lagos that saturates at $\tau = 32$, and $m \geq 4$.

3.1.3 Correlation dimension

Another relevant method for studying the underlying dynamics or internal dynamics of a system is to have a good knowledge of its dimension. The correlation dimension gives a good approximation of this as suggested by Grassberger and Procaccia (1983a, b). The correlation dimension D is defined as

$$D = \lim r \to 0 \frac{\ln C(r)}{\ln r}. \quad (6)$$

The term $C(r)$ is the correlation sum for radius r where for a small radius r the correlation sum can be seen as $C(r) \sim r^d$ for $r \to 0$. The correlation sum is dependent on the embedding dimension m of the reconstructed phase space and is also dependent on the length of the time series N as follows:

$$C(r) = \frac{2}{N(N-1)} \sum_{i=1}^{N} \sum_{j=i+1}^{N} \Theta(r - ||y_i - y_j||) \quad (7)$$

where Θ is the Heaviside step function, with $\Theta(H) = 0$ if $H \leq 0$ and $\Theta(H) = 1$ if $H > 0$. The correlation dimension was computed using the Theiler algorithm approach, with Theiler window w at 180. The Theiler window was chosen to be approximately equal to the product of m and τ. A similar approach to the computation of correlation dimensions was used by Unnikrishnan and Ravindran (2010) to determine the correlation dimension of detrended TEC data for some stations in India, which lies within the equatorial region, like Nigeria. The correlation dimension for data taken for the quietest day of October 2011 and the most disturbed day of October 2011 from Lagos GPS TEC measuring station are represented by Fig. 7. The correlation dimension saturates at $m \geq 5$ for the quietest day of the month and at $m \geq 4$ for the most disturbed day. In this illustration the most disturbed day of this month falls within the storm period of October 2011. The use of quiet and disturbed day classifications in the month of October 2011 enables us to compare the quiet and storm periods together while comparing the quiet days with some relatively disturbed days.

3.1.4 Computation of Tsallis entropy and other entropy measures

Entropy measures are very important statistical techniques that can be used to describe the dynamical nature of a system. Tsallis entropy can be used to describe the dynamical complexity of a system and also to understand the nonlinear dynamics like chaos that may exist in a natural system. The use of entropy measure as a method to describe the state of a physical system has been employed in information theory for decades. Since entropies allow us to describe the state of disorderliness in a system, one can generalize this same concept to characterize the amount of information stored in

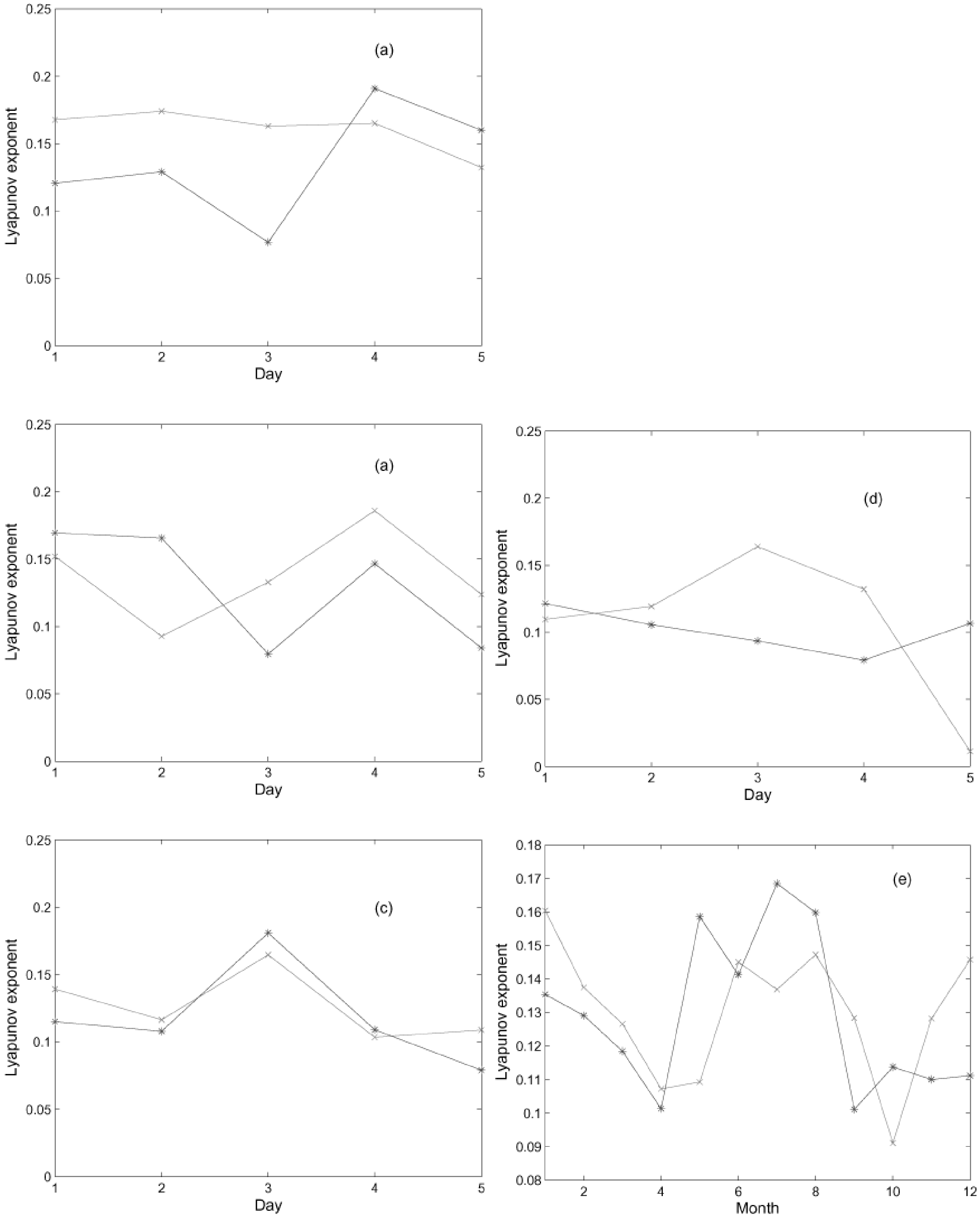

Fig. 8. Lyapunov exponents for the five quietest days (red) and the five most disturbed days (blue) for **(a)** January, **(b)** February, **(c)** March, and **(d)** April. **(e)** Monthly mean values of Lyapunov exponents for the entire year at Birnin Kebbi.

Table 1. Results of the surrogate data test for Lyapunov exponents for TEC data for the quietest days of October 2011 at Lagos station.

Original Data	Surrogate Data
0.1118	0.3397 ± 0.0325
0.0896	0.3493 ± 0.0594
0.1382	0.4091 ± 0.0684
0.1162	0.2967 ± 0.0588
0.2232	0.3914 ± 0.0207

Table 2. Results of the surrogate data test for Lyapunov exponents for TEC data for the most disturbed days of October 2011 at Lagos station.

Original Data	Surrogate Data
0.0563	0.3314 ± 0.0433
0.0370	0.1641 ± 0.0323
0.1763	0.3426 ± 0.0424
0.0872	0.2772 ± 0.0193

more general probability distributions (Kantz and Schreiber, 2003; Balasis et al., 2009). The concept of information theory is basically concerned with these principles. Information theory gives us an important approach to time series analysis, if our time series, which is a stream of numbers, is given as a source of information such that these numbers are distributed according to some probability distribution, and transitions between numbers occur with well-defined probabilities. One can deduce the same average behavior of the system at a different point and for the future.

The term entropy is used in both physics and information theory to describe the amount of uncertainty or information inherent in an object or system (Kantz and Schreiber, 2003). The state of an open system is usually associated with a degree of uncertainty that can be quantified by the Boltzmann–Gibbs entropy, a very useful uncertainty measure in statistical mechanics. However, Boltzmann–Gibbs entropy cannot describe non-equilibrium physical systems with large variability and multifractal structure such as the solar wind (Burlaga et al., 2007; Balasis et al., 2008). One of the crucial properties of the Boltzmann–Gibbs entropy in the context of classical thermodynamics is extensivity, namely proportionality with the number of elements of the system. The Boltzmann–Gibbs entropy satisfies this prescription if the subsystems are statistically (quasi-)independent, or typically if the correlations within the system are essentially local. In such cases the system is called extensive. In general, however, the situation is not of this type and correlations may be far from negligible at all scales. In such cases, the Boltzmann–Gibbs entropy is nonextensive (Balasis et al., 2008, 2009). These genralizations above were propsed by Tsallis (1988, 1998, 1999), who was inspired by the probabilistic description of multifractal geometries. He introduced an entropy measure by presenting an entropic expression characterized by an index q that leads to a nonextensive statistics,

$$S_q = k\frac{1}{q-1}(1-\sum_{i=1}^{W} p_i^q), \tag{8}$$

where p_i are the probabilities associated with the microscopic configurations, W is their total number, q is a real number, and k is Boltzmann's constant. The value q is a measure of the nonextensitvity of the system: $q \to 1$ corresponds to the standard extensive Boltzmann–Gibbs statistics. This is the basis of the so-called nonextensive statistical mechanics, which generalizes the Boltzmann–Gibbs theory. The entropic index q characterizes the degree of nonadditivity reflected in the following pseudoadditivity rule:

$$S_q(A+B) = S_q(A) + S_q(B) + (1-q)S_q(A)S_q(B). \tag{9}$$

The cases $q > 1$ and $q < 1$ correspond to subadditivity (or subextensivity) and superadditivity (or superextensivity), respectively, and $q = 1$ represents additivity (or extensivity). For subsystems that have special theory probability correlations, extensivity is not valid for Boltzmann–Gibbs entropy, but may occur for S_q with a particular value of the index q. Such systems are sometimes referred to as nonextensive (Boon and Tsallis, 2003; Balasis et al., 2008, 2009). The parameter q itself is not a measure of the complexity of the system, but measures the degree of nonextensivity of the system. It is the time variations of the Tsallis entropy for a given $q(S_q)$ that quantify the dynamic changes of the complexity of the system. Lower S_q values characterize the portions of the signal with lower complexity. In this presentation we estimate S_q on the basis of the concept of symbolic dynamics and by using the technique of lumping (Balasis et al., 2008, 2009).

Considering the fact that Tsallis entropy has been used extensively for magnetospheric studies to obtain interesting results for the dynamical complexity by Balasis et al. (2008, 2009), we find it necessary to consider its application to the study of ionospheric dynamics. It is also necessary to compare the results obtained from the computation of Tsallis entropy with those of the Lyapunov exponent. A comparison of the relationship between the values of the Lyapunov exponent and Tsallis entropy was carried out to show their relationship as measures of complexity. This is based on the fact that Tsallis entropy has been linked to a significant degree of response to the edge of chaos and chaotic regime dynamical systems due to its non-extensive nature (Baranger et al., 2002; Anastasiadis et al., 2005), and it has been linked to weak chaos and the vanishing largest Lyapunov exponent (Kalogeropoulos, 2012, 2013). It has been established that the Lyapunov exponent varies directly as the Tsallis entropy (complexity) of a system, based on the variation of the entropic index q introduced by Tsallis (1988) and the nature of the system's dynamics. Baranger et al. (2002) were able to show that the non-extensive case of Tsallis entropy has been

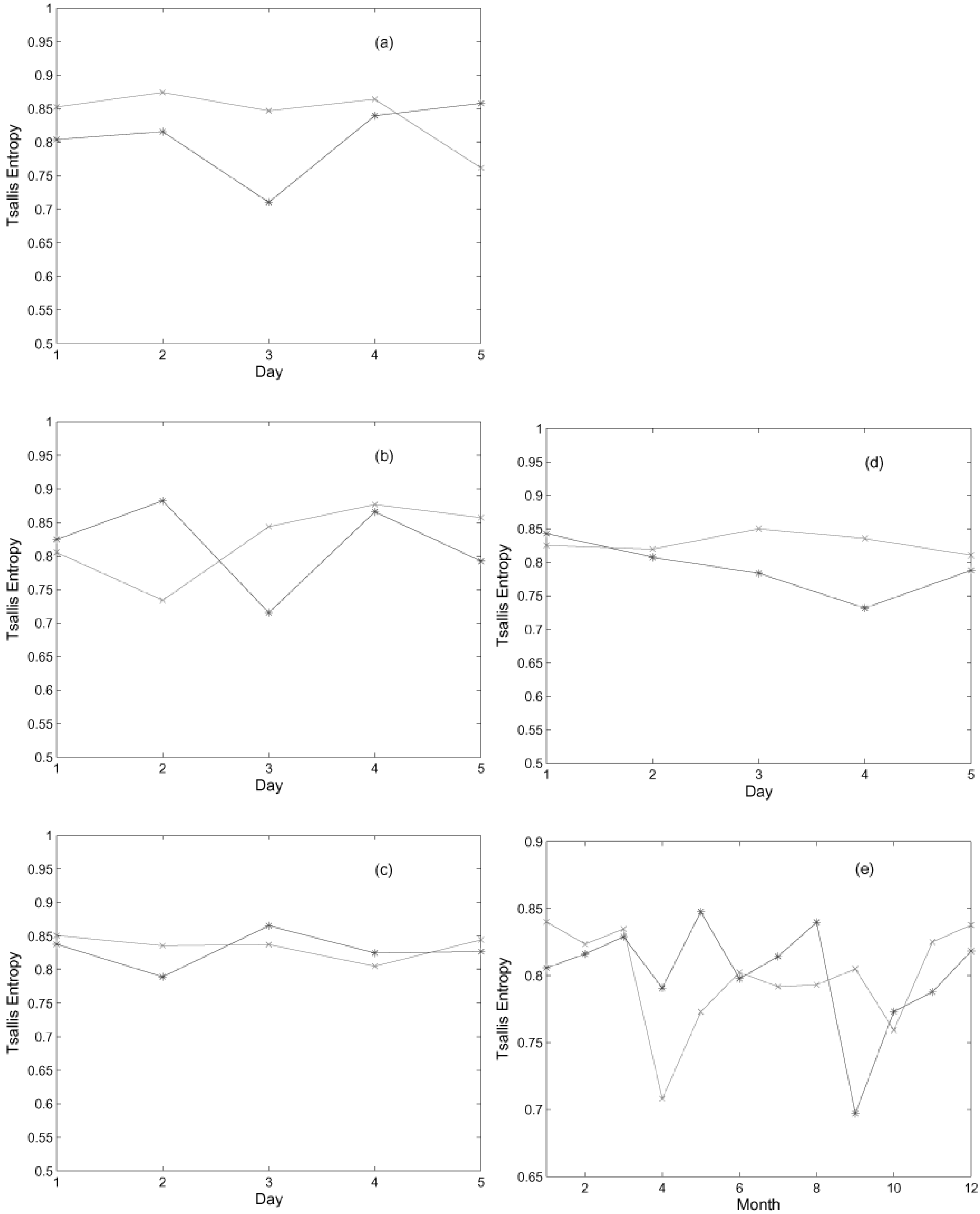

Fig. 9. Tsallis entropy for the five quietest days (red) and the five most disturbed days (blue) for **(a)** January, **(b)** February, **(c)** March, and **(d)** April. **(e)** Monthly mean values of Tsallis entropy for the entire year at Birnin Kebbi.

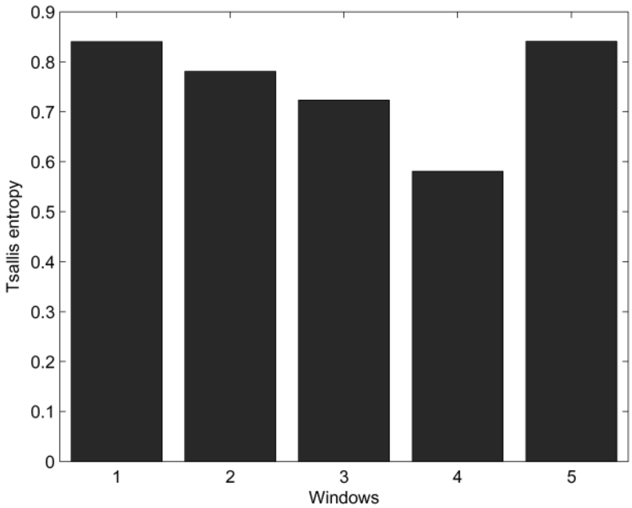

Fig. 10. Entropy values for five window intervals for the most quiet day in January 2011 from detrended TEC taken from Birnin Kebbi.

found to vary directly as Kolmogorov–Sinai generated from Lyapunov exponents for logistic maps and dynamical systems in the threshold of chaos where $\lambda = 0$, with direct variation when $q = 1$ during a chaotic regime. They were able to show that for all cases of the positive Lyapunov exponent λ there will be an average exponential increase of any small initial distance, which can be given as

$$\xi(t) = \frac{x_t - x_t'}{x_0 - x_0'} \tag{10}$$

for $\xi(t) = \exp(\lambda t)$ where x_t and x'_t are positions of two initially closed trajectories. They were able further to relate q to the exponential increase in small distances at the edge of chaos $\lambda = 0$ as

$$\xi(t) = [1 + (1-q)\lambda_q t]^{1/(1-q)}. \tag{11}$$

A similar Tsallis generalization was made for Lyapunov exponents in Coraddu et al. (2006), further explaining that the exponential behavior for the chaotic regime is recovered for $q \to 1$

$$\lim_{q\to 1} \exp(\lambda_q t) = \exp(\lambda t). \tag{12}$$

It was stated in their paper that a large class of generalized exponentials shows similar behavior. However, Anastasiadis et al. (2005) explored different q index values for complex networks for $\lambda < 0$ (periodic case) or $\lambda = 0$ (edge of chaos) and $\lambda > 0$ (chaotic regime), where they found $q = 2$ to be appropriate for a well-distinguished variation in Tsallis entropy between chaos and edge of chaos regimes.

The values of these entropy measures were also computed in order to study the dynamical complexity of the system under observation (the ionosphere). This will help us to obtain

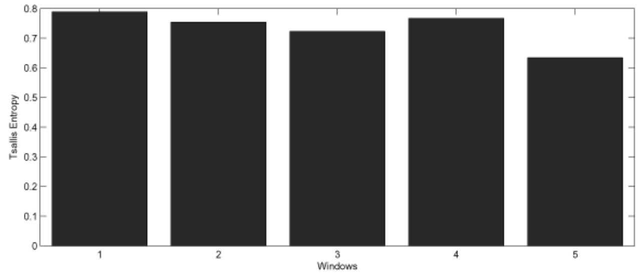

Fig. 11. Entropy values for five window intervals for the most disturbed day in January 2011 from detrended TEC taken from Birnin Kebbi.

some more information on the state of the system. The values of Tsallis entropy were computed for the five quietest days and five most disturbed days of the month also according to the International Quiet Days (IQD) and International Disturbed Days (IDD) classification by Geoscience Australia. The computed values of Tsallis entropy representing the dynamical complexity of a system for the quiet and disturbed days are shown in Fig. 9a–d, with monthly averages for the year in Fig. 9e. The entropy measures were also computed for some daily data, which were split into five time window intervals to be able to study the changes in the pattern of the entropy values in these intervals within a day (Figs. 10 and 11).

3.1.5 Nonlinearity test using surrogate data

The test for nonlinearity using the method of surrogate data according to Kantz and Schreiber (2003) has been proven to be a good test for nonlinearity in time series describing a system. It has been accepted that the method of a surrogate data test could be a successful tool for the identification for nonlinear deterministic structure in experimental data (Pavlos et al., 1999). This method involves creating a test of significance of difference between a linearly developed surrogate and the original nonlinear time series to be tested. The test is done by carrying out the computation of the same quantity on both surrogates and the original time series and then checking for the significance of difference between the results obtained from the surrogates with the original data. Theiler et al. (1992) suggested the creation of surrogate data by using Monte Carlo techniques for accurate results. According to this method, typical characteristics of data under study are compared with those of stochastic signals (surrogates), which have the same auto-correlation function and the power spectrum of the original time series. One can safely conclude from the test of significance carried out on the surrogate and the original data that a stationary linear Gaussian stochastic model cannot describe the process under study provided that the behavior of the original data and the surrogate data are significantly different.

In this work 10 surrogate data were generated from the original data set. The geometrical and dynamical characteristics of the original data were then compared with those of the surrogates using the statistical method of significance of difference, which can be defined as

$$S = \frac{\alpha_{\mathrm{Surr}} - \alpha_{\mathrm{Original}}}{\sigma}. \quad (13)$$

Where α_{Surr} is the mean value of the computed quantity for the surrogate data and $\alpha_{\mathrm{Original}}$ is the same quantity computed for the original TEC data, σ is the standard deviation of the same quantity computed for the surrogate data. The significance of difference considered for the null hypothesis to be rejected here is greater than 2, which enables us to reject the null hypothesis that the original TEC data describing the ionospheric system can be modeled using a Gaussian linear stochastic model with confidence greater than 95 %. The surrogate data test for all stations used in this study shows that the Lyapunov exponent of the surrogate data for the selected days in October are shown in Tables 1 and 2. The results show that the surrogate data test for the Lyapunov exponent shows a significance of difference greater than 2 for all the selected days for all the stations. Similar results were obtained for mutual information, fraction of false nearest neighbors and correlation dimension. This result gives us the confidence to reject the null hypothesis that the data used cannot be modeled using a linear Gaussian stochastic model, which shows that the system is a nonlinear system with some level of determinism. Figure 12 shows the plots comparing the mutual information plotted against time delay for the original detrended data blue with the mutual information for the surrogate data for TEC data measured at Lagos for the quietest day of March 2011, while Fig. 13 compares the fraction of false nearest neighbors for the same set of data. Table 1 shows the values of Lyapunov exponents for both original detrended and its surrogate data for TEC measured in Lagos during the quietest days of October 2011.

3.1.6 Contour/spatial plots

The contour plots for Lyapunov exponents and Tsallis entropies for quiet and disturbed days were plotted against the months of the year. This will enable us to see the behavior. The plots were done to show the variation of the values of these parameters with variations in quietness or disturbance and also to show how this varies with each month. These plots are made to show the response of the two parameters to storms and other variations describing changes in the internal dynamics of the ionosphere. These plots will also reveal the extent of the similarities in the response of these two quantifiers to ionospheric changes. The contour plots were made in this work to show the effect of the degree of quietness and monthly variation of solar activities on the variation of the two quantifiers. Figures 14 and 15 show the contour plots for Lyapunov exponents and Tsallis entropy for 2011 at Birnin Kebbi station.

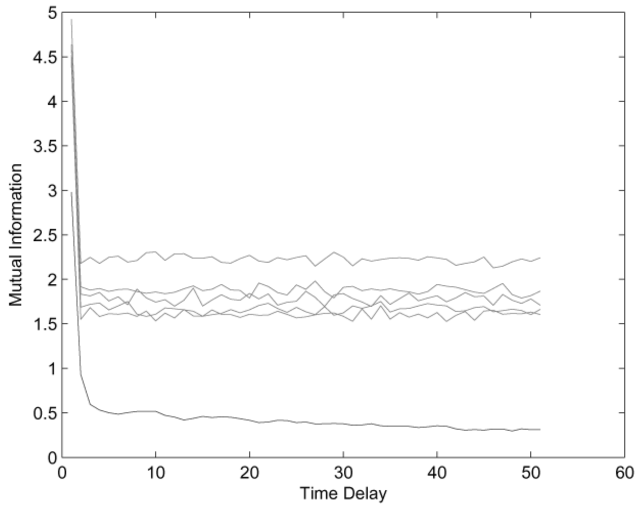

Fig. 12. The plots comparing the mutual information plotted against time delay for the original detrended data (blue curve) with the mutual information for 5 surrogate data (red curves) for TEC data measured at Lagos for the quietest day of March 2011.

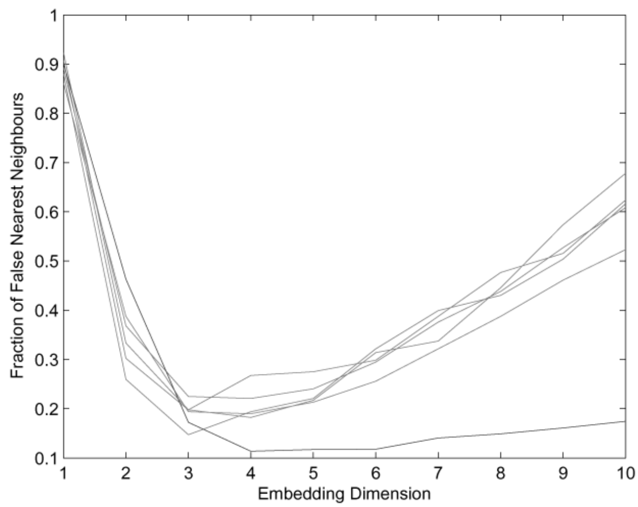

Fig. 13. The plots comparing the fraction of false nearest neighbors plotted against the embedding dimension for the original detrended data (blue curve) with the mutual information for 5 surrogate data (red curves) for TEC data measured at Lagos for the quietest day of March 2011.

4 Discussion

The results presented in this work show that the ionosphere shows a great degree of complexity for different times of the day and for different geophysical conditions. The time series plot in Fig. 1 shows the dominant characteristics of the ionosphere. The time series plot shows the rise in TEC to peak at the sunlit hours of the day; however, it can be seen that the rising to the peak of the ionosphere, which is the dominant dynamics during the day, make it impossible to see the

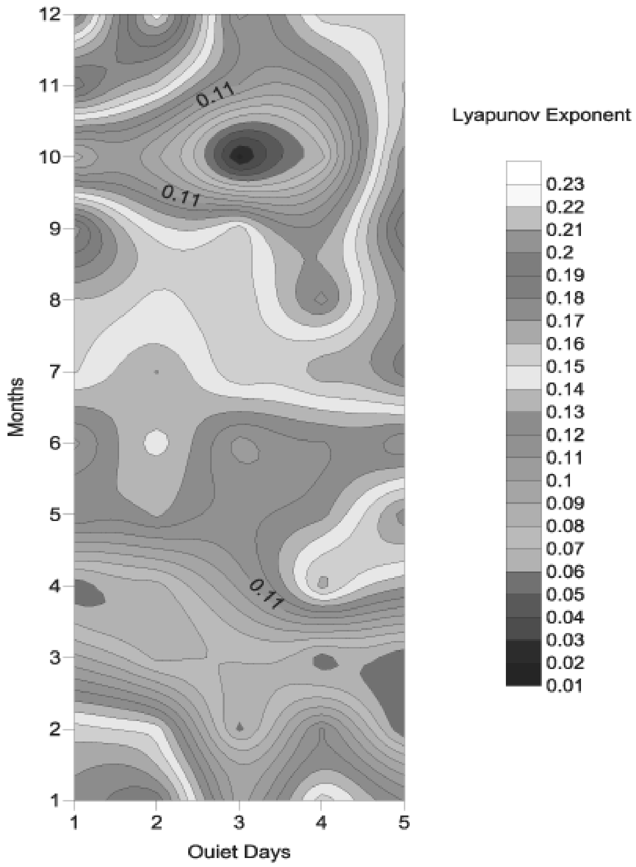

Fig. 14. Contour plot of Lyapunov exponents at Enugu station for quiet days of 2011.

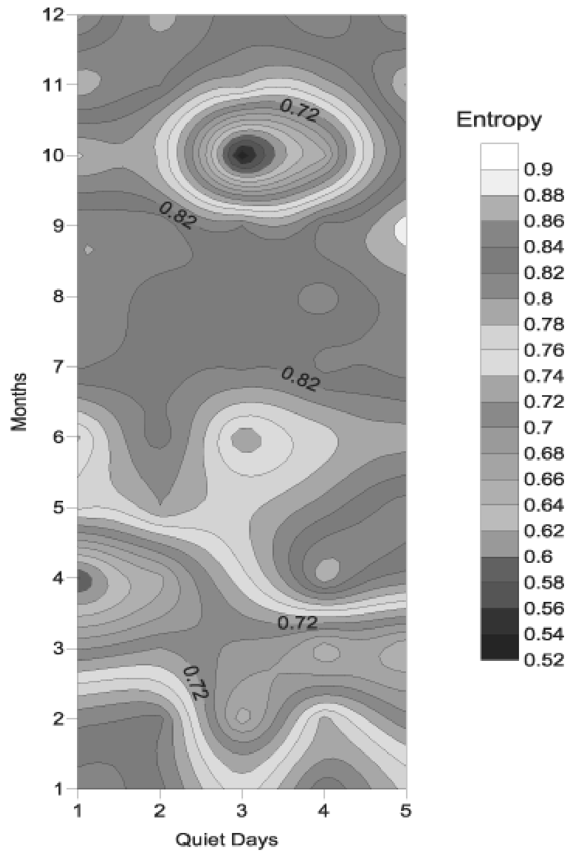

Fig. 15. Contour plot of Tsallis entropy at Enugu station for quiet days of 2011.

internal dynamics of the system from the TEC time series plot. It can be seen that the TEC time series curve is not a smooth curve with tiny variations, which probably describes a part of the internal dynamics. These visible tiny variations around the edges of the time series plot can be regarded as the rate of change of TEC, which is a phenomenon that can describe the influence of scintillations in the ionosphere. These variations are however more obvious during the night time between the 1100th and 1440th min of the day. It should be noted here that scintillations have been described as a night-time phenomenon. The detrended data shows the internal dynamics of the system more clearly, with a pattern similar to the values around the night period mentioned earlier. The values after sunset at night time in Fig. 1 show a pattern similar to the detrended TEC plot. It has been established that TEC does not decrease totally throughout the night as expected normally through the simple theory of the fact that TEC builds up during the day, but it shows some anomalous enhancements and variations and this can occur under a wide range of geophysical conditions (Balan and Rao, 1987; Balan et al., 1991; Unnikrishnan and Ravindran, 2010).

The positive values of the Lyapunov exponent indicate the presence of chaos (Wolf et al., 1985; Rosenstein et al., 1993; Hegger et al., 1999; Kantz and Schreiber, 2003). The ionosphere is a dynamic system controlled by many parameters, including acoustic motions of the atmosphere electromagnetic emission and variations in the geomagnetic field. Because of its extreme sensitivity to solar activity, the ionosphere is a very sensitive monitor of solar events. The ionosphere is that part of the upper atmosphere where free electrons occur in sufficient density to have an appreciable influence on the propagation of radio frequency electromagnetic waves. This ionization depends primarily on the Sun and its activity. Ionosphere structure and peak densities in the ionosphere vary greatly with time (sunspot cycle, seasonally and diurnally), with geographical location (polar, auroral zones, mild latitudes, and equatorial regions), and with certain solar-related ionospheric disturbances. During and following a geomagnetic storm, the ionospheric changes around the globe as observed from the ground site can appear chaotic (Fuller-Rowell et al., 1994; Cosolini and Chang, 2001; Unnikrishnan and Ravindran, 2010). The presence of chaos is indicated by the positive values of the Lyapunov exponent found in all the computations for all the TEC values obtained for the selected days for all the measuring stations used in this work. This can be expected, as it agrees with results from previous works that show that there is a reasonable presence of chaos in the ionosphere, even in the midst of the influence of stochastic drivers like solar winds (Bhattacharyya, 1990; Wernik and Yeh, 1994; Kumar et al., 2004; Unnikrishnan et al., 2006a, b; Unnikrishnan, 2010).

However, these values vary from day to day due to variations in ionospheric processes for different days on the same latitude as seen in Fig. 8a–d, with 8e showing the monthly averages for the entire year. There are also latitudinal variations due to spatial variations in the various ionospheric processes taking place simultaneously. The ionosphere is said to have a complex structure due to these varying ionospheric processes. One would expect all stations to have higher values of the Lyapunov exponent during quiet days; however, the values are sometimes higher for disturbed days, and for some stations the reverse may be the case. The values of Lyapunov exponents are however always lower for storm periods. The higher values of Lyapunov exponents during the quiet or relatively disturbed days indicate that the rate of exponential growth in infinitesimal perturbations in the ionosphere leading to chaotic dynamics might be of a higher degree during those days compared with days with lower values of Lyapunov exponents. The variation along the latitude also shows the inconsistency and complexity of the ionospheric processes. This is why for the same day of the month the values of Lyapunov exponents vary from one station to another.

The results of the correlation dimension show that the values computed are within the range of 2.8 to 3.5, with the lower values occurring mostly during the storm periods. The lower dimension during the storm periods compared with the quiet days may be due to the effect of stochastic drivers like strong solar winds and solar flares that occur during geomagnetic storms on the internal dynamics of the ionosphere; this must have been as a result of the fact that the internal dynamics must have been suppressed by the external influence. The restructuring of the internal dynamics of the ionosphere might be responsible for low-dimension chaos during storms and also the lower values of other measures like the Lyapunov exponents. The relatively disturbed day might however have a higher dimension so long as it is not a storm period, and sometimes a relatively disturbed day of the month might be a day with storms, and in this case there is usually a lower value of chaoticity and sometimes lower values of the correlation dimension as well. The surrogate data test shows a significance of difference greater than 2 for all the computed measures and we were able to reject the null hypothesis that the ionospheric system can be represented with a linear model for all the data used from the stations. However, we found that the lower significance of difference corresponds to the lower values of Lyapunov exponents during storms and extremely disturbed periods. This may be due to the rise in stochasticity during the storm period as a result of drop in values of computed quantities like Lyapunov exponents. Our ability to reject the null hypothesis for all stations shows however the presence of determinism and also that the underlying dynamics of the ionosphere is mostly nonlinear.

Tsallis entropy, which is an information theory approach derived from statistical mechanics, has been discovered to be one of the best entropy measures that can be used to describe magnetospheric dynamics, especially using D_{st} time series. The major explanation for this is the fact that Tsallis entropy is based on non-extensive statistical mechanics (Balasis et al., 2009). The ionosphere is a system that can also be described by similar dynamical processes as the magnetosphere, as both systems are also coupled (Unnikrishnan, 2010). Some of the investigations carried out on the chaoticity of both the ionosphere and the magnetosphere have yielded similar results for storm and quiet periods (Unnikrishnan, 2008, 2010). It implies that one can use similar methods to study the ionosphere and magnetosphere, and this informs the use of Tsallis entropy to study the ionosphere in this work, since it has been used to describe the magnetosphere.

Tsallis entropy is also studied for the five quietest and five most disturbed days, just as the Lyapunov exponent and the result show a similar variation to that of the Lyapunov exponent, as seen in Fig. 9a–d with the monthly averages for the entire year in Fig. 9e. The daily five window intervals for Tsallis entropy show no extensive increase in values of Tsallis entropy, but rise and fall due to changes in the internal dynamics ad dynamical complexity of the ionosphere as seen in Figs. 10 and 11. These changes can indicate some level of determinism in the ionosphere. The rise and fall in the computed values of Tsallis entropy daily five window intervals indicate the interplay between stochasticity and determinism (Unnikrishnan and Ravindran, 2010). The Tsallis entropy was able to show the deterministic behavior of the ionosphere considering its response during storm periods compared with other relatively quiet periods, as the rapid drop in values of Tsallis entropy during storms show that there is a transition from higher complexity during quiet periods to lower complexity during storms. This response in the values of Tsallis entropy is similar to the response of Lyapunov exponent values during storms. This reaction to storms of entropy computed for TEC was also described by the reaction of Tsallis entropy computed for D_{st} during storm periods (Balasis et al., 2008, 2009). This shows the deterministic nature of the ionosphere as mentioned before.

The reaction to storms may be due to the influence of stochastic drivers like strong solar winds flowing into the system as a result of solar flare or CMEs that produce the geomagnetic storms. Although there is always an influence of corpuscular radiation in the form of solar wind flowing from the Sun into the ionosphere, the influence is usually low for days without storms coming compared with days with geomagnetic storms as a result of solar flares, CMEs, etc. However, during months with high solar activities within the year, that is, during the equinoxial months, it can be seen that there is a drop in the values of Tsallis entropy that can also be clearly seen in the contour plots for quiet days. The presence of chaos even at quiet periods in the ionosphere may be due to the internal dynamics and inherent irregularities of the ionosphere, which exhibit nonlinear properties. However, this inherent dynamics may be complicated by external factors like geomagnetic storms. This may be the main reason for the drop in the values of Lyapunov exponents and Tsallis entropy during storms. According to Unnikrishnan et al. (2006a, b), geomagnetic storms are extreme forms of space weather, during which external driving forces, mainly due to solar wind, subsequent plasmasphere–ionosphere coupling, and related disturbed electric field and wind patterns, will develop. This in turn creates many active degrees of freedom with various levels of coupling among them, which alters and modifies the quiet time states of the ionosphere during a storm period. This new situation developed by a storm may modify the stability/instability conditions of the ionosphere, due to the superposition of various active degrees of freedom.

The contour plots show a lower value for March and September equinoxes in months in which storms were recorded for all stations. This can also describe the influence of solar wind (the major external factor) on the ionosphere, since a stronger influence of solar wind on the ionosphere is expected for those periods of the year. It is obvious from the results that to a great extent the influx of solar wind modifies the ionosphere or that it influences the internal dynamics of the ionosphere during geomagnetic storms (Unnikrishnan et al., 2006a, b; Unnikrishnan and Ravindran, 2010).

There are also many variations in the internal dynamics of the ionosphere that could lead to changes in chaotic behavior. The variations of Lyapunov exponents during quiet days might be a result of different variations in the intrinsic dynamics of the ionosphere. Variation patterns at different stations for the same quiet day might also be due to the same reason. It can be affirmed that the ionosphere is a complex system that varies with a short latitudinal or longitudinal interval such that even stations with one or two degrees of latitudinal differences might record different values on the same day for both quiet and disturbed periods. The same might also occur for storm periods. This is illustrated by the different pattern of variation of TEC recorded from different stations within such a close range, as used in this study.

One might experience a more sporadic rate of change in TEC as seen in the time series plots, and this is also reflected in the TEC as a result of irregularities in the internal dynamics of the ionosphere, and these might be as a result of plasma bubbles. Irregularities develop in the evening hours at F region altitudes of the magnetic Equator, in the form of depletions, frequently referred to as bubbles. The edges of these depletions are very sharp, resulting in a large time rate of TEC in the equatorial ionosphere, even during magnetically quiet conditions. The large gradient of the equatorial ionization persists in the local post-sunset hours till about 21:00 LT (DasGupta et al., 2007; Unnikrishnan and Ravindran, 2010). The TEC data for one station might experience an extremely sharp rate of change in TEC that may be due to some plasma bubbles in that region, while the TEC from the other station stays normal. These variations in the various internal dynamics like plasma bubbles leading to scintillation can cause variations in the dynamical response of the TEC. Hence, the irregular variation in the values of the Lyapunov exponent and Tsallis entropy even in quiet periods for two relatively close stations may be due to these irregularities. This might also be responsible for the quiet days in the same station having lower values of the Lyapunov exponent compared with higher values recorded for disturbed days without the external influence of storms.

As mentioned earlier, the disturbed day might be without a storm for months without a record of a geomagnetic storm, but might be the most disturbed day relative to the other days of the same month according to the IQD and IDD classification. Higher values during such a disturbed day compared with lower values of quiet days show that the variation might be due to the inherent internal dynamics causing such variations. This can also be affirmed from Unnikrishnan and Ravindran (2010), who stated that in the deterministic picture, irregularity can be generated automatically by the nonlinearity of the intrinsic dynamics, and an ionospheric scintillation is caused by irregularities of the ionospheric electron density along the signal propagation path.

The variations of these chaos and dynamical complexity parameters might also be a result of the anomalous TEC enhancements that might occur at night. This is because there can be anomalous TEC enhancements even at various geophysical conditions, as recorded by Balan and Rao (1987); Balan et al. (1991). These effects can also be seen more clearly in the Tsallis entropy values for the five window period for quiet days of January 2011, because the night-time value is higher and also shows a much higher series of fluctuations during this period compared with other periods. As mentioned in Unnikrishnan and Ravindran (2010), the irregular changes in the dynamical characteristics of TEC from the results of the Lyapunov exponent and Tsallis entropy may also be due to the collisional Raleigh–Taylor instability that may give rise to a few large irregularities in L band measurements (Rama Rao et al., 2006; Sripathi et al., 2008). All these can be seen as internal factors responsible for variations in the dynamical response of TEC, as recorded from the values of the Lyapunov exponents and Tsallis entropy completed for days without storms, which might be quiet or disturbed according to classification and which could also account for higher values of these qualifiers during disturbed days compared with quiet days. During storms however the values were much lower.

The five window interval was able to show the detailed dynamical behavior of the ionosphere within those intervals of the day. It was also able to show the response to rate of change in TEC for those intervals of the day. A drop in Tsallis entropy values shows a response to a very sharp fluctuation, showing the rate of change in TEC that can be seen for the segmented TEC plots and the detrended TEC segments. Sudden drops in Tsallis entropy values at evening and night time might also be due to night-time TEC enhancement or other interval factors like plasma bubbles as mentioned before, since this can occur for all geophysical conditions. From the results of the daily five window interval computation of the Tsallis entropy, we can infer that the enhancement and the sporadic rate of change of TEC at any time of the day (even at night), as seen in the detrended time series, can lead to sudden changes in the values of Tsallis entropy.

All the daily values of Tsallis entropy correlate positively with the values of Lyapunov exponents at values between 0.78 and 0.81. The contour plots also show similarities for both Lyapunov exponents and Tsallis entropy (Figs. 12 and 13). This indicates that both the values of Lyapunov exponents and Tsallis entropy can be used together as measures to describe the determinism and dynamical complexity of the ionosphere.

5 Conclusions

The chaotic behaviour and dynamical complexity of low-latitude ionospheric behavior over some parts of Nigeria was investigated using TEC time series measured at three different stations, namely Birnin Kebbi (12°32′ N, 4°12′ E and 0.62° N geomagnetically), Enugu (6°26′ N, 7°30′ E and −3.21° N geomagnetically) and Lagos (6°27′ N, 3°23′ E and −3.07° N geomagnetically) within the low-latitude region. The time series data obtained from the GPS data measurement were studied for chaoticity using phase space reconstruction techniques, computation of Lyapunov exponents, correlation dimension, and Tsallis entropy, and implementation of the surrogate data test was carried out. However, the Tsallis entropy was extensively used for the study of dynamical complexity of the ionospheric system described by the TEC data. The detrending analysis of the TEC data was carried out to reduce the influence of the diurnal variations so as to be able to reflect the inherent dynamics of the system for all the days considered in this work before it could be subjected to the various nonlinear time series analyses mentioned above for quantification of deterministic chaos and to show the presence of dynamical complexity. The detrended time series were subjected to further analysis for phase space reconstruction from which the choice of time delay of 30 was obtained, and an embedding dimension of 5 was considered in this study. The computed values of Lyapunov exponents show that there was the presence of chaos, since all the results are positive values. The correlation dimension computed was observed to be between the range of 2.8–3.5, where the lowest values were recorded for the storm periods, showing a transition to a lower dimension during the storm period of October 2011.

The results of Tsallis entropy show that the changes in the ionospheric response to disturbances may be due to geomagnetic storms and other phenomena like changes in the internal irregularities (like plasma bubbles or scintillations) of the ionosphere. The response of the Tsallis entropy to various changes in the ionosphere also shows the deterministic nature of the system. The results of the Tsallis entropy show a lot of similarities to those of the ionosphere. Both results show that equinoxial months and other months with geomagnetic storms indicate low values of the Lyapunov exponent, and the same was observed for Tsallis entropy. The results of the values of the Lyapunov exponent were expected to be lower for the days of the months in which a storm was recorded, and it was found to be the same, which agrees with previous works by other investigators. A similar pattern of results was obtained for the computed values of Tsallis entropy. However, the values of the two quantifiers for most disturbed days of some months from the IDD classification were higher than those of the quietest from the IQD classification. This shows that a more relatively disturbed day of the month without storms might be more chaotic.

The random variations in the values of chaoticity in the detrended TEC describing the internal dynamics of the ionosphere as seen in the result obtained from both the Lyapunov exponent and Tsallis entropy show that the ionosphere is not totally deterministic but also has some elements of stochasticity. The variation in the dynamical complexity can be seen clearly from the rise and fall in the values of Tsallis entropy, which demonstrates an inconsistent pattern. It can also be seen that the results of Tsallis entropy following the same pattern with the Lyapunov exponent show that both quantifiers can be used simultaneously and comparatively as measures of chaos and dynamical complexity, as the correlation of all the values obtained for both quantities gives values between 0.78 and 0.81, and observation of their similarities in response to changes in the ionosphere can also be deduced from the contour plots.

Considering the response of the chaotic quantifiers and entropy to the changes in the geophysical conditions, namely quiet, disturbed and storm periods, it can be seen that the ionospheric internal dynamic is chaotic, with some measure of complexity. However, change in the ionospheric response due to external influence mainly occurs due to stochastic drivers like solar wind. The intermittent variations between stochasticity and determinism from the sudden rise and fall in the value of the chaotic quantifiers and entropy measures show the dynamical complexity of the ionosphere, and these changes may occur not only due to changes in the three geophysical conditions, namely the quiet, disturbed and storm periods, but also due to changes as a result of the irregularities in the internal system of the ionosphere.

The knowledge of being able to characterize the ionospheric behavior using Lyapunov exponents and Tsallis entropy comparatively in work and the similarities in their response to the dynamical changes in the ionosphere show their ability to measure levels of determinism when used to complement each other. The ability of these two quantifiers to describe the ionospheric response to changes in geophysical conditions and also their ability to describe the changes in the internal dynamics of the ionosphere indicate that these quantifiers might be useful as indices for the description of the state of the ionospheric conditions in the near future. The relationship between these two quantifiers as established from this work calls for more research in using the two qualifiers to enable proper description and characterization of the state of the ionosphere. Therefore there is a need for further work on the use of these two parameters as indices that can describe the state of the ionosphere from time to time.

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