Computing the Shapley Value for Ride-Sharing and Routing Games

Extended Abstract

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ABSTRACT

Ride-sharing services are gaining popularity and are crucial for a sustainable environment. A special case in which such services are most applicable, is the last mile variant. In this variant it is assumed that all the passengers are positioned at the same origin location (e.g. an airport), and each have a different destination. One of the major issues in a shared ride is fairly splitting of the ride cost among the passengers.

In this paper we use the Shapley value, which is one of the most significant solution concepts in cooperative game theory, for fairly splitting the cost of a shared ride. We consider two scenarios. In the first scenario there exists a fixed priority order in which the passengers are dropped-off (e.g. elderly, injured etc.), and we show a method for efficient computation of the Shapley value in this setting. Our results are also applicable for efficient computation of the Shapley value in routing games. In the second scenario there is no predetermined priority order, and we show that the Shapley value cannot be efficiently computed in this setting.

KEYWORDS

Ride-Sharing; Cost Allocation; Shapley Value

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1 INTRODUCTION

On-demand ride-sharing services, which group together passengers with similar itineraries, can be of significant social and environmental benefit, by reducing travel costs, road congestion and CO_2 emissions. Indeed, the National Household Travel Survey performed in the U.S. in 2009 [17] revealed that approximately 83.4% of all trips in the U.S. were in a private vehicle (other options being public transportation, walking, etc.). The average vehicle occupancy was only 1.67 when compensating for the number of passengers. The deployment of autonomous cars in the near future is likely to increase the spread for ride-sharing services, since it will be easier and cheaper for a company to handle a fleet of autonomous cars that can serve the demands of different passengers.

Most works in the domain of ride-sharing are dedicated to the assignment of passengers to vehicles, or to planning optimal drop-off routes [1, 9, 12, 16]. In this paper we study a fair allocation of the cost of the shared ride in the last mile variant [4]. That is, we analyze the cost allocation when all passengers are positioned at the same origin location. We concentrate on the Shapley value [18] as our notion of fair cost allocation. The Shapley value is widely used in cooperative games, and is the only cost allocation satisfying efficiency, symmetry, null player property and additivity. The Shapley value has been even termed the most important normative division scheme in cooperative game theory [20]. However, the Shapley value depends on the travel cost of a ride of each subset of the passengers. Therefore, as stated by Özener and Ergun [13], "In general, explicitly calculating the Shapley value requires exponential time. Hence, it is an impractical cost-allocation method unless an implicit technique given the particular structure of the game can be found".

There are two possible general structures of the last-mile ride-sharing problem. In some cases there is a priority order in which the passengers are dropped-off. Such prioritization may be attributed to the order in which the passengers arrived at the origin location, or the frequency of passenger usage of the service; the latter is similar to the different boarding groups on an aircraft. Other rationales for prioritization may include urgency of arrival or priority groups in need (e.g. elderly, disabled, pregnant women, and the injured). Clearly, in such cases, the prioritization is preserved when determining the travel cost of a ride with a subset of the passengers. We denote this problem as the *prioritized ride-sharing problem*. Indeed, in some scenarios there is no predetermined prioritization order. In such cases it is assumed that a ride with a subset of the passengers is performed using the shortest (or cheapest) path that traverses their destinations. We denote this problem as the *non-prioritized ride-sharing problem*.

The prioritized and the non-prioritized ride-sharing problems are closely related to traveling salesman games [15]. In these games, a service provider makes a round-trip along the locations of several sponsors, where the total cost of the trip should be distributed among the sponsors. Specifically, the prioritized ride-sharing problem is similar to the fixed-route traveling salesman game, also known as routing game [21], while the non-prioritized ride-sharing problem is similar to the traveling salesman game. Most of the works on traveling salesman games concentrated on finding an element of the core, a solution game concept which is different from the Shapley value. One exception is the work of Yengin [21], who has studied the Shapley value of routing games and has conjectured

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that there is no efficient way for computing the Shapley value in routing games.

In this paper, we show an efficient computation of the Shapley value for the prioritized ride-sharing problem. Our method is based on smart enumeration of the components that are used in the computation of the Shapley value. Furthermore, our approach can be generalized to routing games, and we thus also provide an efficient way for computing the Shapley value in routing game. We then move to analyze the non-prioritized ride-sharing problem and show that, unless P=NP, there is no polynomial time algorithm for computing the Shapley value.

We note that the term ride-sharing is used in the literature with different meanings. We consider only the setting where the vehicle operator does not have any preferences or predefined destination. Instead, the vehicle's route is determined solely by the passengers' requests. In addition, the context of our work is that the assignment of the passengers to the vehicle has already been determined, either by a ride-sharing system or by the passengers themselves, and we only need to decide on the cost allocation. Since we focus on the case where the assignment has already been determined, we do not consider the ability of passengers to deviate from the given assignment and join a different vehicle, which is acceptable since either they want to travel together or no other alternative exists.

To summarize, the contributions of this paper are two-fold:

(1) We show an efficient method for computing the Shapley value of each user in a shared-ride when the priority order is predetermined. Our solution entails that the Shapley value can be computed in polynomial time in routing games as well, which is in contrast to a previous conjecture made.
(2) We show that there exist no polynomial algorithm for computing the Shapley value of the non-prioritized ride-sharing problem (unless P=NP).

2 RELATED WORK

The ride-sharing cost allocation problems that we study are closely related to traveling salesman games [15]. Specifically, the prioritized ride-sharing problem is similar to the fixed-route traveling salesman game [2, 6, 15], also known as routing game [21].

One variant of routing game is the fixed-route traveling salesman problems with appointments. In this variant the service provider is assumed to travel back home (to the origin) when she skips a sponsor. This variant was introduced by Yengin [21], who also showed how to efficiently compute the Shapley value for this problem but stated that his technique does not carry over to routing games.

The prioritized ride-sharing problem can also be interpreted as a generalization of the airport problem [11] to a two dimensional plane. In the airport problem we need to decide how to distribute the cost of an airport runway among different airlines who need runways of different lengths. In our case we distribute the cost among passengers who need rides of different lengths and destinations. It was shown that the Shapley value can be efficiently computed for the airport problem, however achieving efficient computation of the Shapley value in our setting requires a different technique.

The problem of fair cost allocation was also studied in the context of logistic operation. In this domain, shippers collaborate and bundle their shipment requests together to achieve better rates from a carrier [8]. The Shapley value was also investigated in this domain of collaborative transportation [7, 19]. In particular, Özener and Ergun [13] stated that "we do not know of an efficient technique for calculating the Shapley value for the shippers' collaboration game". Indeed, Fiestras-Janeiro et al. [5] developed the line rule, which is inspired by the Shapley value, but requires less computational effort and relates better with the core. However, the line rule is suitable for a specific inventory transportation problem. Özener [14] describes an approximation of the Shapley Value when trying to simultaneously allocate both the transportation costs and the emissions among the customers. Overall, we note that the main requirements from a cost allocation in the context of logistic operation is stability, and an equal distribution of the profit, since the collaboration is assumed to be long-termed. The type of interaction is our setting is inherently different, as it is an ad-hoc short term collaboration.

In another domain, Bistaffa et al. [3] introduce a fair payment scheme, which is based on the game theoretic concept of the kernel, for the social ride-sharing problem (where the set of commuters are connected through a social network).

3 RESULTS

We first assume that the passengers are ordered according to some predetermined priority order, and we show that we can efficiently compute the payment for every passenger using the Shapley value. Unlike other related work [15], we do not require that the priority order will be the optimal order that minimizes the total cost. For the exact definitions and proofs, see [10].

Theorem 3.1. *The Shapley value in the prioritized ride-sharing problem can be computed in polynomial time.*

We note that the prioritized ride-sharing problem is very similar to the setting of routing games [15]. Indeed, our approach can be generalized to routing games.

Theorem 3.2. *The Shapley value in routing games can be computed in polynomial time.*

Note that this is an unexpected result, since it refutes the conjecture in [21] that there is no efficient way for computing the Shapley value in routing games.

In essence, the computation of the Shapley value for the prioritized ride-sharing problem and routing games could be done efficiently since most of the travel distances cancel out, and only a polynomial number of terms remain in the computation. Unfortunately, this is not the case with the non-prioritized ride-sharing problem, where the Shapley value cannot be computed efficiently unless $P = NP$.

Theorem 3.3. *There is no polynomial time algorithm that computes the Shapley value for a given passenger in the prioritized ride-sharing problem unless* $P = NP$.

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