Irresolute Approval-based Budgeting

Extended Abstract

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ABSTRACT

In participatory budgeting, citizens can take part in the decision on which projects a city should spend money. Formally, the input is a set of items, each having a certain cost, while agents can express their preferences. The task is to choose a set of items respecting a given bound. Recently Talmon and Faliszewski [10] introduced a framework for budgeting based on approval votes. This paper revisits the introduced methods axiomatically from an irresolute point of view, especially showing that two of the proposed methods coincide. The study is complemented by approximation results.

KEYWORDS

participatory budgeting; voting; approximation

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1 INTRODUCTION

In participatory budgeting (PB), citizens are directly involved in the process of collective decision making at municipal or even global level. More precisely, the participants may express their preferences, by voting on which multitude of proposals (at non-uniform cost) public funds should be spent. We consider proposals either fully funded or rejected, in contrast to e.g. Freeman et al. [6], where funds may be divided non-discretely. Due to the rise of digital democracy, such processes are relevant to a large group of people, and the formal framework may be used to make decisions in different contexts. Participatory budgeting may be interpreted as a generalization of multiwinner elections, where each alternative occupies a fixed amount of seats. Following this generalization, there are various approaches to model voters' preferences. Goel et al. [7], Fluschnik et al. [5], and Benade et al. [4] consider assigning each alternative a utility, while Lu and Boutilier [9] consider participation by ranking alternatives. Benade et al. [3] evaluate multiple approaches in an empirical study. An overview of current research on participatory budgeting from a computational social choice perspective is given in the book chapter by Aziz and Shah [2].

Many cities, like Paris for example, that actually conduct PB rely on approval votes, where the voters may simply vote for some (possibly restricted) subset of the alternatives. We will also use approval-based preferences over the set of alternatives as it was proposed by Aziz et al. [1]. In particular, we will expand the framework by Talmon and Faliszewski [10], where satisfaction functions are used to evaluate how good a budget represents voters' preferences.

We contribute by interpreting the given model for irresolute budgeting scenarios, where the result may be a set of feasible winning budgets instead of one distinct bundle. Hence, we generalize studied axioms to irresolute budgeting methods by slightly altering their definition. Notably, we will show that two methods introduced by Talmon and Faliszewski [10] actually coincide irrespective of the used tie-breaking methods. Furthermore, we interpret rules that rely on greedy approaches as approximations and study their performance in contrast to optimal solutions.

The choice to focus on irresolute rules is motivated by the practical application of this framework. Although unique solutions are desirable, ties occur naturally, and breaking ties without participants' consent is at risk of losing either transparency or credibility. For preserving democratic deliberation, it is reasonable to assume that the tie-breaking will be made by the municipality. In extreme cases, breaking ties differently may result in disjunct budgets, which indicates the power, that the tie-breaking authority has. This might naturally lead to a conflict of interest when tie-breaking is not further specified and a possible waste of resources when tie-breaking is fixed priorly. Overall, most real-world campaigns are conducted in multiple stages to guarantee a favorable and realizable outcome. Hence, reaching a consensus that reflects the communities' preferences more precisely by adding a top-layer (i.e. deliberately breaking ties) is exactly in the spirit of PB.

2 PRELIMINARIES

We adopt the framework by Talmon and Faliszewski [10]. Hence, we consider a budgeting scenario as quadruple $E = (A, V, c, \ell)$, consisting of m items $A = \{a_1, \dots, a_m\}$, a function $c\colon A \longrightarrow \mathbb{N}$ assigning a cost to each item, n voters $V = \{v_1, \dots, v_n\}$ each balloting with a set of approved items $A_v \subseteq A$ for $v \in V$, and a budget limit $\ell \in \mathbb{N}$. We denote the set of items from a budget $B \subseteq A$, also approved by voter v as $B_v = A_v \cap B$. In this paper we use composite budgeting methods $\mathcal{R}^r_f$ as defined by Talmon and Faliszewski [10], but interpret them as irresolute procedures. Hence, each method $\mathcal{R}^r_f$ takes any budgeting scenario E as input and outputs a nonempty set of winning budgets $\mathcal{R}^r_f(E) \subseteq 2^A \setminus \{\emptyset\}$. This is done by applying a budgeting rule r, respecting a satisfaction function $f : 2^A \times 2^A \longrightarrow \mathbb{N}$. We adopt proposed satisfaction functions f, also introduced by Talmon and Faliszewski [10], to derive the satisfaction of a voter from her approval ballot, focussing on either the quantity $f(A_v, B) = |B_v|$, the cost of approved items that are budgeted $f(A_v, B) = c(B_v)$

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(where slightly abusing notation it holds $c(B) = \sum_{b \in B} c(b)$ for every bundle $B \subseteq A$)), or the presence of at least one approved item in the budget $f(A_v, B) = \mathbb{1}_{|B_v|>0}$.

Similarly to the satisfaction functions, we adopt the definition of max rules, greedy rules, and proportional greedy rules, additionally we introduce hybrid greedy rules. The formal definition with respect to a given satisfaction function f is as follows.

- **Max rules ($\mathcal{R}^m_f$):** $\mathcal{R}^m_f(E) = \arg\max_{B \subseteq A} \sum_{v \in V} f(A_v, B)$, while respecting the budget limit $\sum_{b \in B} c(b) \leq \ell$.
- **Greedy rules ($\mathcal{R}^g_f$):** Starting with $B = \emptyset$ iteratively extend B by $a \in A \setminus B$, maximizing $\sum_{v \in V} f(A_v, B \cup \{a\})$, such that $\sum_{b \in B} c(b) \leq \ell$.
- **Proportional Greedy rules ($\mathcal{R}^p_f$):** Similar to the greedy rule, maximize $(\sum_{v \in V} f(A_v, B \cup \{a\}) - \sum_{v \in V} f(A_v, B))/\ c(a)$ iteratively, starting with $B = \emptyset$.
- **Hybrid Greedy rules ($\mathcal{R}^h_f$):** Arbitrarily select $B_g \in \mathcal{R}^g_f(E)$ and $B_p \in \mathcal{R}^p_f(E)$ and output the budget with maximum satisfaction $\arg\max(\sum_{v \in V} f(A_v, B_g), \sum_{v \in V} f(A_v, B_p))$ as $\mathcal{R}^h_f(E)$.

We interpret all three greedy variants as irresolute rules, by considering every budget as winning, that may result from breaking the ties in each iteration. We now show that two of the considered budgeting methods coincide.

Theorem 2.1. *$\mathcal{R}^g_{|B_v|}$ and $\mathcal{R}^p_{c(B_v)}$ are equivalent, i.e. they always output the same set of winning budgets.*

Proof. First note that both above binary satisfaction functions f respectively map to an unary function f', where $f(A_v, B) = f'(B_v)$. While maximizing iteratively we may ignore constant factors $f'(B_v)$ carried over from previous iterations, assuming f' is additive. Hence in each iteration, the greedy rule $\mathcal{R}^g_f$ is selecting an item a maximizing $\sum_{v \in V} f'(A_v \cap \{a\})$ while the proportional greedy rule $\mathcal{R}^p_f$ selects item a maximizing $\sum_{v \in V} f'(A_v \cap \{a\})/c(a)$. Further for any f' with $f'(\emptyset) = 0$ it follows that

$$\sum_{v \in V} f'(A_v \cap \{a\}) = |\{v \in V \mid a \in A_v\}| \cdot f'(\{a\}).$$

Note that $|B_v|$ and $c(B_v) = \sum_{b \in B_v} c(b)$ are indeed additive and map to zero for $B_v = \emptyset$. By applying above implications, we conclude that both $\mathcal{R}^g_{|B_v|}$ and $\mathcal{R}^p_{c(B_v)}$ iterate by selecting item a maximizing the value $|\{v \in V \mid a \in A_v\}|$, since $|\{a\}| = c(\{a\})/c(a) = 1$. □

This theorem holds irrespective of the used tie-breaking, since in each iteration the same items may be chosen. Hence, also in the setting of Talmon and Faliszewski [10], both rules are equivalent.

3 APPROXIMATION AND PROPERTIES

We interpret given greedy approaches as approximations and study their performance in contrast to optimal solutions (i.e. max rules).

Proposition 3.1. *$\mathcal{R}^g_{|B_v|}$, $\mathcal{R}^g_{\mathbb{1}_{|B_v|>0}}$, $\mathcal{R}^p_{c(B_v)}$, $\mathcal{R}^p_{|B_v|}$ and $\mathcal{R}^p_{\mathbb{1}_{|B_v|>0}}$ do not have a constant approximation factor.*

This can be shown by counter examples, where the approximation factor is inversely proportional to the budget limit. In contrast, it can be shown that the newly introduced hybrid greedy rules have a $(1 - 1/\sqrt{e})$-approximation.

Proposition 3.2. *For all three satisfaction functions f considered here and every $B_m \in \mathcal{R}^m_f(E)$ and $B_h \in \mathcal{R}^h_f(E)$, it holds that $\sum_{v \in V} f(A_v, B_h)/ \sum_{v \in V} f(A_v, B_m) \geq 1 - 1/\sqrt{e}$.*

Above proposition follows by a similar $(1 - 1/\sqrt{e})$-approximation due to Khuller et al. [8] and the insight, that each max rule can be modeled as a special case of the budgeted maximum coverage problem.

Now, we recap some of the proposed axiomatic properties by Talmon and Faliszewski [10] to study them irrespective of the used tie-breaking rule. Hence, we slightly adapt the properties in order to handle irresolute rules.

Definition 3.3. Let $E = (A, V, c, \ell)$ be a budgeting scenario with $B \in \mathcal{R}(E)$. The following axiomatic properties are satisfied by a budgeting rule $\mathcal{R}$, if for every modified budgeting scenario E' (as defined below) there exists a budget $B' \in \mathcal{R}(E')$, meeting a requirement as defined:

- **Limit Monotonicity:** For $E' = (A, V, c, \ell + 1)$, where for all $a \in A$ it holds $c(a) \neq \ell + 1$, we require $B \subseteq B'$.
- **Discount Monotonicity:** For $b \in B$ and $E' = (A, V, c', \ell)$ with $c'(a) = c(a)$ for every $a \in A \setminus \{b\}$, and $c'(b) = c(b) - 1$, we require $b \in B'$.
- **Splitting Monotonicity:** For $a \in B$ and every E', where a is split into a set of items A', an extended cost function satisfying $c(a) = \sum_{a' \in A'} c(a')$, and exactly those voters approving a, approve all items in A', we require $A' \cap B' \neq \emptyset$.
- **Merging Monotonicity:** Let $A' \subseteq B$, such that for each $v \in V$ it holds either $A_v \cap A' = \emptyset$ or $A' \subseteq A_v$. For E', where A' is merged into a new item a, an extended cost function satisfying $c(a) = \sum_{a' \in A'} c(a')$, and the voters approving a are exactly those who approved A', we require $a \in B'$.

Our results are consistent with those for resolute methods and summarized in Table 1.

Table 1: Axiomatic properties of budgeting methods. Results are generalized from Talmon and Faliszewski [10]. Deviations are marked by ▲ (see Theorem 2.1), new results by ♣.

$\mathcal{R}^r_f$	m	g	p	h	m	g	p	h	m	g	h
	$\lvert B_v \rvert$				$\mathbb{1}_{\lvert B_v \rvert>0}$				$c(B_v)$		
Limit M.	x	x	x	x♣	x	x	x	x♣	x	x	x♣
Discount M.	✓	✓▲	✓	x♣	✓	✓	✓	x♣	x	x	x♣
Splitting M.	✓	✓	✓	✓♣	✓	✓	✓	x♣	✓	x	x♣
Merging M.	x	✓▲	x	x♣	✓	✓	x	x♣	✓	✓	x♣

When considering resolute methods, there are underlying assumptions, which might not be resolved easily. Some of the considered axioms might be violated if the tie-breaking scheme depends on the cost or the total quantity of items budgeted. Even the application of linear mechanisms might not be trivial, as splitting or merging items might interfere with the order.

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