Mechanism Design for School Choice with Soft Diversity Constraints

Extended Abstract

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ABSTRACT

We study the controlled school choice problem where students may belong to overlapping types and schools have soft target quotas for each type. We formalize fairness concepts for the setting that extends fairness concepts considered for restricted settings without overlapping types. Our central contribution is presenting a new class of algorithms that takes into account the representations of combinations of student types. The algorithms return matchings that are non-wasteful and satisfy fairness for same types. We further prove that the algorithms are strategyproof for the students and yield a fair outcome with respect to the induced quotas for type combinations. We experimentally compare our algorithms with two existing approaches in terms of achieving diversity goals and satisfying fairness.

KEYWORDS

School Choice; Fairness; Diversity Constraints; Overlapping Types; Soft Bounds

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1 INTRODUCTION

Incorporating diversity constraints, transparency and fairness into systems and mechanisms are some of the prominent concerns in artificial intelligence. These concerns are also prevalent in matching markets where there has been increased attention to school choice problems that take into account affirmative action and diversity concerns when matching students to schools. One particular model of school choice with diversity constraints is *controlled school choice* [1], in which students are associated with a set of types. In recent years, algorithms for matching with diversity goals have been deployed in many places including in Israel [5] and India [3, 10]. Typically, the diversity goals are achieved by setting minimum and maximum target representation of students [4, 7].

If diversity constraints are considered as hard bounds, there may not exist an outcome that fulfills all minimum quotas, and a fundamental tension between fairness and non-wastefulness arises [4]. There are challenges on the computational front as well: it is NP-hard to check whether there exists a feasible or stable matching for the school choice problem with diversity constraints [2]. The recent literature on controlled school choice problems treats diversity constraints as *soft bounds* which are soft goals that schools attempt to achieve [4, 6, 8, 9].

Most papers in controlled school choice assume that each student is associated with only one type. In reality, students may be associated with multiple types. For example, a student could be both female and aboriginal. In this paper, we study the controlled school choice problem where students may have overlapping types, and diversity constraints are viewed as soft bounds. The research question we consider is *how to design mechanisms that cater to diversity objectives while still satisfying desirable fairness, non-wastefulness and strategy-proofness properties?*

2 PRELIMINARIES

An instance I^T of the school choice problem with diversity constraints consists of a tuple $(S, C, q_C, T, \underline{\eta}, \mathcal{X}, \succ_S, \succ_C)$ where S and C denote the set of students and schools respectively[1]. The capacity vector $q_C = (q_c)_{c\in C}$ assigns each school c a capacity q_c. The type space is denoted by $T = \{t_1, ..., t_k\}$. For each student s, we use $T(s) \subseteq T$ to represent the subset of types to which student s belongs. For each school c, we use $\underline{\eta}_c^t$ to represent the minimum quota for type t. Let $\underline{\eta}_c = (\underline{\eta}_c^t)_{t\in T}$ denote the type-specific minimum quota vector of school c and let $\underline{\eta}$ be a matrix consisting of all schools' type-specific minimum quotas.

Each contract $x = (s, c)$ consists of a student-school pair representing that student s is matched to school c. Let $\mathcal{X} \subseteq S \times C$ denote the set of available contracts. Given any $X \subseteq \mathcal{X}$, let X_s be the set of contracts involving student s, let X_c be the set of contracts involving school c and let X_c^t be the set of contracts involving type t and school c.

Each student s has a strict preference ordering $\succ_s$ over $\mathcal{X}_s \cup \{\emptyset\}$ where $\emptyset$ is a null contract representing the option of being unmatched for student s. A contract (s, c) is *acceptable* to student s if $(s, c) \succ_s \emptyset$. Let $\succ_S = \{\succ_{s_1}, ..., \succ_{s_n}\}$ be the preference profile of all students S. Each school c has a strict priority ordering $\succ_c$ over $\mathcal{X}_c \cup \{\emptyset\}$ where $\emptyset$ represents the option of leaving seats vacant for school c. A contract (s, c) is *acceptable* to school c if $(s, c) \succ_c \emptyset$. Let $\succ_C = \{\succ_{c_1}, ..., \succ_{c_m}\}$ be the priority profile of all schools.

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[1] To simplify the presentation, we focus on minimum quotas only for the rest of the paper, as was the focus of Kurata et al. [8]. The impossibility result in Theorem 2.1 carries over to maximum quotas, and our new algorithms can be extended to cater to maximum quotas.

An outcome (or a matching) X is a subset of $\mathcal{X}$. An outcome X is *feasible* (under soft bounds) for I^T if i) each student s is matched with at most one school, i.e., $|X_s| \leq 1$, and ii) the number of students matched to each school c does not exceed its capacity, i.e., $|X_c| \leq q_c$. A feasible outcome X is *individually rational* if each contract $(s, c) \in X$ is acceptable to both student s and school c. Without loss of generality, we focus on acceptable contracts. Given a feasible outcome X, student s claims an empty seat of school c if $(s, c) \succ_s X_s$ and $|X_c| < q_c$. A feasible outcome is *non-wasteful* if no student claims an empty seat.

Next, we propose a binary relation to facilitate the comparison of the contribution made by two students in terms of meeting minimum targets of schools. Given a feasible outcome X, let $\underline{V}^X_c = \{t \in T |\ \underline{\eta}^t_c > |X^t_c|\}$ denote the set of types that are undersubscribed at school c. Given a feasible outcome X and two students s, s' with $(s, c) \notin X$ and $(s', c) \notin X$, i) $s \trianglerighteq^X_c s' \Leftrightarrow T(s) \cap \underline{V}^X_c \supseteq T(s') \cap \underline{V}^X_c$; ii) $s \triangleright^X_c s' \Leftrightarrow s \trianglerighteq^X_c s'$ and $s' \not\trianglerighteq^X_c s$; iii) $s \sim^X_c s' \Leftrightarrow s \trianglerighteq^X_c s'$ and $s' \trianglerighteq^X_c s$.

Given an instance I^T and a feasible outcome X, student s has *justified envy* towards student s' if i) $(s, c) \succ_s \{X_s\}$, $(s', c) \in X$ and ii) for the outcome $X' = X \setminus \{(s', c)\}$, either (a) $s \triangleright^{X'}_c s'$, or (b) $s' \not\triangleright^{X'}_c s$ and $(s, c) \succ_c (s', c)$ holds. An outcome is *fair* if no student has justified envy towards any student.

Theorem 2.1. *When each student has multiple types, the set of fair and non-wasteful outcomes could be empty, even if there are only two types.*

3 A CLASS OF ALGORITHMS GDA-TC

In this section, we propose a new class of algorithms *Generalized Deferred Acceptance for Type Combinations (GDA-TC)* that yield non-wasteful and fair outcomes for students of same types. The general idea is to eliminate overlapping types by creating a new set U corresponding to type combinations of T so that each student is associated with exactly one type combination. Then we establish new quotas $\underline{\delta}$ for type combinations U and incorporate the induced quotas into the choice function Ch^{TC}_c of schools. We employ the GDA algorithm with choice function Ch^{TC}_c to determine the outcome. All these procedures consist of our new class of algorithms GDA-TC, as shown in Algorithm 1.

Require: $I^T = (S, C, q_C, T, \underline{\eta}, \mathcal{X}, \succ_S, \succ_C)$
Ensure: An outcome $X \subseteq \mathcal{X}$

1: Create a set of type combinations U from types T.
2: Determine quotas $\underline{\delta}$ for type combinations U.
3: Incorporate quotas $\underline{\delta}$ into choice function Ch^{TC}_c.
4: Run GDA with choice function Ch^{TC}_c.

Algorithm 1: GDA-TC

The GDA algorithm works in much the same way as the original deferred acceptance algorithm does: each student first selects one contract involving her favorite school that has not rejected her yet; then schools choose a set of contracts among the proposals and reject others. Repeat this procedure until no more contract is rejected by any school. There are different ways to establish quotas for type combinations U and each different method specifies one particular algorithm of GDA-TC. For instance, we can invoke linear programming to divide minimum quotas $\underline{\eta}$ for types T into minimum quotas $\underline{\delta}$ for type combinations U. We refer to this algorithm as GDA-TC-LP that makes use of linear programming.

Determining quotas for type combinations Let $\underline{\delta}_c = (\underline{\delta}^u_c)_{u \in U}$ denote a minimum target vector of school c where each element $\underline{\delta}^u_c$ is the minimum target quota of type combination u. Let $\underline{\delta} = (\underline{\delta}_c)_{c \in C}$ be a matrix consisting of minimum target quota of each type combination for each school. We can calculate the vector $\underline{\delta}_c$ through the following linear programming:

$$\min \quad \sum_{u \in U} \underline{\delta}^u_c \tag{1}$$

$$\sum_{u \in U^t} \underline{\delta}^u_c \geq \underline{\eta}^t_c, \quad \forall c \in C, \forall t \in T \tag{2}$$

$$\underline{\delta}^u_c \geq 0, \quad \forall u \in U \tag{3}$$

$$\underline{\delta}^u_c \times |S^v| = \underline{\delta}^v_c \times |S^u|, \quad \forall c \in C, \forall u, v \in U \tag{4}$$

Specifying choice functions for schools Next we briefly explain how the choice function Ch^{TC}_c of schools works: Given a set of contracts X, the choice function Ch^U_c traverses the set of contracts X_c involving school c twice in accordance with the priority order of school c: in the first round, it selects a set of contracts without exceeding any minimum quota for type combinations and the capacity q_c of school c; in the second round, it selects a set of contracts without exceeding the capacity only.

	GDA-TC	GDA-PMA	DA-OT
Fairness	✗	✗	✗
KHIY-fairness	✗	✗	✓
Fairness for same types	✓	✗	✓
KHIY-non-wastefulness	✗	✗	✗
Non-wastefulness	✓	✓	✓
Strategy-proofness	✓	✗	✓

Table 1: Comparison of our new algorithm GDA-TC with two existing algorithms GDA-PMA and DA-OT.

We compare with two existing algorithms designed for school choice with multiple types by Kurata et al. [8] and Gonczarowski et al. [5]. Table 1 summarizes the properties satisfied by each algorithm. We also undertake the experimental comparative analysis of school choice algorithms with overlapping types allowed. Our generated data uses similar features as the private data set used by Gonczarowski et al. [5]. The experimental results show that our new algorithm performs well across several axes, including fairness, diversity goals as well as running time. Especially, it outperforms the other two algorithms in terms of consistently satisfying a reasonable relaxation of targets representations. Note that although DA-OT additionally satisfies KHIY-fairness, it performs the worst in achieving diversity goals and running time.

In conclusion, GDA-TC-LP is a suitable alternative algorithm to GDA-PMA and DA-OT. It outperforms DA-OT in terms of achieving diversity goals and returns a much more balanced outcome. It also has satisfies several important properties that GDA-PMA does not.

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