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Machine learning analysis and modeling of interest rate curves

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Abstract. The present research deals with the review of the analysis and modeling of Swiss franc interest rate curves (IRC) by using unsupervised (SOM, Gaussian Mixtures) and supervised machine (MLP) learning algorithms. IRC are considered as objects embedded into different feature spaces: maturities; maturity-date, parameters of Nelson-Siegel model (NSM). Analysis of NSM parameters and their temporal and clustering structures helps to understand the relevance of model and its potential use for the forecasting. Mapping of IRC in a maturity-date feature space is presented and analyzed for the visualization and forecasting purposes.

1 Introduction

Interest rate curves (IRC) are fundamental objects in economics and finance. They are widely used in financial engineering and risk management. Therefore the analysis, modeling and forecasting of IRC are very important. By definition, the IRC is the relation between the interest rate (cost of borrowing) and the time to maturity of the debt for a given borrower in a given currency. The main objectives of the present paper are the following: 1) analysis of IRC variance-covariance matrix evolution the coherency in the behavior of interest rates of different maturities using moving window approach in order to avoid problems with non-stationarity; 2) comprehensive analysis and patterns detection in a parametric feature space composed of the parameters of NSM widely used for the prediction of IRC; and 3) to revise recently proposed by the authors forecasting and reconstruction of missing IRC by applying spatial statistics and machine learning. Traditionally IRC are considered either as fixed in time curves modeled using no-arbitrage principle, or by applying equilibrium models by modeling the dynamics of the intravenous rate using affine models. Real case study is based on the recent evolution of Swiss franc (CHF) IRC.

Typical CHF IR curves for some fixed days are presented in Figure 1 (top) and daily evolution (from 1998 to 2006) of interest rates for different maturities is shown in Figure 1 (bottom). This period of the study is quite interesting because it represents different market conditions. The IRC considered are composed of LIBOR interest rates (maturities from 1 week up to 1 year) and of swap interest rates (maturities from one year to 10 years). In this research we use the following maturities: 1 week, 1, 2, 3, 6 and 9 months; 1, 2, 3, 4, 5, 7 and 10 years.

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Interest rates depend on time and on maturity which defines their term structure. In general, IRC follow some well known stylized facts [1]: the average yield curve is increasing and concave; the yield curve assumes a variety of shapes through time, including upward sloping, downward sloping, humped, and inverted humped; IR dynamics is persistent, and spread dynamics is much less persistent; the short end of curve is more volatile than the long end; long rates are more persistent than short. In a more general setting IRC can be considered as functional data.

In Figure 2 temporal evolution of cross-correlations between different maturities (correlation matrix) is presented as a time series. Window of 50 business days was used for the computations. The observed patterns of correlations change from very high positive (cooperative) behavior to quite dispersed patterns: correlations can be found between [-0.8; +1].

Some analysis of IRC by using self-organizing Kohonen maps (SOM) was presented in [2]. Curves were considered in a 13 dimensional feature space composed from 13 maturities. Temporal clustering corresponding to different market conditions (e.g., bullish, bearish, transitional) was observed.

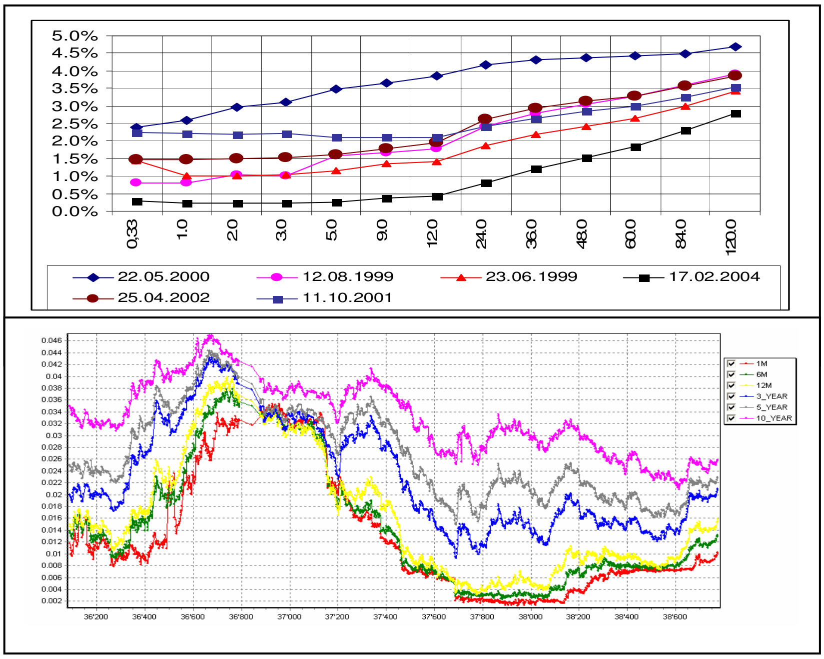

Fig. 1: Examples of interest rate curves (top) and time series for some maturities (bottom).

2 Modeling

As it was mentioned above, modeling of IRC is of great importance for financial industry. In [3] the Nelson-Siegel model for modeling IR curve was proposed. In [1] this model was extended to a 3 factors dynamic model. These factors correspond to long-term, short-term and medium-term IR behaviour: *Level = (maturity 10 years), Slope = [(maturity 10 years) – (maturity 3 months)], Curvature = [2*(maturity 2 years) – (maturity 3 moths) – (maturity 10 years)].* In [1] these parameters were modelled as a linear auto-regressive time series using historical data and have demonstrated efficiency of this methodology. Weekly IR data were considered.

The study of the evolution of local cross-correlations between NSM factors and their clustering are important for understanding of their contribution to the final characterization of the curves. As it can be seen in Figure 3, there are some windows when all factors are highly and positively correlated. It means that all of them move coherently which can correspond to 1) market behaviour; 2) temporal clustering [2,4].

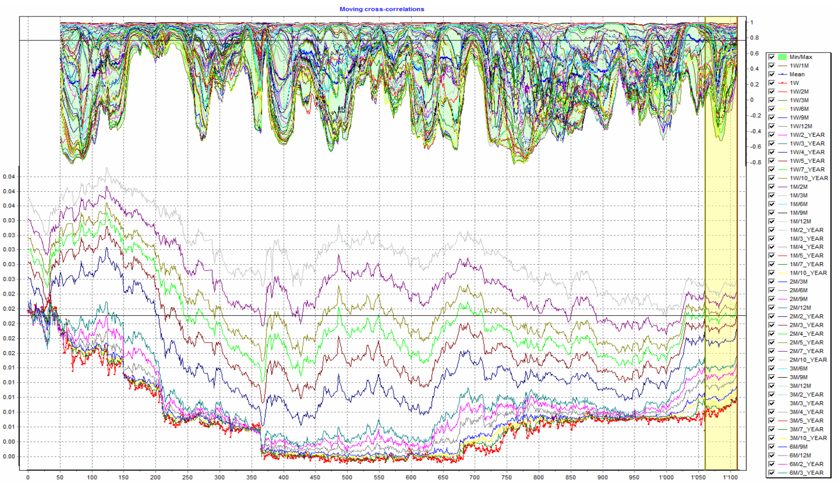

Fig. 2: Cross-correlations between maturities (up) estimated in a window of 50 days and time series of maturities (2000-2006) (bottom).

In this paper clustering in the space of NSM parameters is studied using Gaussian Mixture Model. Usually Mixture Models are used for the *density estimation* of the data. Density estimation is the construction of an estimate, based on observed data, of an unobservable underlying *probability density function (p.d.f.)*. Mixture Model estimates density distribution in a form of a linear combination of some simple functions (called components, units, or kernels):

$$p(x) = \sum_{j=1}^{m} p(x \mid j) P(j)$$

Such representation of a p.d.f. is called a *mixture distribution* [5]. *P(j)* are mixing coefficients. In a Bayesian framework, *P(j)* can be considered as *prior* probabilities of any data point having been generated from component ***j*** of the mixture.

In Figure 4 phase trajectories in two-dimensional spaces (level-slope; slope-curvature; level-curvature) are presented as well as the result of SOM based analysis (four clusters are shown). According to these figures it is evident that there is also temporal clustering. GMM was applied to study clustering and the results are given in Figure 5. These patterns of clustering can be used to develop nonlinear models of the evolution of parameters and then to forecast interest rate curves as it was done with AR model in [1]. In this paper we apply another approach.

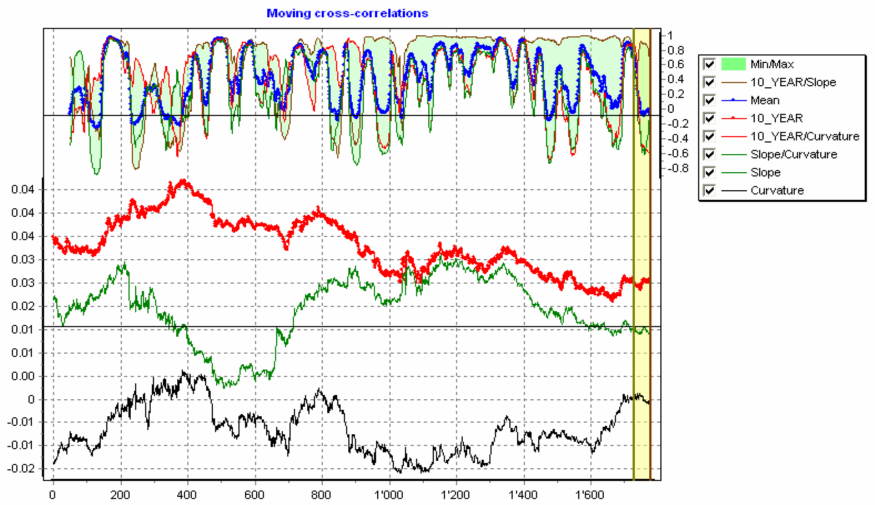

Fig. 3: Time series of Nelson-Siegel 3 factors model parameters (bottom) and their cross-correlations estimated in a window of 50 days (up).

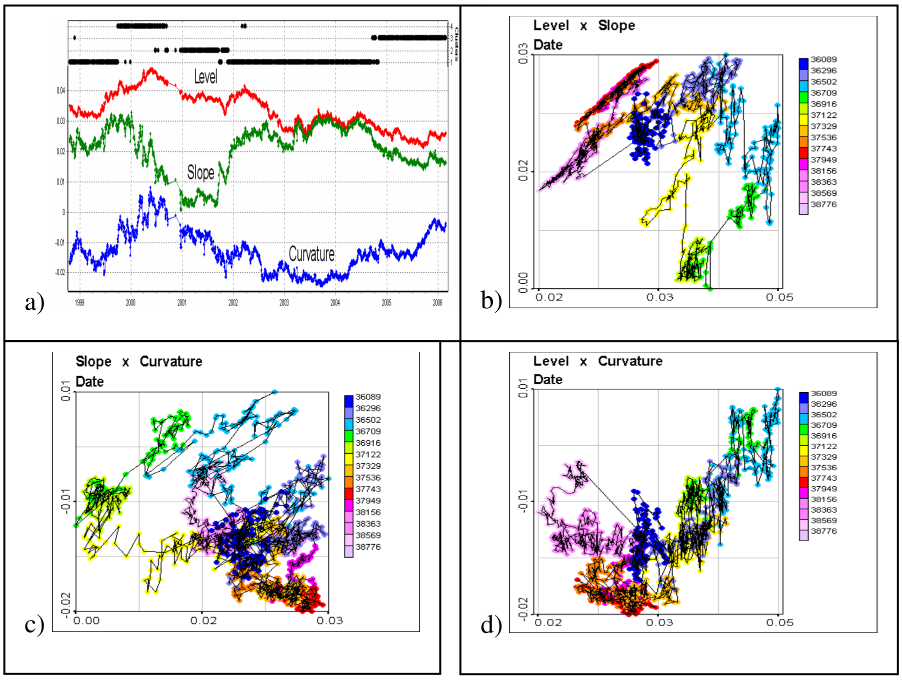

Fig. 4: Temporal evolution and clustering by SOM (4 clusters represented by dots) of NS model parameters (a), (b)-(d) – 2D graphs of temporal evaluations of factors (presented by colour scale).

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3 Mapping and predictions of interest rates

Mapping of interest rates can be used for different purposes [6,7]: 1) visualisation of interest rate curves as a two-dimensional image; 2) completion/interpolation of the part of image in order to fill missing data/curves; and 3) forecasting/extrapolation of IRC. In our case mapping is an embedding of all interest rate curves into a two-dimensional feature-space composed of maturity and date. In this space we have dispersed measurements at some points in time and for some maturities, according to raw data. The space can be filled/completed by using either traditional interpolation geostatistical tools, or by applying machine learning algorithms [6,7]. After the interpolation a two dimensional representation of IRC estimated at any maturity and for all dates is given. If we apply also an extrapolation technique, the forecasting of IRC can be performed.

An example of interest rates mapping by using simple linear interpolation and multilayer perceptron is given in Figure 6 (bottom). Two dimensional images after mapping are easier and better for understanding and interpretation of data. Linear interpolation reflects also some temporal information “flows” between different maturities. Mapping of IRC can be a basis for the development of visual analytics and exploratory tools.

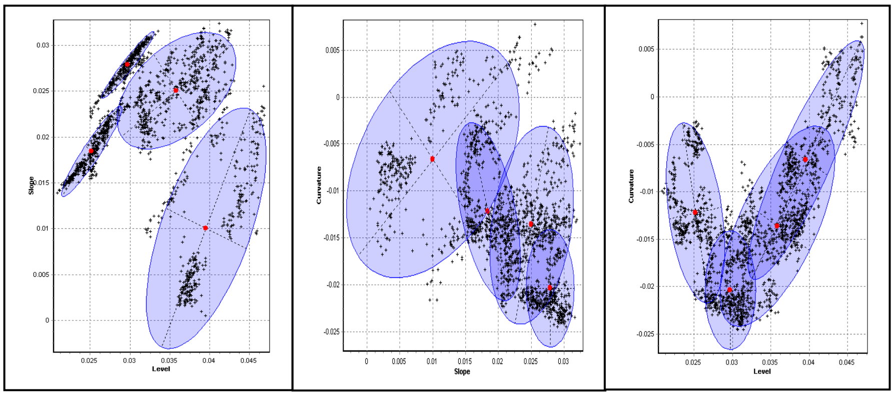

Fig. 5: 2D presentation of NSM 3 factors and 4 clusters found by GMM.

In the present research two interesting results following from IRC mapping are given. In Figure 6 (up-left) a reconstruction (missing curves) of two typical curves is presented. This can be considered as in-sample forecasting. Of course, more interesting is an out-of-sample forecasting of IR curves. Forecasting of the curve in a three business weeks is shown in Figure 6 (up-right).

4 Conclusions

In computational finance machine learning has a great potential and should be widely used from the comprehensive exploratory data analysis to the presentation/visualization of modeling results. In the present paper some topics were

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considered with promising results: analysis of local cross-correlations along with clustering in model's parameter space and mapping of interest rate for the visualization, missing data recovering and forecasting purposes. Future research will elaborate joint methodology by combining both approaches. An important topic deals with the connection of these results with the financial instruments and risk management.

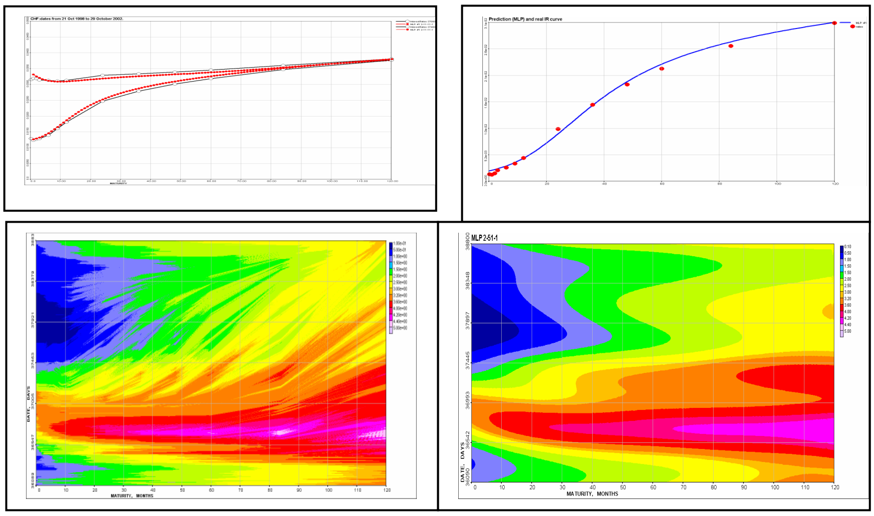

Fig. 6: 2D mapping (interest rates vs. X=maturities, Y=date) modeled by linear interpolation (bottom-left) and 2-50-1 MLP model (bottom-right) and missing curves reconstruction (top-left) and forecasting of interest rate curve (top-right).

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