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Shear Strength Prediction using Dimensional Analysis and Functional Networks

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Abstract. This paper presents a three steps methodology for predicting the failure shear effort in concrete beams. In the first step, dimensional analysis is applied to obtain several sets of dimensionless variables; in the second step, functional and neural networks are used to estimate a relation between those variables and, in the last step, the failure shear effort is recovered from the relations learnt. Finally, the performance of the methodology was validated using data from shear strength experiments.

1 Introduction

Traditionally, structural analysis of the elements of reinforced concrete has been accomplished empirically, and also employing theoretical formulas based on the experimental designs and the results obtained from them. One property of reinforced concrete, shear strength, follows analogous developments. During the last 50 years, several experiments [6, 10] allowed to fit theoretical models and to act as a base for obtaining the formulas of the several available codes and recommendations [1, 9] which limited predictive capability guarantees, however, enough structural security. In these experimental tests, the fundamental element is the beam, which is subjected to the action of increasing loads. The beam presents resistant mechanisms to the flexure and shear efforts originated from these loads. If an adequate selection of the appropriate variables involved is done, the failure shear effort of the beam could be predicted. In this case,

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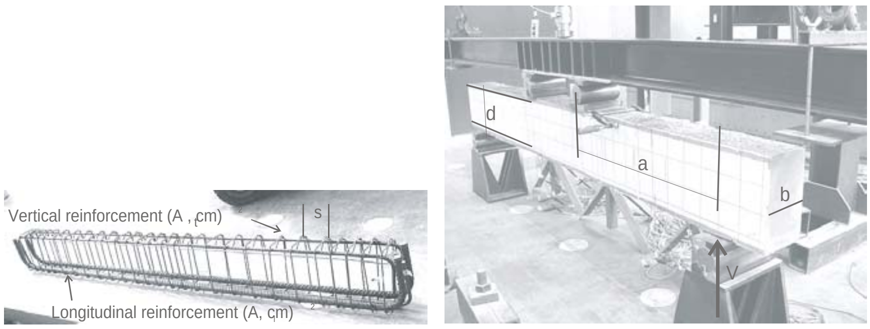

(a) Longitudinal and transversal reinforcement

(b) Geometric parameters and failure shear effort

Figure 1: Variables involved in failure shear effort

the principal variables playing a role are: a) the concrete strength (f_c, MPa), b) the yield strength used as reinforcement (f_y, MPa), c) the longitudinal reinforcement ratio in the shape of bars (ρ_l, %), d) the vertical shear reinforcement ratio in the shape of stirrups (ρ_v, %), e) a series of geometrical parameters such as the depth of the member (d, m), its width (b, m) and the shear span to depth ratio (a/d, adimensional) and, f) finally, the failure shear effort (V, kN). The name of each variable together with their units are shown in parenthesis.

Fig.1 a) shows the reinforcement before manufacturing the beam. Fig.1 b) shows the meaning of some of the variables detailed above. The experimental data used in this work belong to transversal reinforcement beams, and can be recovered from the database of the University of Illinois at Urbana-Champaign (Shear Data Bank, http://cee.ce.uiuc.edu/Kuchma/sheardatabank/). The available set of experimental data allows the use of artificial intelligence techniques, such as neural and functional networks. These networks have been extensively used for many years and they have been revealed as useful tools to derive non-linear relations between the input and the output variables involved in a given problem [2, 5, 8]. Therefore, they can be used to estimate the non-linear mapping g relating the previous variables as expressed by the following equation:

$$V = g(b, d, f_c, f_y, \rho_l, \rho_v, a/d) \; . \tag{1}$$

Moreover, the Π–Theorem, a fundamental theorem of dimensional analysis, can be used to find a simpler equivalent relation of g with a reduced set of dimensionless variables that reproduce the same physical relation.

In this work, the failure shear effort has been estimated using a three steps methodology based on dimensional analysis and functional and neural networks. In a first step, dimensional analysis is employed to reduce the dimension of the input space, using a set of dimensionless variables instead of the original variables. In a step further, functional and neural networks are used to learn the function that relates these variables, and, in the last step, the failure shear effort, V, is recovered from the function learnt.

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2 The proposed methodology

For the sake of simplicity, consider a problem with $n-1$ input variables, $X_1, \cdots, X_{n-1}$, and one output variable, X_n. This set of variables is represented using s fundamental magnitudes, $M_1, \cdots, M_s$. The proposed methodology will be applied as follows:

First step: Obtaining the dimensionless ratios. First, the number of input variables will be reduced. The following algorithm is automatically applied to determine all the sets of dimensionless ratios ensuring that the number of ratios is less than the number of variables:

1. Write the variables in terms of fundamental magnitudes. The variables are expressed in terms of the fundamental magnitudes as:

$$X_j = \prod_{i=1}^{s} M_i^{a_{ij}}; \;\; j = 1, 2, \ldots, n, \tag{2}$$

   where a_{ij} are the exponents associated with variable j, and the fundamental magnitude i. The elements a_{ij} form the matrix $\mathbf{A}_{s\times n}$ shown in Table 1.

Table 1: Matrix **A**: Representation of the variables in terms of fundamental magnitudes.

	X_1	X_2	$\ldots$	X_n
M_1	a_{11}	a_{12}	$\ldots$	a_{1n}
M_2	a_{21}	a_{22}	$\ldots$	a_{2n}
$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$
M_s	a_{s1}	a_{s2}	$\ldots$	a_{sn}

2. Determine the number of dimensionless ratios. The Buckingham Π–Theorem, a fundamental theorem used in dimensional analysis [3, 4], allows to determine the number of dimensionless ratios involved in a given problem. This theorem says that there exist $n-r$ dimensionless monomials by means of which the problem can be represented being r the rank of the matrix **A**.

   A submatrix **C** of **A** leading to the rank is calculated. The indices of the columns (input variables) of the matrix **A** that form the submatrix **C** compound the set $\mathcal{B}$, analogously, the set $\mathcal{F}$ is formed by indices of rows.

   It is necessary to choose the variables among the $n-1$ input variables, so that only one of the dimensionless ratios would contain the output variable in order to recover it later.

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3. Reduce dimensionality. Build a matrix $\mathbf{B}$ by removing from $\mathbf{A}$ the rows not in $\mathcal{F}$ and the columns in $\mathcal{B}$.

4. Change basis. Calculate the matrix $\mathbf{D} = \mathbf{C}^{-1}\mathbf{B}$, that gives the variables in terms of the new basic variables (those in $\mathcal{B}$).

5. Build the dimensionless ratios. Using the Π–theorem, the ratios are selected as:

$$\pi_k = \frac{X_k}{\prod\limits_{\ell \in \mathcal{F}} (X_\ell)^{d_{\ell k}}}; \forall k \notin \mathcal{B} \tag{3}$$

   where $d_{\ell k}$ are the elements of matrix $\mathbf{D}$.

It is important to notice that, in the step 2 of this algorithm, several submatrices $\mathbf{C}$ could be selected leading to different sets of dimensionless ratios. So, the algorithm is automatically repeated from step 2 until all the possible sets are obtained.

Second Step: Estimating the dimensionless output. Once the dimensionless ratios are known, neural and functional networks are employed to estimate the dimensionless ratio which includes the dimensional output, π_q, using all the others ratios as inputs, i.e., the function g' will be estimated as follows:

$$\pi_q = g'(\pi_1, \pi_2, \ldots, \pi_{q-1}). \tag{4}$$

Third Step: Recovering the dimensional output. The last step consists in recovering the original dimensional output X_n, using equation (3), from π_q as:

$$X_n = \pi_q \left(\prod_{\ell \in \mathcal{F}} (X_\ell)^{d_{\ell n}} \right) \tag{5}$$

3 Simulations

The proposed methodology was applied to estimate the failure shear effort (V) of a concrete beam. After applying the first step, the sets of ratios in Table 2 were obtained. Functional and neural networks were used to estimate the dimensionless output ratio, π_6. For the case of functional networks, the following minimization problem was proposed:

$$Minimize\ Q = \sum_{i=1}^{m} \left(\pi_{6i} - \sum_{k_1=d_{01}}^{d_1} \sum_{k_2=d_{02}}^{d_2} \cdots \sum_{k_5=d_{05}}^{d_5} C_{k_1,k_2,\cdots,k_5} \pi_{1i}^{k_1} \pi_{2i}^{k_2} \cdots \pi_{5i}^{k_5} \right)^2 \tag{6}$$

where $C_{k_1,k_2,\cdots,k_5}$ are the parameters to learn, m is the number of samples, d_{0t} and d_t with $t = 1, 2, 3$ and 5 were obtained experimentally as 0 and 2,

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Table 2: Sets of dimensionless ratios

Number of the set	π_1	π_2	π_3	π_4	π_5	π_6
1	$\frac{d}{b}$	$\frac{f_c}{f_y}$	ρ_l	ρ_v	$\frac{a}{d}$	$\frac{V}{b^2 f_y}$
2	$\frac{b}{d}$	$\frac{f_c}{f_y}$	ρ_l	ρ_v	$\frac{a}{d}$	$\frac{V}{d^2 f_y}$
3	$\frac{d}{b}$	$\frac{f_y}{f_c}$	ρ_l	ρ_v	$\frac{a}{d}$	$\frac{V}{b^2 f_c}$
4	$\frac{b}{d}$	$\frac{f_y}{f_c}$	ρ_l	ρ_v	$\frac{a}{d}$	$\frac{V}{d^2 f_c}$

respectively, while d_{04} and d_4 were obtained as -1 and 1. The high number of parameters of this model may overfit the network, so a restriction was added permitting only combinations of two variables. Moreover, for the sake of comparison, functional and neural networks were also employed to estimate the original function g in equation (1). In this case, the same functional model was employed (without using dimensional analysis), however, better results were obtained with the following approximation:

$$Minimize\ Q = \sum_{i=1}^{m} \left(\sum_{j=1}^{7} \sum_{k=d_{0j}}^{d_j} C_{jk} X_{ji}^{k} \right) \tag{7}$$

where X_{ji} is the i sample of the j original input variable, d_{0j} and d_j with $j = 1, 2, 3$ and 6 were obtained experimentally as 0 and 2, respectively and with $j = 4, 5$ and 7 were obtained as -1 and 1.

In the case of neural networks, a multilayer perceptron with several neurons in its hidden layer was used employing or not dimensional analysis. The best results were obtained using 5 hidden neurons, a regularized mean squared cost function, the Levenberg-Marquardt learning algorithm and the Mackay's bayesian framework to adapt the hyperparameters.

Table 3: Mean Normalized Mean Squared Errors over 30 simulations for Functional Networks (FN) and Neural Networks (NN)

		Approach				
		0	1	2	3	4
FN	Test	0.1789	0.0988	0.5283	0.1277	0.2535
	Validation	0.8460	0.5962	0.5768	0.3943	0.5076
NN	Test	0.1361	0.2291	0.2197	0.2212	0.3461
	Validation	2.9265	1.4570	2.8299	1.1047	1.1943

A ten-fold cross-validation was employed, running 30 simulations using different initial values. Moreover, a new set with 12 samples was obtained later from [7] allowing to accomplish a further validation of the trained systems. The mean results obtained for test and validation data using functional networks (FN) and neural networks(NN) without using the proposed methodology (approach 0) and using it (approaches 1 to 4) are shown in Table 3. It can be

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observed that approaches 1 to 4 achieve better validation results than the approach 0. Moreover, the test results are better for approaches 1 and 3, when functional networks are used. Also, the functional networks obtain better results than the neural networks. Furthermore, the results obtained using the ACI theoretical formulas [1] are 0.44 and 3.15 for test and validation data, respectively, so the methodology proposed also improves these results.

4 Conclusions

In this work, the failure shear effort was estimated using a new methodology based on dimensional analysis and functional networks. The proposed methodology uses dimensional analysis to reduce the number of inputs of a functional and/or neural networks showing a better performance than when they are applied without the previous step of dimensional analysis.

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