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Diminishing marginal utility and the teaching of economics:
A note

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This paper discusses how utility can be taught in undergraduate courses in microeconomics so that to illustrate total and marginal utility, the law of diminishing marginal utility, and consumer rationality. Diminishing marginal utility is essential in describing rational consumer behavior, overconsumption, and oversaturation to students of economics. We demonstrate a quadratic and a logarithmic total utility with the subsequent forms and shapes of marginal utility. From what it seems there is no contradiction between diminishing marginal utility in the univariate context of consuming one good and the indifference curve as the multivariate case of two goods consumed.

Introduction

Undergraduate students of economics studying the theory of the consumer learn that his utility of consuming a good is positive and increases, as he consumes more of that good. This is a simple way to express that more is valued much higher than less or that more is better than less. The so-called *Law of diminishing marginal utility* was first formulated by Herman Gossen (1854) who stated:

> "The magnitude of one and the same satisfaction, when we continue to enjoy it without interruption continually decreases until satisfaction is reached."

Marshall (1920, 1961) continued this line of thinking, giving a more technical definition. He stated that the additional benefit which a person derives from a given increase of his stock of a thing diminishes with every increase in the stock that he already has. If this concept is put in the context of total utility and marginal utility, then two results follow:

1. Total utility, as the happiness derived of consuming some commodity increases, when the consumer consumes more of it.
2. The marginal utility, as the speed with which total utility increases or the satisfaction which every next unit consumed brings to the consumer, decreases.

These two outcomes lead us to believe that a maximum of the total utility function would potentially be reached, that the marginal utility of some particular unit consumed may be negative, and that the total utility might bend over and eventually start to decline beyond that

point of consumption. Economics is defined as the study of the allocation of limited economic resources to satisfy the unlimited human needs. It is presumed that human needs are unlimited and that more is better than less. This is implied in microeconomics in the context of the indifference curve where it is demonstrated graphically that higher indifference curves are preferable to lower ones which depict lower levels of utility. On the one hand, it seems like a person's demand cannot be satiated. On the other, Gossen (1854) is talking about the person being over-satisfied and overfilled with a good. This is not a contradiction – it is believed that cumulative human needs are insatiable and a man desires maximum wealth, but the consumption of a single good can become excessive and there is a limit to which one can consume that good. If, beyond the point of satiation which Gossen is referring to, total utility does not fall but remains relatively constant for infinitesimal changes in the consumption of the commodity in question, then how could total utility be explained to students?

The concept of diminishing marginal utility is the subject of debate in the teaching of undergraduate economics. Some scholars believe that diminishing marginal utility is inconsistent with the theory of ordinal and cardinal utility (Lin and Peng, 2019). A different debate gravitates around whether demand curves should be derived from diminishing marginal utility and whether demand is a function of it. Dittmer (2005) argues that demand curves should not be derived from diminishing marginal utility in introductory textbooks and suggests that introductory textbooks should instead treat demand in relation to diminishing marginal value or the diminishing marginal rate of substitution. Silbeberg (1990) believes that the concept of diminishing marginal utility is irrelevant in modern economics. Silbeberg (1990, p. 38, note 6) also sees diminishing marginal utility as an "archaic, and for the most part abandoned assertion."

To us these are essentially two different debates.[1] While we do not necessarily object to the argument that the demand curve does not reflect diminishing marginal utility and should not be derived thereof, we believe that the diminishing marginal utility of one good is an essential concept which should be taught to beginning students of economics. This paper is a reconsideration of the *Law* formulated by Gossen and Marshall and an attempt to pay tribute to their ideas. Furthermore, we recommend a way to teach marginal utility in introductory courses in microeconomics with a smooth transition from the univariate case with one good where the *Law* applies to the introduction of the indifference curve with two commodities involved, that is, the multivariate context. In this sense, the paper is pedagogically oriented emphasizing some of the most essential elements of consumer theory which beginning students of economics should be familiar with. We try to rehabilitate the concept of diminishing marginal utility in its most general meaning, in the context of consuming one good only and without reference to the law of demand. The demand curve reflects the quantity the consumer would buy at difference price levels and, hence is a function of price, rather than of marginal utility, but this does not imply that the concept of diminishing marginal utility is wrong and should not be discussed in undergraduate economics courses.

Some fields which rely on diminishing marginal utility are game theory and choice under uncertainty. Von Neumann and Morgenstern (1944) studied gambling and insurance using different attitudes to risk. We argue that there is a place for diminishing marginal utility in economics beyond the area of finance, risk aversion and the theory of wealth. To us there is a place for diminishing marginal utility not only in financial economics, uncertainty, and risk but also in describing the ordinary consumption of the consumer in the conditions of certainty.

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[1] The literature on both debate issues is extensive, dating back to the 1920s and 1930s, and we do not attempt to thoroughly review it.

Through Gossen's *Law of diminishing marginal utility* students realize that overconsumption could be a threat to the wellbeing of the consumer. Undergraduate students can be taught that a commodity could be a good, if consumed in normal quantities, and a bad, if consumed excessively. Unlike the concept of indifference curves which is taught in terms of two commodities consumed and the utility of the consumer being a function of those two, the concept of diminishing marginal utility is univariate. The student can see the contribution of one product alone in the consumption basket and its effect on total utility. Furthermore, the concept of normal and excessive consumption, i.e. goods versus bads, could be transferred to the concept of the indifference curve where the slope is negative in the well-behaved section.

Today it becomes clear that Gossen had a complete theory of the economic behavior of individuals. According to Van Daal (1995) it is unfair to consider him only a precursor of later pioneers in the marginalist school such as Jevons, Menger and Walras. Gossen developed a coherent economic theory which neoclassical economists repeated later, putting it in a more technical framework. The reason why Gossen's substantive contribution to economics has been undermined or neglected is perhaps the fact that his writings contained many religious beliefs and statements. Gossen is especially credited for his "second law," by which the agent maximizes his utility by equalizing the marginal utilities of a set of goods consumed weighted by their prices (Steiner, 2011). According to Steiner (2011) Jevons and Walras were surprised that an unknown, self-taught economist had formulated their ideas two decades earlier and so they had to put their work in a formal perspective in order to make it original and distinguishable from that of Gossen.

The section that follows discusses how to best present total and marginal utility to undergraduate students of economics. It is essential for beginning students who have no prior background in economics or applied mathematics to understand these concepts in very basic terms. We discuss two modes, a quadratic and a logarithmic total utility, with their caveats. Then we relate the simple case of the total utility of a single good to that of two goods, x and y. We discuss the marginal rate of substitution in relation to the change in the ratio of the two marginal utilities, as the consumer changes the consumption of x and y along the indifference curve. The paper ends with conclusions.

Teaching total and marginal utility

When discussing the total and marginal utility of a good textbooks use a quadratic total utility function, as presented in Figure 1. The requirement is for the function of total utility to have a maximum and the marginal utility to be declining, that is, for a concave total utility function to obtain.
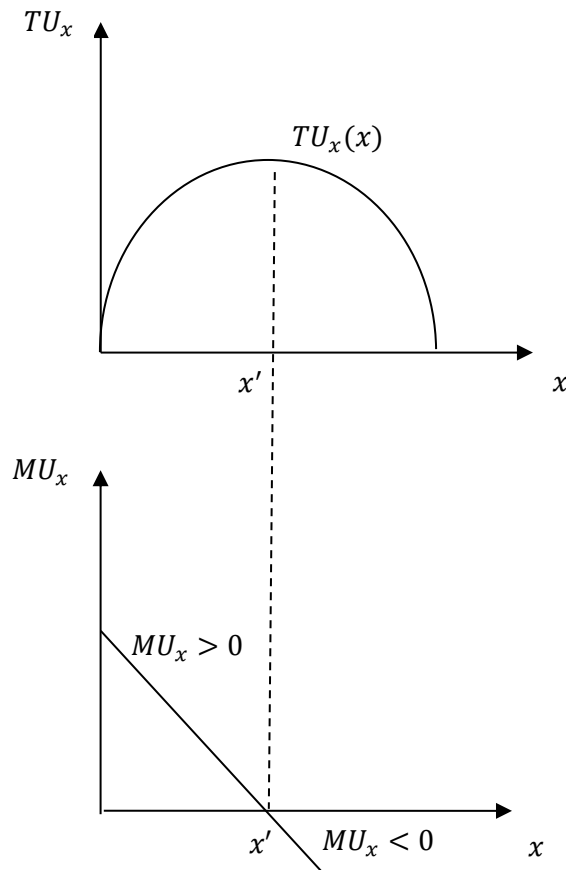
Figure 1. Quadratic total utility

Under these premises, marginal utility is a linear, negatively sloped function with a slope of $MU_x' = TU_x''(x') = 2a$, where $a < 0$ since total utility is presumed to be a quadratic function of the general type $TU_x(x) = ax^2 + bx + c$. With a strictly concave total utility function there are no flat segments. With such a quadratic function, there is one extremum obtained at

$MU_x = TU'_x = 2ax' + b = 0$, or
$$x' = -\frac{b}{2a}$$
implying that $b > 0$ for a positive quantity of good x consumed and $c = 0$. The last result follows from the fact that with zero consumption of the good the consumer would derive zero total utility. The quadratic total utility gives the curvature of marginal utility which is declining at a constant rate but is initially positive, showing that at the beginning the good affects consumer happiness favorably, and then negative, demonstrating that good x turns into a bad eventually, decreasing the total utility of the consumer. Thus, there is a point of saturation beyond which it is irrational for the consumer to keep on consuming.

Some texts describe a logarithmic, rather than a quadratic, total utility function, as shown in Figure 2.
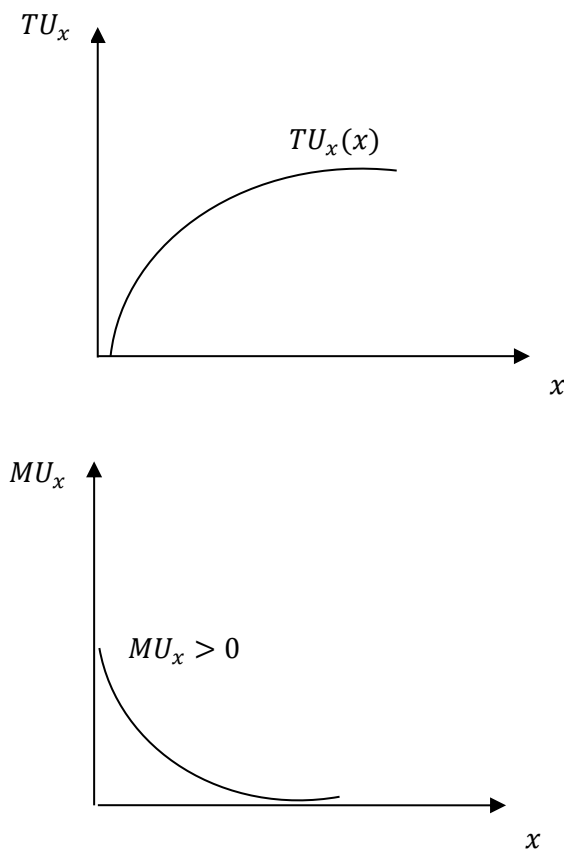
Figure 2. Logarithmic total utility

This form of the utility function satisfies the two conditions stated initially, that is, that the total utility is increasing, as the consumer consumes more of x, and the marginal utility is decreasing. This satisfies the *Law of diminishing utility*, but two things are worth noting. First, the total utility does not have a declining section in the logarithmic form, that is, marginal utility can never become negative but reaches the x-axis asymptotically. This precludes the possibility for good x to be a bad and have an adverse effect on the consumer's health. The total utility thus reaches a flat section and does not fall below that, excluding the possibility for relating this analysis to positively sloped indifference curves, overconsumption, or the indifference curve map. It also seems that such a logarithmic total utility is more relevant to finance, wealth maximization, portfolio theory or attitude to risk and not so much to pure consumption. Second, marginal utility is a nonlinear, decreasingly decreasing function, unlike the quadratic case where it is linear. Total utility thus can be presented as

$$TU_x = log_b x$$

where for the base b we have $b > 1$. Thus, x and TU_x are positively related where TU_x starts at $x > 0$, as in Figure 3. For the consumer experiencing such a total utility function this implies that he will not start deriving satisfaction before a certain level of consumption is reached or the consumer needs to consume a minimum amount of x so that his total utility begins to grow. This is different from the quadratic case where TU_x starts at zero. However, TU_x could start at zero or values close to the origin of the coordinate system under a different form of the function, for instance, $TU_x = log_b x + a$ where $a > 0$, as illustrated by the dotted line in Figure 3.
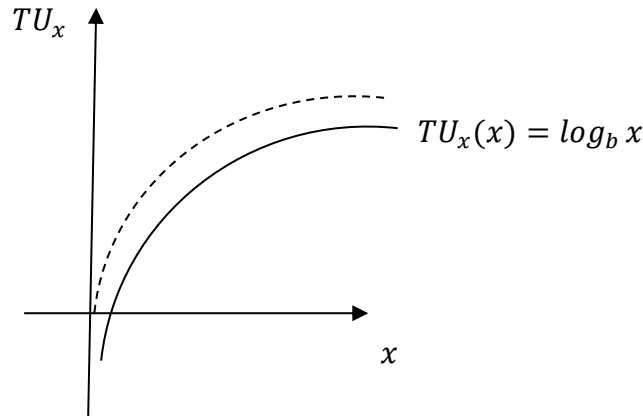
Figure 3. Forms of logarithmic total utility

Analyzing the logarithmic utility function further, we obtain marginal utility as

$$MU_x = \frac{dTU_x}{dx} = \frac{d \log_b x}{dx} = \frac{1}{x \ln b}$$

which is a decreasing function in x. In the case of a more sophisticated utility function such as $TU_x = \log_b f(x)$, the marginal utility function is

$$MU_x = \frac{dTU_x}{dx} = \frac{d \log_b f(x)}{dx} = \frac{f'(x)}{f(x) \ln b}$$

In the special case of natural logs and the natural exponential number e the function becomes

$$MU_x = \frac{dTU_x}{dx} = \frac{d \ln x}{dx} = \frac{1}{x}$$

and with a natural logarithmic total utility function such as $TU_x = \ln_b f(x)$, the marginal utility function is

$$MU_x = \frac{dTU_x}{dx} = \frac{d \ln f(x)}{dx} = \frac{f'(x)}{f(x)}$$

These results confirm the nonlinear form of marginal utility and the slow rate at which it decreases, as the consumption of x increases. This implies that a satiation point for the consumer can hardly be reached or will be reached at infinitely large values of x, as opposed to the quadratic case. The slopes of marginal utility in the simple cases of a natural and general logarithmic function are, respectively,

$$MU'_x = -\frac{1}{x^2}, \text{ and}$$
$$MU'_x = -\frac{1}{x^2 \ln b}$$

Such a setting would perhaps be more appropriate to the utility of wealth and money, as used in finance and portfolio theory. The concept of quadratic total utility is more appropriate to ordinary consumption. The ongoing debate about the diminishing marginal utility and the derivation of the demand curve sets diminishing marginal utility for all goods as a necessary condition for the diminishing marginal rate of substitution. The *Law of diminishing marginal*

utility is essential for the learning of economics and can be taught in relation to the concept of the indifference curve. In this sense, there is no contradiction between the two concepts, that of marginal utility and that of the indifference curve. On the contrary, they supplement each other in a course in introductory microeconomics. When dealing with marginal utility in the univariate context, i.e., when a single good is consumed, it is essential to stress this adverse relationship to students. The consumption of x is increasing but its marginal utility is falling and vice versa.[2] Thus, commodities in large supply or abundance, consumed in larger volumes, bring lower marginal utility to the consumer compared to other commodities in short supply.

According to the diamond-water paradox introduced by Adam Smith (1776) the commodities which are scarce such as diamonds are most valuable. Smith observed that people cannot live without water, yet diamonds are much more valuable to them. From the perspective of marginal utility, the first drops of water are valuable for life. But because of the abundance of water and their scarcity diamonds are considered more valuable. Because of its abundance water has much lower marginal utility in a person's consumption basket. Because of their scarcity diamonds have a higher marginal utility for the consumer.[3] This dependence between the marginal utility and the abundance of the good is relevant to the indifference curve. Along the indifference curve, where the consumer consumes two goods, x and y, his utility of consuming these two goods is constant, or

$$dTU = MU_x dx + MU_y dy = 0$$

This already takes us to the two-variable case, from the single-variable one of total utility as a function of x alone. The differential equation gives the slope of the indifference curve, or

$$MRS = \frac{dy}{dx} = -\frac{MU_x}{MU_y}$$

Based on the consumption of the two goods and whether the consumer will substitute x for y or y for x the marginal rate of substitution will change. In accordance with the single-variable *Law of diminishing utility* the marginal utility of x will fall, as the consumer increases his consumption of x. To keep his utility constant on the indifference curve, he will have to reduce his consumption of y which will further increase the marginal utility of y. Thus, MU_x will decrease, while MU_y will increase which reduces the marginal rate of substitution. In the other case, when x is decreased and y is increased, the opposite happens. Due to the *Law of diminishing marginal utility*, MU_x increases and MU_y decreases making the indifference curve steeper. Therefore, the *Law of diminishing marginal utility* does not violate conventional consumer theory in the part on indifference curves but rather supports it.

We proceed with a discussion of the indifference curve map where the *Law* becomes handy again. Figure 4 illustrates an indifference curve map with the good section of well-behaved indifference curves in quadrant I. This is the section of convex and negatively sloped indifference curves where the two marginal utilities MU_x and MU_y are positive and x and y are

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[2] Exceptions could be some atypical goods (for instance, addictive goods) for which total utility might increase increasingly, that is, be convex.

[3] In fact, Smith distinguished between "value of use" and "value of exchange" apparently implying utility (benefit) and cost (price). The paradox thus reinforces the argument that demand is a function of price, that is, the result of scarcity, rather than utility. Thus, diminishing marginal utility and the demand curve, as the price of the good, are essentially two different things.

consumed at normal amounts, that is, the two commodities are goods. In quadrant II the consumption of x is excessive at x'' and, therefore, its marginal utility is negative. Thus, x is a bad and y is a good. This gives a positive slope of the indifference curve. In quadrant III both x and y are bads, having negative marginal utilities MU_x and MU_y, respectively. This gives a negative slope of the indifference curve, but the curve is concave. Finally, in quadrant IV the consumer consumes a normal quantity of x but an excessive amount of y, which is why its marginal utility MU_y is negative and the marginal rate of substitution is positive.
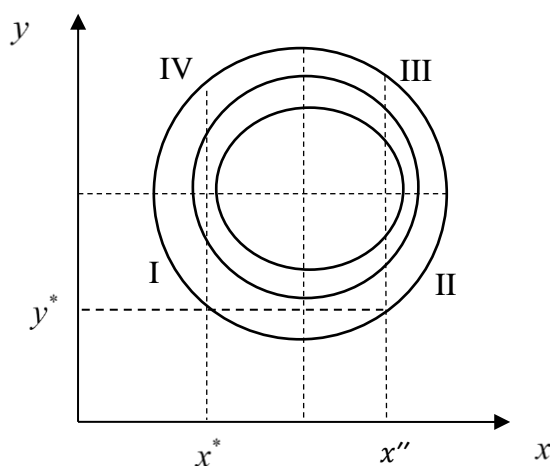

Figure 4. Diminishing marginal utility and the indifference curve map

Note that the analysis of marginal utility need not involve budget considerations where the consumer must allocate his budget over the two goods whose prices are p_x and p_y, respectively. In the presence of a budget constraint he must set his optimal consumption bundle of x and y just where the marginal rate of substitution equals the relative commodity price, that is,

$$-\frac{MU_x}{MU_y} = -\frac{p_x}{p_y}$$

One does not have to refer to prices to present diminishing marginal utility, but students understand this essential concept as part of rational human behavior in economics.

Conclusion

A professor teaching introductory economics would find it handy to explore different techniques of teaching diminishing marginal utility to undergraduate students. While to some authors the concept is contradictory and outdated, we believe that the *Law of diminishing marginal utility*, as defined by Gossen, is essential in describing rational versus irrational behavior, utility versus disutility, goods versus bads. It is important to explain overconsumption and oversaturation to students of economics and diminishing marginal utility is a way to demonstrate those. We believe there is still room for the *Law of diminishing marginal utility* in the teaching of economics, at least at the introductory or intermediate level and that diminishing marginal utility should not be thrown out of textbooks.

Professors teaching undergraduate economics could present utility as either quadratic or logarithmic. The first fits better with the ordinary consumption of a good, whereas the logarithmic type seems more relevant to portfolio theory and the study of risk aversion. In our paper we have presented how to teach both approaches to utility, with the peculiarities of the marginal utilities associated with them. We also relate the univariate case to the multivariate one with two goods consumed along the indifference curve. We do not see a contradiction between the univariate *Law of diminishing marginal utility* and the two-variable indifference curve. Proper utility functions should be chosen which are consistent with the *Law*. This would bring much more clarity into the methodology of teaching total and marginal utility with their possible forms, shapes, and dependences.

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