Reinforcement Learning

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Introduction to Machine Learning (CS771A)

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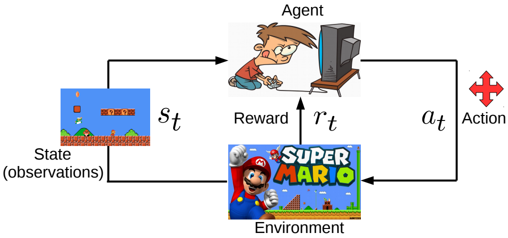

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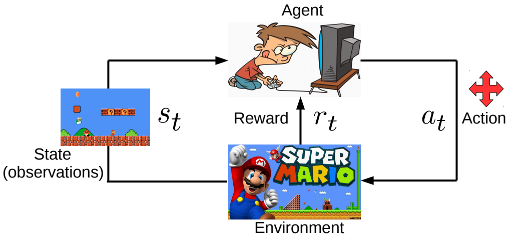

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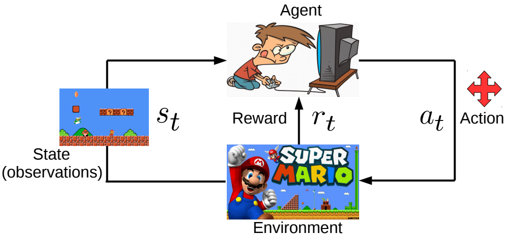

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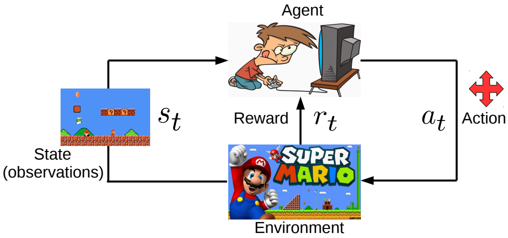

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- Many applications: Robotics, autonomous driving, computer game playing, online advertising, etc.

Markov Decision Processes (MDP)

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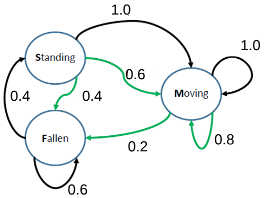

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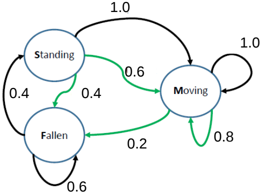

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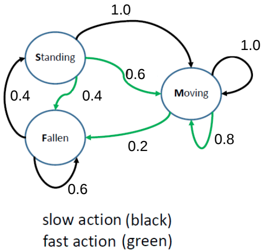

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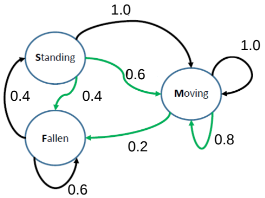

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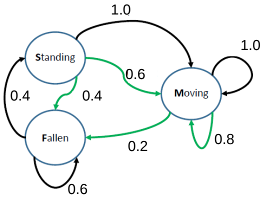

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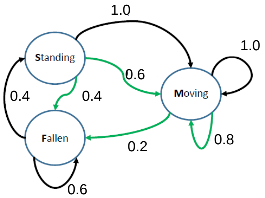

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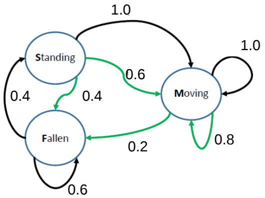

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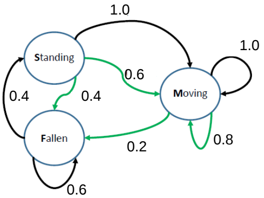

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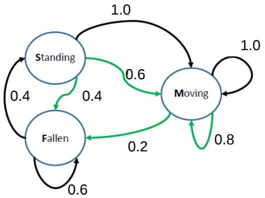
- P_{sa} and R may be unknown (may need to be learned)



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- The expectation $\mathbb{E}$ is w.r.t. all possibilities of the initial state s_0

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  - **Policy Iteration:** Iterate between learning the optimal policy π^* and learning V^*
  - **Q Learning** (a variant of value iteration)

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- **Note:** The inner loop can update $V(s)$ for all states simultaneously, or in some order

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$$P_{sa}(s') = \frac{\#\text{ of times we took action } a \text{ in state } s \text{ and got to } s'}{\#\text{ of times we took action } a \text{ in state } s}$$

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- Likewise, the expected reward $R(s)$ in state s can be computed
  - $R(s)$ = average reward in state s across all the trials

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Note: Step 3 can be made more efficient by initializing V with values from the previous iteration

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- How to handle these continuous state spaces?

Discretization

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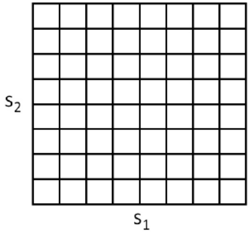

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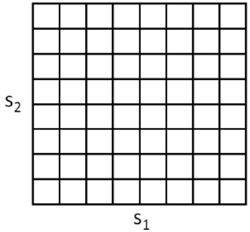

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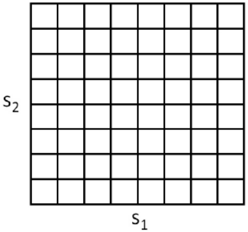

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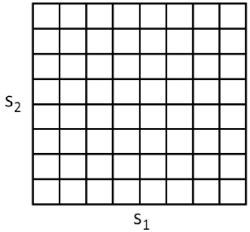

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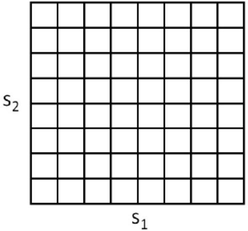

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- To do so, we will need (an approximate) model of the underlying MDP

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- A and B can be estimate from the trial data
- Can also make the function stochastic/noisy: $s_{t+1} = As_t + Ba_t + \epsilon_t$ where $\epsilon_t \sim \mathcal{N}(0, \Sigma)$ is the random noise (Σ can also be learned)

Approximating the MDP Model

- Can also learn nonlinear functions $s_{t+1} = f(s_t)$

$$s_{t+1} = A\phi_s(s_t) + B\phi_a(a_t)$$

- Any nonlinear regression algorithm can be used here

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&= R(s) + \gamma \max_a \mathrm{E}_{s' \sim P_{sa}}[V(s')]
\end{aligned}
$$

- Note: sum replaced by integral (since the state space S is continuous)
- Want a model for $V(s)$. Let's assume $V(s) = \theta^\top \phi(s)$
- We would need some training data in order to learn θ

$$\{V(s^i), \phi(s^i)\}_{i=1}^m$$

Approximating the Value Function

- We will use "Fitted Value Iteration" methods
- Recall the value iteation

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- We will generate such training data and learn θ in an alternating fashion

Fitted Value Iteration: The Full Algorithm

1. Randomly sample m states $s^{(1)}, s^{(2)}, \ldots s^{(m)} \in S$.
2. Initialize $\theta := 0$.
3. Repeat {

    For $i = 1, \ldots, m$ {

    For each action $a \in A$ {

    Sample $s'_1, \ldots, s'_k \sim P_{s^{(i)}a}$ (using a model of the MDP).

    Set $q(a) = \frac{1}{k}\sum_{j=1}^{k} R(s^{(i)}) + \gamma V(s'_j)$

    // Hence, $q(a)$ is an estimate of $R(s^{(i)}) + \gamma \mathrm{E}_{s' \sim P_{s^{(i)}a}}[V(s')]$.

    }

    Set $y^{(i)} = \max_a q(a)$.

    // Hence, $y^{(i)}$ is an estimate of $R(s^{(i)}) + \gamma \max_a \mathrm{E}_{s' \sim P_{s^{(i)}a}}[V(s')]$.

    }

    // In the original value iteration algorithm (over discrete states)

    // we updated the value function according to $V(s^{(i)}) := y^{(i)}$.

    // In this algorithm, we want $V(s^{(i)}) \approx y^{(i)}$, which we'll achieve

    // using supervised learning (linear regression).

    Set $\theta := \arg\min_\theta \frac{1}{2}\sum_{i=1}^{m} \left(\theta^T \phi(s^{(i)}) - y^{(i)}\right)^2$

Fitted Value Iteration

- Other nonlinear regression algorithms can also be used

$$V(s) = f(\phi(s))$$

  where f is a nonlinear function (e.g., modeled by a Gaussian Process)

- Note: Fitted value iteration is not guaranteed to converge (though, in practice, mostly it does)
- The final output is V (an approximation to V^*)
- V implicitly represents our policy π. The optimal action

$$\arg\max_a \mathbb{E}_{s' \sim P_{sa}}[V(s')]$$

Other Topics related to RL

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  - Doesn't assume the reward function to be known. Learns it

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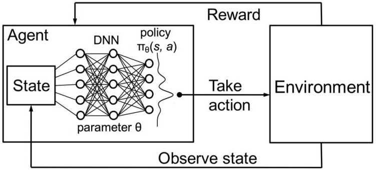
- Deep Reinforcement Learning



Summary

- Basic introduction to Reinforcement Learning
- Looked at the definition of a Markov Decision Process (MDP)
- Looked at methods for learning the MDP parameters from data
  - Easily and exactly for the discrete state-space case
  - Using function approximation methods in the continuous case
- Looked at methods for Policy Learning
  - MDP Learning and Policy Learning usually done jointly