

**MLAMBDA:  
MATLAB Package for Integer Least Squares  
Ambiguity Determination**

*Users' Guide*

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# 1 Introduction

Let the sets of all real and integer  $m \times n$  matrices be denoted by  $\mathbb{R}^{m \times n}$  and  $\mathbb{Z}^{m \times n}$ , respectively, and the sets of real and integer  $n$ -vectors by  $\mathbb{R}^n$  and  $\mathbb{Z}^n$ , respectively.

Given a real least square estimate  $\mathbf{a} \in \mathbb{R}^n$  of the integer ambiguity vector and its covariance matrix  $\mathbf{W} \in \mathbb{R}^{n \times n}$ , this MATLAB package provides a function to produce  $p$  integer least squares estimates of the integer ambiguity vector, i.e., it produces  $p$  optimal solutions to the integer least squares problem (in quadratic form)

$$\min_{\mathbf{x} \in \mathbb{Z}^n} (\mathbf{x} - \mathbf{a})^T \mathbf{W}^{-1} (\mathbf{x} - \mathbf{a}), \quad (1)$$

in the sense that  $\mathbf{x}^{(j)} \in \mathbb{Z}^n$  is the  $j$ -th optimal solution if its corresponding value of the objective function is the  $j$ -th smallest (some of these  $p$  values can be equal). Here  $p$  is a parameter to be provided by a user and its default value is 1. This package mainly uses the reduction algorithm proposed in [1] and the search algorithm given in [2]. The implementation of the search algorithm uses an idea given in [3].

If your integer least squares problem is in the standard form:

$$\min_{\mathbf{x} \in \mathbb{Z}^n} \|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2^2, \quad (2)$$

where  $\mathbf{A} \in \mathbb{R}^{m \times n}$  has full column rank, we strongly suggest that you use our MATLAB package MILES, which can be downloaded from the same web page, as a transformation from (2) to (1) may not be numerically reliable.

The purpose of this document is to show how to use this package.

## 2 System Requirements

The package has been fully tested on PC, Linux and Mac with MATLAB R2016a, and should work on any platform supporting MATLAB. For the system requirements of running MATLAB, please refer to:

<http://www.mathworks.com>

## 3 Installing Package

The package is provided as a compressed file with extension “zip”. To extract the package, an uncompress tool should be used. Suppose that the package is extracted to “C:/MLAMBDA” on Windows, then you can enter the directory and use the provided functions or examples. If you want to use the package in a directory rather than “C:/MLAMBDA”, you have to add “C:/MLAMBDA” to the MATLAB path.

## 4 Citation

MLAMBDA is a freely available software package provided on the Internet. If you are using it in research work in GNSS to be published, please include explicit mention of our work in your publication. You may state something like:

“To fix integer ambiguities, we use MLAMBDA, a MATLAB package for integer least squares ambiguity determination [1][2][3]”

with the following corresponding entries in your bibliography:

- [1] X.-W. Chang, MLAMBDA: MATLAB Package for Integer Least Squares Ambiguity Determination, <http://www.cs.mcgill.ca/~chang/software.php>, May 2016.
- [2] X.-W. Chang, X. Yang, and T. Zhou. MLAMBDA: A modified LAMBDA method for integer least-squares estimation. *Journal of Geodesy*, 79:552–565, 2005.
- [3] M. Al Borno, X.-W. Chang, and X. Xie. On “decorrelation” in solving integer least-squares problems for ambiguity determination. *Survey Review*, 46:37-49, 2014.

## 5 Support

The MLAMBDA project supports the package in the sense that reports of errors or poor performance will gain immediate attention from the developers. Any comment and suggestion for improvement of the code or the document is also welcome. It may still be possible to improve the efficiency by using some programming tricks, but for research purpose we try to keep the code simple and clear. Error reports and also descriptions of interesting applications and other comments should be sent to:

Prof. Xiao-Wen Chang  
Email: [chang@cs.mcgill.ca](mailto:chang@cs.mcgill.ca)

## 6 Routines

The package includes four routines `ldlp.m`, `reduction.m`, `search.m` and `mlambda.m`. The driver routine `mlambda.m` calls `reduction.m` and `search.m`, and `reduction.m` calls `ldlp.m`.

The package provides a script M-file `example.m` to show how to use this package to solve an ILS problem in the form of (1).

## 7 Troubleshooting

Here we list some problems a user may encounter.

### 7.1 Common errors in calling MLAMBDA routines

A user should always carefully read the leading comments of a routine before using it. The leading comments describe what the routine can do and give a detailed description of all input/output arguments. For the benefit of users, here we list the most common programming errors in calling a routine. These errors may cause the MLAMBDA routines or MATLAB to report a failure, or may lead to wrong results without a warning message.

- Wrong number of arguments

- Arguments in wrong order
- Wrong dimensions for an array argument
- The input matrix is not symmetric positive definite
- MATLAB path is not set up appropriately.

## 7.2 Poor performance in efficiency

One should note that the integer least squares problem is NP-hard. If the dimension of the integer least squares problem is big, then the computation can be very time-consuming. Another thing we should mention is that MATLAB is slower than some high level programming languages, such as C/C++.

## 7.3 Integer overflow

If an integer number produced in the computation is outside of the interval  $[-2^{53} + 1, 2^{53} - 1]$ , then its floating point representation in double precision may not be accurate. This may lead wrong integer solutions. For practical applications, however, this integer overflow phenomena may not be a concern. This package does not check integer overflow and so does not give an warning message if this occurs.

## References

- [1] M. Al Borno, X.-W. Chang, and X. Xie. On “decorrelation” in solving integer least-squares problems for ambiguity determination. *Survey Review*, 46:37-49, 2014.
- [2] X.-W. Chang, X. Yang, and T. Zhou. *MLAMBDA: A modified LAMBDA method for integer least-squares estimation*. *Journal of Geodesy*, 79:552–565, 2005.
- [3] A. Ghasemmehdi and E. Agrell. Faster Recursions in Sphere Decoding, *IEEE Transactions on Information Theory*, 57:3530–3536, 2011.