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GRAPHS WITH CLUSTERS PERTURBED BY REGULAR GRAPHS—A_α-SPECTRUM AND APPLICATIONS

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Abstract

Given a graph G, its adjacency matrix $A(G)$ and its diagonal matrix of vertex degrees $D(G)$, consider the matrix $A_\alpha(G) = \alpha D(G) + (1-\alpha)A(G)$, where $\alpha \in [0,1)$. The A_α-spectrum of G is the multiset of eigenvalues of $A_\alpha(G)$ and these eigenvalues are the α-eigenvalues of G. A cluster in G is a pair of vertex subsets (C,S), where C is a set of cardinality $|C| \geq 2$ of pairwise co-neighbor vertices sharing the same set S of $|S|$ neighbors. Assuming that G is connected and it has a cluster (C,S), $G(H)$ is obtained from G and an r-regular graph H of order $|C|$ by identifying its vertices with the vertices in C, eigenvalues of $A_\alpha(G)$ and $A_\alpha(G(H))$ are deduced and if $A_\alpha(H)$ is positive semidefinite, then the i-th eigenvalue of $A_\alpha(G(H))$ is greater than or equal to i-th eigenvalue of $A_\alpha(G)$. These results are extended to graphs with several pairwise disjoint clusters $(C_1,S_1),\ldots,(C_k,S_k)$. As an application, the effect on the energy, α-Estrada index and α-index of a graph G with clusters when the edges of regular graphs are added to G are analyzed. Finally, the A_α-spectrum of the corona product $G \circ H$ of a connected graph G and a regular graph H is determined.

Keywords: cluster, convex combination of matrices, A_α-spectrum, corona product of graphs.

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