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Throughput Maximization of Large-Scale Secondary Networks over Licensed and Unlicensed Spectra

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Abstract—**Throughput of a mobile ad hoc network (MANET) operating on an unlicensed spectrum can increase if nodes can also transmit on a (shared) licensed spectrum. However, the transmissions on the licensed spectrum has to be limited to avoid degradation of quality of service (QoS) to primary users (PUs). We address the problem of how the nodes of a MANET or secondary users (SUs) should spread their transmissions on both licensed and unlicensed spectra to maximize network throughput, and characterize 'throughput gain' achieved in such spectrum sharing systems. We show that the gain can be significant and is increasing in the density of the SUs. The primary and secondary users are modeled as two independent Poisson point processes and their performance is evaluated using techniques from stochastic geometry.**

I. INTRODUCTION

The emergence of new network paradigms including Internet of Things (IoT) [1], Device to Device communications (D2D) [2] continue to raise the demand for scarce wireless spectrum. To maintain cost effectiveness, most of these networks operate on unlicensed spectrum, e.g. ISM band, which is reserved internationally. But, this spectrum band is limited and may not suffice to guarantee good network performance. On the other hand, most of the terrestrial wireless communication spectrum is licensed for proprietary usage. However, this static allocation is inefficient and lot of this spectrum is under-utilized [3], e.g. uplink channels in cellular networks [4]. In this work, we are interested in the performance (success rate) improvement that a large-scale wireless ad hoc network can achieve by simultaneously using both licensed spectrum (shared) and unlicensed spectrum for packet transmission.

Spectrum sharing is key to alleviate spectrum shortage and improve throughput in mobile ad hoc networks. One promising technology, which has been extensively explored, is the cognitive radio networks where unlicensed users (secondary) opportunistically transmit on a spectrum owned by licensed users (primary). The secondary users (SUs) either transmit on the unused portion of the licensed spectrum (overlay) or transmit on the same spectrum as the primary users (PUs) provided their transmissions do not affect the performance of PUs (underlay). In underlay networks, the SUs continuously sense the primary spectrum to detect idle transmission slots, thus their performance depends on PUs' traffic pattern.

In applications such as the Internet of Things (IoT), a large number of heterogeneous devises need to continuously transmit critical data requiring seamless connectivity among them. If a licensed spectrum is available for spectrum sharing, some of these devices (SUs) can transmit on a licensed spectrum (provided they do not degrade performance of PUs) while the rest transmit on the unlicensed spectrum. Then the key question is what fraction of the SUs should transmit on each channel at any given instant. Clearly, more SUs transmitting on the licensed spectrum may degrade quality of service (QoS) of the PUs. On the other hand, more SUs transmitting on the unlicensed spectrum can lead to a higher outage rate due to increased intra channel interference. Therefore, we investigate how should the secondary users/traffic be split across licensed and unlicensed spectra so that the density of successful transmissions (throughput in short) for SUs increases without violating QoS of PUs.

In LTE-U technology, cellular network operators propose to use unlicensed spectrum in addition to their licensed spectrum to boost coverage. We address the reverse problem of unlicensed spectrum users sharing a licensed spectrum to improve their throughput. Our goal in this work is to characterize the *throughput gain* of a secondary network by using both the licensed/primary channel (PC) and unlicensed/secondary channel (SC) simultaneously. Throughput gain of a secondary network is defined as the ratio of highest achievable throughput using both PC and SC and that achievable using SC alone. At each time slot, a secondary device selects either PC or SC for transmission which decides what fraction of SUs use PC or SC in that slot. Furthermore, the amount of secondary transmissions on PC should be such that the outage probability of each PU remains below some threshold (QoS guarantee).

We assume that the secondary network is a mobile ad hoc network (MANET) and the primary network is another MANET operating on a proprietary spectrum, e.g. FlashLinQ [5]. We model the PUs and SUs as homogeneous independent Poisson point processes and use techniques from stochastic geometry to derive performance metrics. Stochastic geometry has been widely used to analyze the performance of mobile ad hoc networks [6], cognitive radio networks [7], [8] and cellular networks [9], often leading to tractable analysis while giving performance comparable to that observed in reality [10].

In our setting, the strategy of each SU is to select a channel (PC or SC) for transmission in each time slot. A network operator[1] interested in providing high connectivity in the secondary network would assign a channel selection strategy that maximizes overall density of successful trans-

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[1] Network operator can be a regulator or a device manufacturer that sets the protocol of the secondary users.

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missions (network throughput) for SUs while maintaining QoS guarantee for PUs. However, due to decentralized nature of MANETs, the SUs may not follow the strategy assigned by the operator and selfishly select channels to maximize their throughput leading to loss in performance of secondary network and also degradation of QoS in primary network. To mitigate the loss due to non-cooperation, we consider a pricing based de-incentivizing mechanism where the network operator charges the SUs for transmissions on the PC. We define a channel selection game among the SUs where utility of each SU is defined in terms of weighted difference of success probability and transmission costs and study *symmetric Nash equilibria* (SNE) of the game. Though the game involves infinite number of players, focus on SNE allows the game to be treated as a two-person game with closed form expressions for equilibrium[2]. The price can be then used to control fraction of the SUs transmitting on the PC.

The summary of our contributions and results are as follows:

- We model spectrum sharing between two MANETs, one operating on a licensed spectrum and another on an unlicensed spectrum and characterize the throughput gain achieved in the system. Our benchmark for gain is the highest throughput achieved by SUs when all of them transmit only on SC and use ALOHA protocol.
- We show that when QoS requirement of PUs is ‘relaxed’, the throughput gain can be significant and increases with the density of SUs (Theorem 1).

All the proofs are given in the technical report [11].

A. Related works

Several papers including [12]–[17] study performance of co-existing heterogeneous networks under outage constraints. We discuss papers that consider underlay networks as we do.

In [15], the authors study transmission-capacity trade-off of a network where a MANET shares uplink of a cellular network; transmission capacity is defined as the highest density of PUs and SUs that can co-exist without violating the outage constraints. It is shown that the capacity region of co-existing network is a triangle. In [17], the authors study co-existance of two MANETs and evaluate the transmission capacity of the secondary network under outage constraints. The authors in [13] study single hop transport capacity (STC) of two MANETs that co-exist, where STC involves both the transmission distance and transmission capacity. They consider different distribution on the distance between transmitter-receiver pair. In [12], the authors consider spectrum sharing between D2D devices and cellular networks. A D2D device either transmits directly to other devices or uses cellular network. The authors derive rate expression and analyze achievable rates for D2D devices.

As discussed above, the papers on spectrum sharing analyze the highest density of SUs and PUs that can co-exist without violating outage constraints. Whereas our work studies how the SUs spread their transmissions across PC and SC to improve their throughput without degrading QoS of PUs.

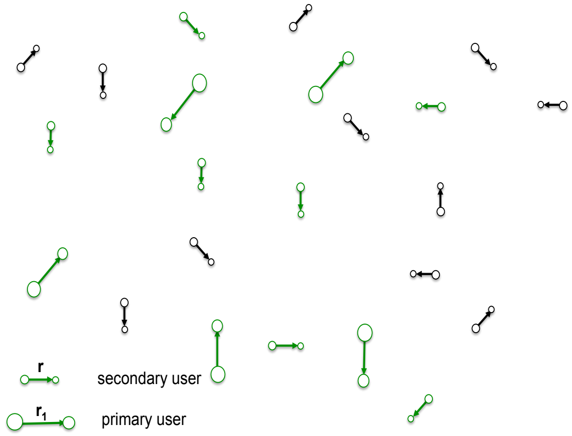

Figure 1: A snapshot of PUs and SUs. Users colored in green transmit of PC and that colored in black transmit on SC.

Paper organization: We begin with spectrum sharing problem between two MANETs in Section II and discuss the model and setup in Subsection II-A. In Section III, we consider a cooperative scenario where all the SUs follow a strategy assigned by a network operator. The non-cooperative case is considered in [11] where we study Nash equilibria of the game and a pricing scheme to achieve it. We end with concluding remarks in Section IV.

II. SPECTRUM SHARING BETWEEN MANETS

In this section we consider two mobile ad hoc networks, one operating on a licensed spectrum and the other on an unlicensed spectrum. The extension to the case where multiple licensed spectra are available for sharing can also treated similarly, but it brings in more combinatorial complexities. Using terminology of cognitive radio networks, we refer to licensed spectrum as primary channel (PC) and transmitters on it as primary users (PUs). Similarly, unlicensed spectrum is referred as secondary channel (SC) and transmitters on it as secondary users (SUs). We assume that PC is of higher bandwidth than SC and is of better quality. The SUs can also transmit on the PC provided they do not degrade PUs' quality of service (QoS). We assume that both SUs and PUs are saturated, i.e., always have a packet to transmit.

A. Model and Setup

Both PUs and SUs are spread in a common geographical area and we assume that they are distributed according to independent homogeneous Poisson point processes (P.p.p) of intensity λ_I and λ_{II} respectively.

We consider the simplified mobile ad hoc network (MANET) model called the Poisson bipolar model proposed in [6] for the SUs and PUs. Each dipole of the MANET consists of a transmitter and an associated receiver. Let $\Phi^{II} := \{X_i^{II}\}_{i\geq 1}$ denote the locations of secondary transmitters that are scattered in the Euclidean plane according to an homogeneous P.p.p of intensity λ_{II}. The set of secondary receivers $\{y_i^{II}\}_{i\geq i}$, where y_i^{II} denotes the receiver associated with transmitter X_i^{II}, are assumed to be distributed uniformly

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[2] It is well known that computing the games is in general a hard problem.

on a circle of radius r centered around its transmitter, i.e., $y_i^{II} = X_i^{II} + rR(\theta_i^{II})$, where θ_i^{II} is uniformly, independently and identically distributed on $[0 \; 2\pi]$, and $R(\theta) = (\cos(\theta), \sin(\theta))$. Let $\Phi^I := \{X_i^I\}_{i \geq 1}$ denote the location of primary transmitters and, like SUs, the receiver of each PU is located uniformly on a circle of radius r_1 centered around its transmitter[3]. Figure 1 represents co-existing MANETs.

Let $n = 0, 1, 2, \cdots$ denote index of time slots with respect to which all nodes are synchronized[4]. We associate with each SU a multi dimensional mark that carries information about decision of which channel to use (PC or SC) and the fading condition at each time slot. Following the notation of [6][Chap. 17], let the sequence $M_i^{II}(n) = \{e_i(n), F_i^{II}(n), F_i^{II-I}(n)\}_{n \geq 0}$ denote the marks associated with SU i, and $M_i^I(n) = \{F_i^I(n), F_i^{I-II}(n)\}_{n \geq 0}$ denote the marks associated with PU i, where

- $e_i = \{e_i(n)\}_{n \geq 0}$ denotes the sequence of channel access decisions of SU i. $e_i(n)$ is an indicator function that takes value 1 if node i decides to transmit on PC in slot n, otherwise it takes value zero. The random variables $e_i(n)$ are assumed to be independently and identically distributed (i.i.d.) in i and n, and independent of everything else.
- $F_i^{II}(n) = \{F_{ij}^{II}(n) : j \geq 1\}$ denotes the sequence of channel conditions between the transmitter of SU i and all the secondary receivers (including its own receiver).
- $F_i^{II-I}(n) = \{F_{ij}^{II-I}(n) : j \geq 1\}$ denotes the channel condition between the transmitter of ith SU and all primary receivers.
- $F_i^I(n) = \{F_{ij}^I(n) : j \geq 1\}$ denotes the sequence of channel conditions between the transmitter of ith PU and all the receivers (including its own receiver).
- $F_i^{I-II}(n) = \{F_{ij}^{I-II}(n), j \geq 1\}$ denotes the sequence of channel conditions between the transmitter of ith PU and all secondary receivers.
- It is assumed that channel conditions are i.i.d. across the nodes and time slots, with a generic distribution on $\mathbb{R}^+$ denoted by F with mean $1/\mu$. The marks are assumed to be independent in space and time.

The probability that the ith SU transmits in time slot n on PC is $p_i := \Pr\{e_i(n) = 1\} = \mathbb{E}[e_i(n)]$. When all the SUs use the same p_i, we drop the subscript i and write it as p. If each SU use PC with probability p and its decisions is independent of everything else, we get a pair of independent Poisson processes at each time slot n, one representing a set of SUs on PC $\Phi_1^{II}(n) = \{X_i^{II}, e_i(n) = 1\}$ and the other representing the rest of SUs on the SC $\Phi_0^{II}(n) = \{X_i^{II}, e_i(n) = 0\}$ with intensities $p\lambda_{II}$ and $(1-p)\lambda_{II}$ respectively. All the SUs transmit at a fixed power P_{II}, and the PUs at a fixed power level of P_I. For notational convenience we write $P = P_I / P_{II}$.

Let $l(x, y)$ denote the attenuation function between any two given points $x, y \in \mathbb{R}^2$. We assume that this function just depends on the distance between the points, i.e., $|x - y|$. With a slight abuse of notation we denote this function as $l(x, y) = l(|x - y|)$. We consider the following form for attenuation

$$l(x, y) = |x - y|^{-\beta} \text{ for and } \beta > 2. \tag{1}$$

B. Coverage probability of a SU

A signal transmitted by ith SU is successfully received in time slot n on SC if the SINR at its receiver is larger than some threshold T_{II}, i.e.,

$$SINR_i^{II}(n) := \frac{P_{II} F_{ii}^{II}(n) l(r)}{I_{\Phi_0^{II}(n)}(y_i^{II}) + W(n)} > T_{II}, \tag{2}$$

where $W(n)$ is the thermal noise power at the receiver and

$$I_{\Phi_0^{II}(n)}(y_i^{II}) = \sum_{X_j^{II} \in \Phi_0^{II}(n) \setminus X_i^{II}} P_{II} F_{ij}^{II}(n) l(|X_j^{II} - y_i^{II}|)$$

denotes the *shot noise* of the P.p.p. $\Phi_0^{II}(n)$ in time slot n. We assume that the noise is an i.i.d. process. A signal transmitted by the SU at X_i^{II} is successfully received in time slot n on PC if SINR at its receiver is larger than a threshold T_I, i.e.,

$$SINR_i^{II-I}(n) := \frac{P_{II} F_{ii}^{II}(n) l(r)}{I_{\Phi_1^{II}(n)}(y_i^{II}) + I_{\Phi^I(n)}(y_i^{II}) + W(n)} > T_I, \tag{3}$$

where

$$I_{\Phi_1^{II}(n)}(y_i^{II}) = \sum_{X_j^{II} \in \Phi_1^{II}(n) \setminus X_i^{II}} P_{II} F_{ij}^{II}(n) l(|X_j^{II} - y_i^{II}|), \tag{4}$$

is the shot noise from SUs on the PC and

$$I_{\Phi^I(n)}(y_i^{II}) = \sum_{X_j^I \in \Phi^I(n)} P_I F_{ji}^{I-II}(n) l(|X_j^I - y_i^{II}|)$$

denote the shot noise from PUs, at y_i^{II} in time n. Since bandwidth of PC is assumed to be much larger than that of SC we set $T_{II} > T_I$. Consider a typical SU at origin with mark $M_i^{II}(0) = (e_i(0), F_i^{II}(0), F_i^{II-I}(0))$ at $n = 0$. The typical node is said to be covered in slot $n = 0$ on SC if (2) holds given that it selects to operate on SC. Then the coverage probability of the typical node is

$$\mathbf{P}_0^{II} \left\{ SINR_i^{II}(0) > T_{II} \;\middle|\; e_i(0) = 0 \right\},$$

where $\mathbf{P}_0^{II}$ denotes the Palm distribution [19][Chap. I] of the stationary marked P.p.p Φ_0^{II}. Note that due to time-homogeneity, this conditional probability does not depend on n. The coverage probability of a typical node when all other nodes use the same channel access decision is evaluated in [20]. Continuing the notation used in [20] we denote this coverage probability (non-outage probability) as $p_c^{II} := p_c^{II}(r, (1-p)\lambda_{II}, T_{II})$. A tagged node is said to be covered in slot $n = 0$ on PC if (3) holds given that it selects to operate on PC. We denote this coverage probability as $p_c^{II-I} := p_c^{II-I}(r, \hat{\lambda}_I, T_I)$ and is given by

$$\mathbf{P}_0^{II-I} \left\{ SINR_i^{II-I}(0) > T_I \;\middle|\; e_i(0) = 1 \right\},$$

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[3] Extension to include random distance between Tx-Rx pairs is straightforward. But it provides little new insights.

[4] Analysis extends to non-synchronous using techniques in [18]

where $\mathbf{P}_0^{II-I}$ denote the Palm distribution of the stationary marked P.p.p $\tilde{\Phi}^I = \Phi_1^{II} + \Phi^I$ with density $\hat{\lambda}_I := \lambda_I + p\lambda_{II}$.

Proposition 1 (prop. 16.2.2 [6]): Let the fading process be Rayleigh distributed and each SU select PC with probability p in each slot. Then, coverage probability of a typical SU on SC and PC, respectively, is

$$\begin{aligned} p_c^{II} &= \mathcal{L}_{I_{\Phi_0^{II}}}(\mu T_{II}/P_{II}l(r))\mathcal{L}_W(\mu T_{II}/P_{II}l(r)) \\ p_c^{II-I} &= \mathcal{L}_{I_{\Phi_1^{II}}}(\mu T_I/P_{II}l(r))\mathcal{L}_{I_{\Phi^I}}(\mu T_I/P_{II}l(r)) \\ &\times \mathcal{L}_W(\mu T_I/P_{II}l(r)), \end{aligned}$$

where $\mathcal{L}_X(s) = \mathbb{E}[e^{-sX}]$ denotes the Laplace transform of random variable X evaluated at s.

Corollary 1: Let the fading process be Rayleigh distributed and $W \equiv 0$. For the path loss model in (1), we have

$$\begin{aligned} p_c^{II} &= \exp\{-(1-p)\lambda_{II}C_{II}\} \\ p_c^{II-I} &= \exp\{-p\lambda_{II}C_I\}\exp\{-\lambda_I P^{2/\beta}C_I\}, \end{aligned}$$

where $C_{II} = r^2 T_{II}^{2/\beta}K(\beta)$, $C_I = r^2 T_I^{2/\beta}K(\beta)$ and $K(\beta) = 2\pi^2/(\beta\sin(2\pi/\beta))$.

C. Coverage probability of a PU

ith PU is said to be covered if SINR at its receiver is larger than threshold T_I. We denote the coverage probability of a typical PU as as $p_c^I := p_c^I(r_1, \hat{\lambda}_I, T_I)$. Followings the steps used for typical PU, it can be evaluated as follows:

Proposition 2 (prop. 16.2.2 [6]): Let each SU transmit on PC with probability p. For Rayleigh fading, the success probability of a typical PU is given as

$$\begin{aligned} p_c^I := \mathcal{L}_{I_{\Phi_1^{II}}}(\mu T_I/P_I l(R))\mathcal{L}_{I_{\Phi^I}}(\mu T_I/P_I l(r_1)) \\ \times\mathcal{L}_W(\mu T_I/P_I l(r_1)). \end{aligned}$$

Corollary 2: Let the fading process be Rayleigh distributed and $W \equiv 0$. For the path loss model in (1), the coverage probability of a PU is

$$p_c^I = \exp\{-p\lambda_{II}P^{-2/\beta}\bar{C}_I\}\exp\{-\lambda_I\bar{C}_I\},$$

where $\bar{C}_I = r_1^2 T_I^{2/\beta}K(\beta)$.

D. Density of Successful transmission of secondary users

Let $d_s^{II}(p)$ denote the spatial density of successful transmissions of the SUs on SC. Since the SUs form a P.p.p of intensity $(1-p)\lambda_{II}$ in each slot on SC, we get $d_s^{II}(p) = (1-p)\lambda_{II}p_c^{II}(r, (1-p)\lambda_{II}, T_{II})$. Similarly, let $d_s^I(p)$ denote the spatial density of the successful transmissions of the SUs on PC. Since the SUs form a P.p.p of intensity $p\lambda_{II}$ in each time slot on PC, we get $d_s^I(p) = p\lambda_{II}p_c^{II-I}(r, \hat{\lambda}, T_I)$. Then, if each SU decides to transmit on PC with probability p independent of others, the total density of successful transmissions of the SUs, denoted as $d_s(p)$, is

$$d_s(p) = d_s^{II}(p) + d_s^I(p).$$

In the following we first consider the case where all the SUs co-operate and aim to maximize their spatial density of success without degrading the quality of service to the PUs. We then consider that the SUs selfishly select the channels and propose a mechanism to improve equilibrium performance.

III. CO-OPERATIVE CASE

In this section we assume that all the SUs belong to a network operator and access PC with a probability assigned by the operator. A fraction of SUs are allowed to transmit on the PC provided their transmissions do not degrade the quality of service guarantee (QoS) for the PUs on PC.

A. Quality of Service guarantee for the PUs

A natural way to guarantee a certain QoS to the PUs is to limit the amount of interference by the SUs on PC, or, alternatively, to maintain a minimum coverage probability of the PUs in presence of SUs. Specifically, we consider that each PU is covered with probability atleast $1-\delta$, where δ determines the predefined QoS.

The objective of the operator is to maximize the density of successful transmissions of the SUs without degrading the QoS requirement of the PUs, i.e.,

$$\begin{aligned} &\text{maximize}_{p\in[0\ 1]} && d_s(p) \\ &\text{subject to} && p_c^I \geq 1-\delta \end{aligned} \tag{5}$$

Performance gain: In the absence of any additional channel, the operator can improve the success density of SUs on SC using contention resolution protocols like ALOHA, CSMA. In ALOHA/CSMA, only a fraction of the SUs transmit in each time slot, while the others remain silent to reduce interference. If an additional channel is available, some of the SUs can transmit on it (provided the QoS constraints are met) and increase the number of SU's concurrent transmissions without increasing intra channel interference on SC We measure this gain in success density of the SUs by comparing maximum success density achievable using both PC and SC to that achievable using SC alone. Specifically, we define SUs success density gain, denoted $G := G(\lambda_{II}, \lambda_I, T_{II}, T_I)$, as:

$$G = \frac{d_s(p_o^*)}{\max_{p\in[0\ 1]} d_s^{II}(p)}, \tag{6}$$

where p_o^* denote global maximum of optimization in (5).

B. Performance Evaluation

For analytic tractability, we focus on the case of Rayleigh fading and neglect channel noise[5] ($W \equiv 0$). From Corollaries (1) and (2), the optimization problem in (5) for the Rayleigh fading is expressed as:

$$\begin{aligned} \text{maximize}_{p\in[0\ 1]} \quad & (1-p)\lambda_{II}\exp\{-(1-p)\lambda_{II}C_{II}\}+ \\ & p\lambda_{II}\exp\{-p\lambda_{II}C_I - \lambda_I P^{\frac{2}{\beta}}C_I\} \qquad (7) \\ \text{subjected to} \quad & \exp\{-p\lambda_{II}P^{\frac{-2}{\beta}}\bar{C}_I - \lambda_I\bar{C}_I\} \geq 1-\delta. \end{aligned}$$

The following simple observation tells when is to possible for both the SUs and PUs to share the PC.

Lemma 1: Given $\delta > 0$, SUs can transmit on the PC iff

$$\lambda_I \leq -\log(1-\delta)/\bar{C}_I.$$

Scaling properties: For notational convenience write $\bar{\lambda}_I := -\log(1-\delta)/\bar{C}_I$ and $\bar{p} := -(\log(1-\delta)+\lambda_I\bar{C}_I)/\lambda_{II}P^{\frac{-2}{\beta}}\bar{C}_I$.

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[5] For the case with non-negligible noise the same analysis holds upto a constant scaling factor that depends on Laplacian of noise random variable.

If $\lambda_I \le \bar{\lambda}_I$, then some SUs are permitted to use PC. On the other hand if $\lambda_I > \bar{\lambda}_I$, SUs are prohibited to use PC. However, notice that $\bar{\lambda}_I$ is inversely proportional to T_I (through $\bar{C}$), and in the later case, SUs can share PC with PUs without degrading their QoS provided T_I decreases. Specifically, if T_I decreases by a factor $a \in (0\ 1]$, then the density of both PUs and SUs can increase by a factor $a^{-2/\beta}$ on PC without affecting the QoS for the SUs. To see this, note that $\bar{p}$ depends on the product $\lambda_I T_I^{2/\beta}$ (through $\lambda_I \bar{C}$) and its value does not change if T_I decreases by a factor $a \in [0\ 1]$ and λ_I increases by a factor $a^{-2/\beta}$. Also, the scaling of T_I increases $\bar{p}$ by a factor of $a^{-2/\beta}$ due to $\bar{C}$ in the denominator.

Let us first consider unconstrained optimization problem ignoring the constraint in (7). It is easy to note that both $d_s^{II}(p)$ and $d_s^{I}(p)$ are quasi-concave in p and each attain a unique maxima. But, their sum need not be quasi-concave and maxima of $d_s(p)$ may not be unique. Some of its properties are listed below.

Proposition 3: Let p^* denote a maximum of $d_s(p)$. If $\lambda_{II} C_I \le 1$, then p^* is unique. Further,

- If $\lambda_{II} C_{II} \le 1$, then $d_s(p)$ is concave in p
- If $\lambda_{II} C_{II} > 1$, then $d_s(p)$ is monotonically increasing for all $p \le 1 - 1/\lambda_{II} C_{II}$ and $p^* \ge 1 - 1/\lambda_{II} C_{II}$.

Proposition 4: Let p^* denote a maximum of $d_s(p)$. If $\lambda_{II} C_I > 1$, then p^* is not unique. Further, any optimum p^* is such that $1/\lambda_{II} C_I \le p^* \le 1 - 1/\lambda_{II} C_{II}$.

Recall that $C_{II} > C_I$ implies $T_{II} > T_I$, and smaller the values of T_I and T_{II}, better the channel quality (with higher success rate). For a given λ_{II}, if $\lambda_{II} C_{II} \le 1$, d_s^{II} is maximized at $p = 0$, i.e., there is no need for SUs to transmit on PC, hence $G = 1$. If $\lambda_{II} C_{II} > 1$, $d_s^{II}(p)$ is maximized at $p = 1 - 1/\lambda_{II} C_{II}$ and the operator benefits by allowing a fraction $1 - 1/\lambda_{II} C_{II}$ of the SUs to transmit on PC. Further, if $\lambda_{II} C_I \le 1$, PC quality is better than that of SC, hence the operator benefits more by allowing more than $1 - 1/\lambda_{II} C_{II}$ fraction of the SUs to transmit on PC as noted in the second part of Proposition (3). On the other hand, if $\lambda_{II} C_I \ge 1$, PC quality is not significantly better than that of SC, and the operator may gain only by allowing a smaller than $1 - 1/\lambda_{II} C_{II}$ fraction of the SUs to transmit on PC (Proposition 4). Thus we focus on scenario where $\lambda_{II} C_{II} > 1$ and $\lambda_{II} C_I \le 1$, i.e., the quality of PC is better compared to that of SC where the operator prefers to place more SUs on PC but is constrained by the QoS requirements for the PUs.

Now we return to the constrained optimization in (7). Note the objective function is concave in the regime of interest and constraint is a convex set. Hence solution of (7) is unique. Its properties are listed below.

Proposition 5: Let $\lambda_{II} C_{II} > 1$ and $\lambda_{II} C_I \le 1$. For a given $\delta > 0$ and $\bar{p}$ the global optimum d_o^* of (5) satisfies

- If $\bar{p} \le 1 - 1/\lambda_{II} C_{II}$, then $p_o^* = \bar{p}$
- If $\bar{p} > 1 - 1/\lambda_{II} C_{II}$, then $p_o^* \ge 1 - 1/\lambda_{II} C_{II}$.

As noted earlier, the operator benefits if at least $1 - 1/\lambda_{II} C_{II}$ fraction of the SUs operate on PC. However, if doing so violates QoS guarantee for PUs, then the operator can place at most $\bar{p}$ fraction of SUs on PC. The following Theorem characterizes the gain in different regimes.

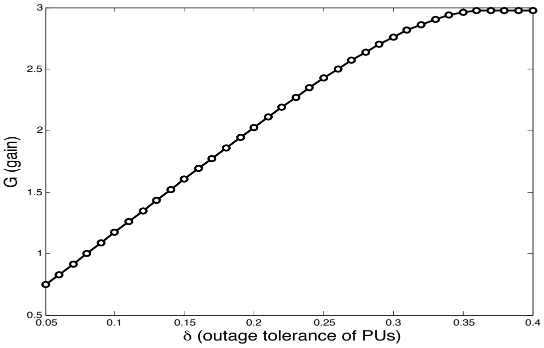

Figure 2: Gain vs. outage . We set $P_{II} = P_I, \lambda_{II} C_{II} = 2, \lambda_{II} C_I = 0.2, \lambda_{II} \bar{C}_I = .5, \lambda_I \bar{C}_I = 0.05$.

Theorem 1: Let $\lambda_{II} C_{II} > 1$ and $\lambda_{II} C_I \le 1$. Write $a = \exp\{-\lambda_I P^{2/\beta} C_I\}$, we have

- If $\bar{p} \ge 1 - 1/\lambda_{II} C_{II}$, then

$$\begin{aligned} &1 + ea\lambda_{II} C_{II} \exp\{-\lambda_{II} C_I\} \ge G \ge \\ &1 + ea(\lambda_{II} C_{II} - 1) \exp\{-(\lambda_{II} C_{II} - 1) C_I / C_{II}\} \end{aligned} \tag{8}$$

- If $\bar{p} < 1 - 1/\lambda_{II} C_{II}$, then

$$G \le 1 + ea(\lambda_{II} C_{II} - 1) \exp\{-(\lambda_{II} C_{II} - 1) C_I / C_{II}\}$$

Both the upper and lower bounds in (8) are increasing in the ratio T_{II}/T_I (through C_{II}/C_I) for a fixed $\lambda_{II} C_{II}$. Thus, the gain G is higher if PC quality improves compared to that of SC quality. We note that for the case $\bar{p} \ge 1 - 1/\lambda_{II} C_{II}$ the lower bound is strictly larger than 1, thus the operator always gains using PC. However, if QoS requirement for the SUs is 'stringent' such that $\bar{p} < 1 - 1/\lambda_{II} C_{II}$, then it may be possible that $G < 1$ and the operator may not gain by using PC. In Figure 2 we plot G as a function of the outage tolerance δ for PUs. As seen, for small values of δ, G is smaller than 1. In this regime the operator should avoid using PC and aim to increase the success density on SC alone using contention resolution protocols.

IV. CONCLUSION

The LTE in unlicensed spectrum (LTE-U) is a proposal where cellular networks would like to use unlicensed spectrum in addition to their licensed spectrum to boost coverage in their cellular networks. In this work we considered the opposite scenario where the users of a MANET (secondary) operating on an unlicensed also transmit on a licensed spectrum to improve their throughput. We characterized the throughput gain of secondary network and showed that it can be significant for large-scale networks.

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REFERENCES

[1] A. Asadi, Q. Wang, and V. Mancuso, “A survey on device-to-device communication in cellular networks,” *IEEE Communication Surveys & Tutorial*, vol. 16, no. 4, 2014.

[2] L. Atzoria, A. Ierab, and G. Morabitoc, “The internet of things: A survey,” *Elsevier journal on Computer Networks*, vol. 54, no. 15, 2010.

[3] F. C. Commission, “Spectrum policy task force,” *Rep. ET Docket no. 02-135*, Nov 2002.

[4] D. Kim and D. G. Jeong, “Capacity unbalance between uplink and downlink in spectrally overlaid narrow-band and wide-band cdma mobile systems,” *IEEE Transactions on Vehicular Technology*, vol. 49, no. 4, July 2000.

[5] X. Wu, S. Tavilda, S. Shakkottai, T. Richardson, J. Li, R. Laroia, and A. Jovicic, “Flashlinq: A synchornous distributed scheduler for peer-to-peer ad hoc networks,” in *48th Allerton conference*, University of Illinois, Urbana Champaign, USA, Sep 2010.

[6] F. Baccelli and B. Blaszczyszyn, *Stochastic Geometry and Wireless Networks Volume 2: APPLICATIONS.* Foundations and Trends in Networking, 2009, vol. 4, no. 1-2.

[7] T. V. Nguyen and F. Baccelli, “A stochastic geometry model for cognitive radio networks,” *The Computer Journal*, vol. 55, no. 5, 2012.

[8] C. Lee and M. Haenggi, “Interference and outage in poisson cognitive networks,” *IEEE Transactions on Wireless Communications*, vol. 11, no. 4, 2012.

[9] J. G. Andrews, A. K. Gupta, and H. Dhillon, “A primer on cellular network analysis using stochastic geometry,” *Arxiv 2016, https://arxiv.org/abs/1604.03183*, 2016.

[10] J. G. Andrews, F. Baccelli, and R. K. Ganti, “A new tractable model for cellular coverage,” in *48th Allerton conference*, University of Illinois, Urbana Champaign, USA, Sep 2010.

[11] “Throughput maximization of secondary networks over licensed and unlicensed spectra,” Tech. Rep., 2016. [Online]. Available: https://www.dropbox.com/s/jsp4f8ok3btt21e/WiOpt17.pdf?dl=0

[12] X. Lin, J. G. Andrews, and A. Ghosh, “Spectrum sharing for device-to-device communication in cellular networks,” *IEEE Transactions of Wireless Communication*, vol. 13, no. 12, 2014.

[13] C. Li and H. Dai, “Transport throughput of secondary networks in spectrum sharing systems,” in *Proceedings of IEEE INFOCOM*, Shanghai, China, April 2011.

[14] P. Pinto, A. Gioregetti, M. Z. Win, and M. Chinai, “A stochastic geometry approach to coexistence in heterogeneous wireless networks,” *IEEE Journal on selected areas in communications*, vol. 27, no. 7, 2009.

[15] K. Huang, V. K. N. Lau, and Y. Chen, “Spectrum sharing between cellular and mobile ad hoc networks: Transmission-capacity trade-off,” *IEEE Journal on selected areas in communications*, vol. 27, no. 7, 2009.

[16] R. W. Heath, M. Kountouris, and T. Bai, “Modeling heterogeneous network interference using poisson point processes,” *IEEE Transactions on Signal Processing*, vol. 61, no. 16, 2013.

[17] J. Lee, S. Kim, J. Andrews, and D. Hong, “Achievable transmission capacity of secondary system in cognitive radio networks,” in *Proceedings of IEEE ICC*, Cape Town, South Africa, May 2010.

[18] B. Blaszczyszyn and P. Múhlethaler, “Stochastic analysis of non-slotted aloha in wireless ad-hoc networks,” in *Proceedings of IEEE INFOCOM*, San Diego CA, USA, Mar 2010.

[19] F. Baccelli and B. Blaszczyszyn, *Stochastic Geometry and Wireless Networks Volume 1: THEORY.* Foundations and Trends in Networking, 2009, vol. 3, no. 3-4.

[20] F. Baccelli, B. Blaszczyszyn, and P. Múhlethaler, “Stochastic analysis of spatial and opportunistic aloha,” *IEEE Journal of Selected Areas in Communication*, vol. 27, 2009.