Sensitivity of Stable Rates in Cognitive Radio Systems to the Sensing Errors

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***Abstract*—We study the sensitivity of stable rates to imperfect sensing in cognitive radio systems comprised of a set of source-destination pairs having different priorities. The adopted cognitive access protocol allows the secondary user not only to exploit the idle slots of the primary user but also to transmit along with the primary user with some probability. This is aimed at achieving full utilization of the shared channel with capture. The abolition of strong primacy, however, requires the secondary user to properly regulate its multi-access probability in order not to impede the primary user's stability guarantee at any stabilizable input demand. To this end, the stability region of the system is characterized which describes the theoretical limit on rates that can be pushed into the system while maintaining the queues stable. Interestingly, we found that even with non-zero sensing error rates, there exists a condition for which we can achieve identical stability region that is achieved with perfect sensing, i.e., the stability is insensitive to the sensing errors. This happens when relatively strong capture effect is present. For the case when the stability is sensitive to the sensing errors, we precisely quantify the loss due to the imperfect sensing in terms of the size of the stability region.**

I. Introduction

Studies on the spectrum usage have revealed that substantial portion of the licensed spectrum is underutilized, which demands for a new technological breakthrough to improve the current spectrum utilization [1]. The cognitive radio communications, a means of opening up licensed bands to unlicensed users, have the potential to become a solution to the spectrum underutilization problem [2]. The high-priority user, often called as the primary, is allowed to access the spectrum whenever it needs, while the low-priority user, called as the secondary, is required to make a decision on its transmission based on what the primary user does.

We start with some background study. In [3], an opportunistic scheduling policy for cognitive access systems was developed. It is based on the collision channel model, in which if more than one user transmits at the same time, none of them are successful. This is too pessimistic in the sense that a transmission may succeed even in the presence of interference, which is called *capture effect* [4]. Furthermore, the activity of the primary user was modeled as a random process which evolves independently of the secondary users. In other words, even if the primary user's packet is lost due to the collision caused by a secondary user, the primary user does not attempt to retransmit the lost packet. Unless the primary user is servicing a certain loss-tolerant application, the lost packets must be retransmitted through a medium access control (MAC) protocol and those retransmissions would certainly affect the primary user's activity. In [5], the unrealistic assumption made in [3] was corrected for a reduced system model consisting of a single primary and secondary user, but the results are derived based on the assumption that the primary user is always stable. In the absence of the knowledge on the network stability region[1], however, it is infeasible to judge the stability of the primary user's queue a priori, and the characterization of the stability region is usually not an easy problem especially when the network nodes are *interacting*, i.e., the service process

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[1] Stability region is defined as the set of arrival rate vectors for which the queues in the system are stable, and a queue is said to be stable if it reaches a steady state and does not drift to infinity. A formal definition is given in Section II.

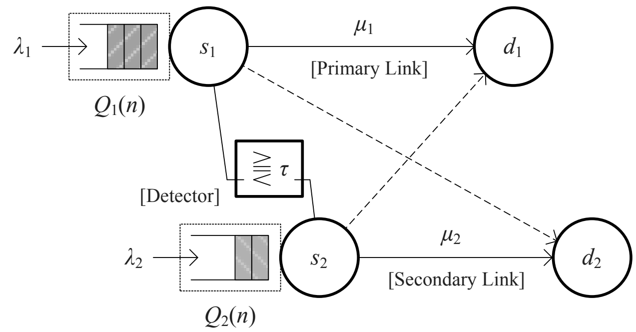

Fig. 1. The cognitive access system model with sensing at the secondary source

of one depends on the status of the others. In [6], such interaction between users was fully taken into account for a similar network model with that considered in [5], and the stability region of the system was obtained using the *stochastic dominance technique*, which was originally introduced in [7].

We notice, however, that all the above-mentioned studies, [3], [5], [6], are based on the ideal assumption that the secondary user always knows the exact activity of the primary user without an error. In reality, however, such knowledge is acquired by sensing the primary user's signal at the secondary user, and it is imperfect because the occurrence of errors is inevitable as long as there exists randomness in the observed signal.

In this paper, we focus attention on the impact of imperfect sensing on which the overall performance of the cognitive radio system depends. The opportunistic cognitive access protocol proposed in [6] is considered again for the system consisting of a single primary and secondary source-destination pair as shown in Fig. 1. Specifically, the primary user transmits uninterruptedly whenever its queue is non-empty. On the other hand, the secondary user first observes the activity of the primary user and, if it is sensed to be idle, it transmits with probability 1 if its queue is non-empty. Otherwise, if the primary user is sensed to be active, the secondary user transmits with some probability p to take advantage of capture although, at the same time, it risks the primary user's success. Our design objective is, therefore, to optimally choose the multi-access probability p of the secondary user so as to maximize its own stable throughput while ensuring the stability of the primary user at given input rate demand and sensing error rates.

Our contributions in this work can be summarized as follows. When compared to the previous work that over-simplified the primary user's activity, the primary user's activity in our work is precisely modeled through the queueing dynamics. Furthermore, the imperfect spectrum sensing, one of the most practical aspects of cognitive access systems, is also incorporated in the model. After that, the impact of imperfect sensing on the stability of the system is precisely analyzed. The remarkable result is that there exists a condition for which we can achieve identical stability region that is achieved with perfect sensing, which fundamentally eliminates the need for the spectrum sensing itself. This is the case when relatively strong capture effect exists. For the case when the condition does not hold, we quantify the loss due to the imperfect sensing in terms of the size of the stability region.

The rest of the paper is organized as follows. In Section II, we present the system model and revisit the notion of stability. In Section III, we describe our main result on the stability region of the cognitive radio system in the presence of spectrum sensing errors. The proof of our main result is presented in Section IV. Finally, we draw some conclusions in Section V.

II. System Model

We consider a system consisting of two source-destination pairs, the primary pair (s_1, d_1) and the secondary pair (s_2, d_2), as shown in Fig. 1. Each source s_i, $i \in \{1, 2\}$, has an infinite size queue for storing the arriving packets of fixed length. Time is slotted and the slot duration is equal to a packet transmission time. The primary user's transmission consists of the preamble symbols followed by the encoded data symbols of a packet, if the primary user transmits during the corresponding time slot. It is assumed that the secondary user knows the exact timing of the primary user's frame and performs sensing on the existence of the primary user's signal during the preamble symbol duration. Once the secondary user decides to transmit, it transmits over the primary user's data symbol duration in a synchronous manner. It is assumed that the acknowledgments (ACKs) on the success of transmissions are sent back from the destinations to the corresponding sources instantaneously and error-free.

Let $Q_i(n)$ denote the number of packets buffered at s_i at the beginning of the n-th slot which evolves according to

$$Q_i(n+1) = \max[Q_i(n) - \mu_i(n), 0] + A_i(n)$$

where the stochastic processes $\{\mu_i(n)\}_{n=0}^{\infty}$ and $\{A_i(n)\}_{n=0}^{\infty}$ are sequences of binary random variables representing the number of services and arrivals at s_i during time slot n, respectively. The arrival process $\{A_i(n)\}_{n=0}^{\infty}$ is modeled as an independent and identically distributed (i.i.d.) Bernoulli process with $E[A_i(n)] = \lambda_i$, and the processes at different nodes are assumed to be independent of each other. The service process $\{\mu_i(n)\}_{n=0}^{\infty}$ depends jointly on the transmission protocol, sensing errors, and the underlying channel model, which governs the success of transmissions. In the considered cognitive access protocol, s_2 adapts its transmission based on the observation made on the activity of s_1. Note that s_1 can be falsely sensed to be active by s_2 when indeed it is idle or falsely sensed to be idle when it is active. These are called *false alarm* and *miss*, and their rates are denoted by ϵ_f and ϵ_m, respectively. Also, denote by $\bar{\epsilon}_f = 1 - \epsilon_f$ and $\bar{\epsilon}_m = 1 - \epsilon_m$, which are probabilities of *correct rejection* and *hit*, respectively. These terminologies were borrowed from [8]. Please refer [9]–[11], and references therein for more details on various spectrum sensing techniques and their performance.

The channel model used in this work is a generalized form of the packet-erasure model, which reflects the effect of fading, attenuation, and interference at the physical layer [12], [13]. Denote with $q_{i|\mathcal{M}}$ the success probability of user s_i when a set $\mathcal{M}$ of users are transmitting simultaneously. It is related to the physical layer parameters through

$$q_{i|\mathcal{M}} = \Pr[\gamma_{i|\mathcal{M}} \geq \theta] \tag{1}$$

where $\gamma_{i|\mathcal{M}}$ denotes the signal-to-interference-plus-noise-ratio (SINR) of the signal transmitted from s_i at the designated receiver d_i given set $\mathcal{M}$ of simultaneous transmitters, and θ is the threshold for the successful decoding of the received signal, which depends on the modulation scheme, target bit-error-rate, and the number of bits in the packet, i.e., the transmission rate. Of course, Eq. (1) is an approximation since it does treat interference as white Gaussian noise, however, it is used widely and represents a compromise between accuracy and cross-layer modeling [14].

We adopted the notion of stability used in [15] where the stability of a queue is equivalent to the existence of a proper limiting distribution. That is, a queue is said to be *stable* if

$$\lim_{n\to\infty} \Pr[Q_i(n) < x] = F(x) \quad \text{and} \quad \lim_{x\to\infty} F(x) = 1.$$

If a weaker condition holds, namely,

$$\lim_{x\to\infty} \liminf_{n\to\infty} \Pr[Q_i(n) < x] = 1$$

the queue is said to be *substable* or bounded in probability. Otherwise, the queue is *unstable*. If $Q_i(n)$ is an aperiodic and irreducible Markov chain defined on a countable space, which is the case considered in this paper, substability is equivalent to the stability and both can be understood as the recurrence of the chain. Both the positive and null recurrence imply stability because a limiting distribution exists for both cases although the latter may be degenerate. Loynes' theorem, as it relates to stability, plays a central role in our approach [16]. It states that if the arrival and service processes of a queue are strictly jointly stationary and the average arrival rate is less than the average service rate, the queue is stable. If the average arrival rate is greater than the average service rate, the queue is unstable and the value of $Q_i(n)$ approaches infinity almost surely. If they are equal, the queue can be either stable or substable but in our case the distinction is irrelevant, as mentioned earlier. Finally, the stability region of the system is defined as the pair of arrival rates (λ_1, λ_2) for which the queues at both s_1 and s_2 are stable by considering all feasible multi-access probability p.

III. Stability in the Presence of Sensing Errors

Define $\Delta_i = q_{i|\{i\}} - q_{i|\{1,2\}}$, $i \in \{1, 2\}$, which is the difference between the success probabilities when s_i transmits alone and when it transmits along with s_j ($j \neq i$). The quantity Δ_i is strictly positive since interference only reduces the probability of success. Let us further define

$$\eta \triangleq q_{1|\{1\}}q_{2|\{1,2\}} + q_{2|\{2\}}q_{1|\{1,2\}} - q_{1|\{1\}}q_{2|\{2\}}$$

which can be viewed as an indicator of the degree of the capture effect. In the case of the collision channel, for instance, it is given by $q_{i|\{i\}} = 1$ and $q_{i|\{1,2\}} = 0$, $\forall i \in \{1, 2\}$ and, thus, $\eta = -1$. On the contrary, in the case of the perfect orthogonal channel with $q_{i|\{i\}} = q_{i|\{1,2\}} = 1$, $\forall i \in \{1, 2\}$, we have $\eta = 1$.

Described below is our main finding, which is a sufficient and necessary condition for the stability of the considered cognitive access system.

Case A: If $\eta \geq 0$, the stability region of the system is given by the union of the following subregions:

$$\mathcal{R}_1^A = \left\{(\lambda_1, \lambda_2) : \lambda_2 \leq f_1(\lambda_1), 0 \leq \lambda_1 \leq I_1^A\right\}$$
$$\mathcal{R}_2^A = \left\{(\lambda_1, \lambda_2) : \lambda_2 \leq f_2(\lambda_1), I_1^A < \lambda_1 \leq q_{1|\{1\}}\right\}$$

where

$$f_1(\lambda_1) = q_{2|\{2\}} - \frac{\Delta_2}{q_{1|\{1,2\}}}\lambda_1,$$

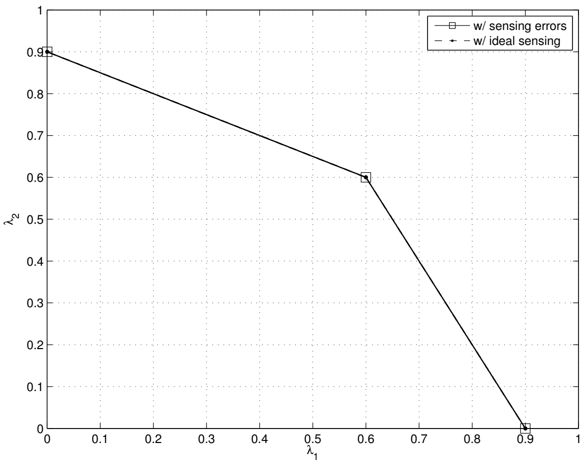

Fig. 2. Illustration of the stability region for Case A (parameter setting: $q_{1|\{1\}} = q_{2|\{2\}} = 0.9$, $q_{1|\{1,2\}} = q_{2|\{1,2\}} = 0.6$ with any positive values of ϵ_m and ϵ_f)

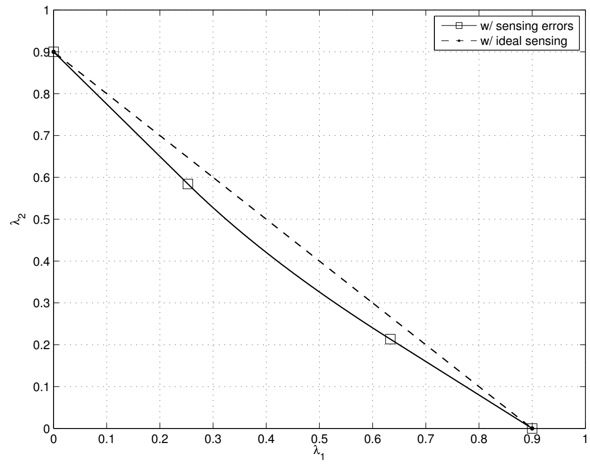

Fig. 3. Illustration of the stability region for Case B (parameter setting: $q_{1|\{1\}} = q_{2|\{2\}} = 0.9$, $q_{1|\{1,2\}} = q_{2|\{1,2\}} = 0.4$, $\epsilon_m = \epsilon_f = 0.3$)

$$f_2(\lambda_1) = \frac{q_{2|\{1,2\}}}{\Delta_1}(q_{1|\{1\}} - \lambda_1),$$

and $I_1^A = q_{1|\{1,2\}}$. The region is depicted in Fig. 2, which is a convex polygon. The entire boundary of the region can be achieved with multi-access probability $p^* = 1$. Note that the stability region does not depend on sensing error rates.

Case B: If $-q_{2|\{2\}}\epsilon_f\Delta_1 \leq \eta < 0$, the stability region is given by the union of the following subregions:

$$\mathcal{R}_1^B = \left\{(\lambda_1, \lambda_2) : \lambda_2 \leq f_1(\lambda_1), 0 \leq \lambda_1 \leq I_1^B\right\}$$
$$\mathcal{R}_2^B = \left\{(\lambda_1, \lambda_2) : \lambda_2 \leq f_3(\lambda_1), I_1^B < \lambda_1 \leq I_2^B\right\}$$
$$\mathcal{R}_3^B = \left\{(\lambda_1, \lambda_2) : \lambda_2 \leq f_2(\lambda_1), I_2^B < \lambda_1 \leq q_{1|\{1\}}\right\}$$

where

$$f_3(\lambda_1) = \frac{(\sqrt{-\eta'} - \sqrt{q_{2|\{2\}}\epsilon_f\lambda_1})^2}{\bar{\epsilon}_m\Delta_1} + \frac{q_{2|\{1,2\}}(q_{1|\{1\}} - \lambda_1)}{\Delta_1},$$

$\eta' = \bar{\epsilon}_m\eta - q_{1|\{1,2\}}q_{2|\{2\}}\epsilon_f$, $I_1^B = \frac{q_{1|\{1,2\}}^2 q_{2|\{2\}}\epsilon_f}{-\eta'}$, and $I_2^B = \frac{-\eta'}{q_{2|\{2\}}\epsilon_f}$. Note that $\eta < 0$ implies $\eta' < 0$ but the converse is not true. The boundary of the subregion $\mathcal{R}_1^B$ is achieved with $p^* = 1$, that of $\mathcal{R}_2^B$ is achieved with

$$p^* = \frac{q_{1|\{1\}} - \epsilon_m\Delta_1 - \sqrt{\frac{-\eta'\lambda_1}{q_{2|\{2\}}\epsilon_f}}}{\bar{\epsilon}_m\Delta_1} \tag{2}$$

and that of $\mathcal{R}_3^B$ is achieved with p^* given by Eq. (2) evaluated at $\lambda_1 = I_2^B$. The region is non-convex as illustrated in Fig. 3.

Case C: If $\eta < -q_{2|\{2\}}\epsilon_f\Delta_1$, the stability region is given by the union of the following subregions:

$$\mathcal{R}_1^C = \left\{(\lambda_1, \lambda_2) : \lambda_2 \leq f_1(\lambda_1), 0 \leq \lambda_1 \leq I_1^C\right\}$$
$$\mathcal{R}_2^C = \left\{(\lambda_1, \lambda_2) : \lambda_2 \leq f_3(\lambda_1), I_1^C < \lambda_1 \leq I_2^C\right\}$$
$$\mathcal{R}_3^C = \left\{(\lambda_1, \lambda_2) : \lambda_2 \leq f_4(\lambda_1), I_2^C < \lambda_1 \leq I_3^C\right\}$$
$$\mathcal{R}_4^C = \left\{(\lambda_1, \lambda_2) : \lambda_2 \leq f_2(\lambda_1), I_3^C < \lambda_1 \leq q_{1|\{1\}}\right\}$$

where

$$f_4(\lambda_1) = q_{2|\{2\}}\bar{\epsilon}_f - \frac{q_{2|\{2\}}\bar{\epsilon}_f - q_{2|\{1,2\}}\epsilon_m}{q_{1|\{1\}} - \epsilon_m\Delta_1}\lambda_1,$$

$I_1^C = \frac{q_{1|\{1,2\}}^2 q_{2|\{2\}}\epsilon_f}{-\eta'}$, $I_2^C = \frac{q_{2|\{2\}}\epsilon_f(q_{1|\{1\}} - \epsilon_m\Delta_1)^2}{-\eta}$, and $I_3^C = q_{1|\{1\}} - \epsilon_m\Delta_1$. As in Case B, the boundary of subregion $\mathcal{R}_1^C$ is achieved with $p^* = 1$, and that of $\mathcal{R}_2^C$ is achieved with p^* given in Eq. (2) which diminishes from one to zero as λ_1 increases from I_1^C to I_2^C. For the boundary of subregions $\mathcal{R}_3^C$ and $\mathcal{R}_4^C$, it is given by $p^* = 0$. The entire region, which is depicted in Fig. 4, is non-convex as in Case B.

Remark 3.1: Consider the case with perfect sensing. By substituting $\epsilon_f = \epsilon_m = 0$ into the descriptions of the stability region given above, we find the stability region for the case with perfect sensing, which reconfirms the previous result obtained in [6]. For comparison's sake, it is also depicted in Fig. 2 to 4 along with the case of imperfect sensing. Most importantly, it is observed from Fig. 2 that the stability region is not affected by the sensing errors when $\eta \geq 0$. This is because the boundary achieving multi-access probability is $p^* = 1$ regardless

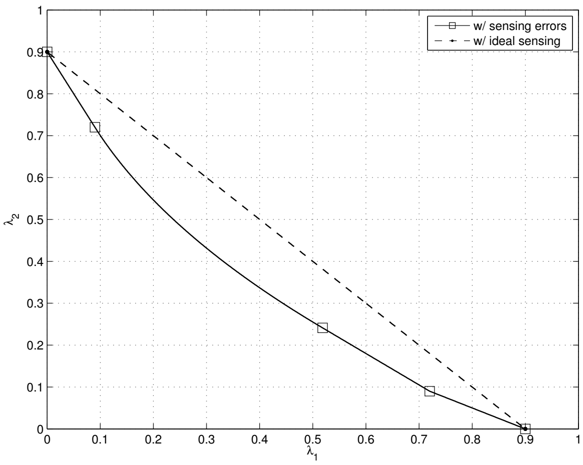

Fig. 4. Illustration of the stability region for Case C (parameter setting: $q_{1|\{1\}} = q_{2|\{2\}} = 0.9$, $q_{1|\{1,2\}} = q_{2|\{1,2\}} = 0.3$, $\epsilon_m = \epsilon_f = 0.3$)

of the values of sensing error rates. In other words, when relatively strong capture effect is present, which is indicated by η, it is beneficial to let the secondary node access the channel persistently and aggressively regardless of the sensing outcome whenever it has non-empty queue. This fundamentally eliminates the need for sensing itself.

IV. Analysis Using the Stochastic Dominance Technique

In this section, we provide details on the derivation of our main results presented in the previous section. In the considered protocol, primary user s_1 transmits independently of the actions made by the secondary user s_2. Secondary user s_2, on the other hand, makes use of the ability to sense before transmitting. The probability that s_1 is sensed to be idle is $\bar{\epsilon}_f$ when s_1 is indeed idle and ϵ_m when s_1 is actually active. Similarly, the probability that s_1 is sensed to be active is $\bar{\epsilon}_m$ when it is indeed active and ϵ_f when it is actually idle. Taking these into account, the average service rates of the users can be written as

$$\begin{aligned}\mu_1 = q_{1|\{1\}} \left(\Pr[Q_2 = 0] + \Pr[Q_2 \neq 0]\bar{\epsilon}_m(1-p)\right) \\ + q_{1|\{1,2\}}\Pr[Q_2 \neq 0]\left(\epsilon_m + \bar{\epsilon}_m p\right) \quad (3)\end{aligned}$$

and

$$\begin{aligned}\mu_2 = q_{2|\{2\}}\Pr[Q_1 = 0](\bar{\epsilon}_f + \epsilon_f p) \\ + q_{2|\{1,2\}}\Pr[Q_1 \neq 0]\left(\epsilon_m + \bar{\epsilon}_m p\right) \quad (4)\end{aligned}$$

where Q_i denotes the steady-state number of packets in the queue at s_i.

Note that the rates of the individual departure processes cannot be computed directly, as they are interdependent, without knowing the stationary probability of the joint queue length process. We bypass this difficulty by using the stochastic dominance technique introduced in [7] and also exploited in [12], [17]–[19]. The essence of the stochastic dominance technique is to decouple the interaction between queues via the construction of a hypothetical system; this hypothetical system operates as follows: i) the packet arrivals at each node occur at *exactly* the same instants as in the original system, ii) the *coin toss* that determines the multi-access by the secondary node has *exactly* the same outcome in both systems, iii) however, one of the nodes in the system continues to transmit dummy packets even when its packet queue is empty. It is obvious that sample-pathwise the queue sizes in the dominant system will never be smaller than their counterparts in the original system, provided the queues start with identical initial conditions. Thus, the stability condition obtained for the dominant system is a sufficient condition for the stability of the original system. It turns out, however, that it is indeed sufficient and necessary, which will be discussed in detail later in this section.

A. First Dominant System: Secondary User Transmits Dummy Packets

Construct a hypothetical system which is identical to the original system except that the secondary user s_2 transmits dummy packets when it decides to transmit but its queue is empty. Thus, s_2 transmits with probability 1 if s_1 is sensed to be idle and with probability p if s_1 is sensed to be active regardless of the emptiness of its queue. Hence, from (3), the average service rate of s_1 is obtained as

$$\mu_1 = q_{1|\{1\}}\bar{\epsilon}_m(1-p) + q_{1|\{1,2\}}\left(\epsilon_m + \bar{\epsilon}_m p\right)$$

which can be rewritten as

$$\mu_1 = q_{1|\{1\}} - \epsilon_m\Delta_1 - \bar{\epsilon}_m\Delta_1 p \quad (5)$$

By Loynes' Theorem, the queue at s_1 is stable if $\lambda_1 \leq \mu_1$, and the content size follows a discrete-time $M/M/1$ model with the arrival rate λ_1 and the service rate μ_1. For a stable input rate λ_1, the queue at s_1 empties out with probability given by

$$\Pr[Q_1 = 0] = 1 - \frac{\lambda_1}{\mu_1} = 1 - \frac{\lambda_1}{q_{1|\{1\}} - \epsilon_m\Delta_1 - \bar{\epsilon}_m\Delta_1 p} \quad (6)$$

By substituting (6) into (4), the average service rate of the queue at s_2 is obtained as

$$\mu_2 = q_{2|\{2\}}\left(\bar{\epsilon}_f + \epsilon_f p\right) + \frac{q_{2|\{1,2\}}\epsilon_m - q_{2|\{2\}}\bar{\epsilon}_f + (q_{2|\{1,2\}}\bar{\epsilon}_m - q_{2|\{2\}}\epsilon_f)p}{q_{1|\{1\}} - \epsilon_m\Delta_1 - \bar{\epsilon}_m\Delta_1 p}\lambda_1 \quad (7)$$

and the queue at s_2 is stable if $\lambda_2 \le \mu_2$. Consequently, for a given multi-access probability p, stable input rate pairs (λ_1, λ_2) are those componentwise less than (μ_1, μ_2).

What is important is that the boundary of the stability region of the dominant system coincides with that of the original system for the range of values of λ_1 that is less than μ_1 given by Eq. (5)[2]. The reason is this: if for some λ_2, the queue at s_2 is unstable in the hypothetical system, then $Q_2(n)$ approaches infinity almost surely. Note that as long as the queue does not empty, the behavior of the hypothetical system and the original system are identical, provided that they start from the same initial conditions, since dummy packets will never have to be used. A sample-path that goes to infinity without visiting the empty state, which is a feasible one for a queue that is unstable, will be identical for both the hypothetical and the original systems. Therefore, the instability of the hypothetical system implies the instability of the original system. This is the so-called *indistinguishability* argument [7].

We now take the closure of the stability region over the multi-access probability p. This can be equivalently done by solving the following optimization problem in which we maximize μ_2 while guaranteeing the stability of the queue at s_1 for a given value of λ_1 as follows:

$$\text{maximize} \quad \mu_2 \quad (8)$$

$$\text{subject to} \quad 0 \le \lambda_1 \le q_{1|\{1\}} - \epsilon_m\Delta_1 - \bar{\epsilon}_m\Delta_1 p \quad (9)$$

$$0 \le p \le 1 \quad (10)$$

where the expression for μ_2 is given in (7).

[2]This condition is sufficient for the stability of the queue at s_1. Thus, in the following, we are only concerned about the stability of the queue at s_2.

To maximize μ_2 over p, we need to understand their relationship. Differentiating μ_2 with respect to p gives

$$\frac{\partial\mu_2}{\partial p} = q_{2|\{2\}}\epsilon_f + \frac{\eta'\lambda_1}{\left(q_{1|\{1\}} - \epsilon_m\Delta_1 - \bar{\epsilon}_m\Delta_1 p\right)^2}$$

where η' was defined as $\eta' = \bar{\epsilon}_m\eta - q_{1|\{1,2\}}q_{2|\{2\}}\epsilon_f$ in the previous section. When $\eta \ge 0$, which is equivalent to the case when $\eta' \ge -q_{1|\{1,2\}}q_{2|\{2\}}\epsilon_f$, we observe that

$$\begin{aligned}\frac{\partial\mu_2}{\partial p} &\ge q_{2|\{2\}}\epsilon_f - \frac{q_{1|\{1,2\}}q_{2|\{2\}}\epsilon_f\lambda_1}{\left(q_{1|\{1\}} - \epsilon_m\Delta_1 - \bar{\epsilon}_m\Delta_1 p\right)^2} \\ &\ge q_{2|\{2\}}\epsilon_f\left(1 - \frac{q_{1|\{1,2\}}}{q_{1|\{1\}} - \epsilon_m\Delta_1 - \bar{\epsilon}_m\Delta_1 p}\right) \\ &\ge 0\end{aligned}$$

where the last inequality follows from

$$\begin{aligned}q_{1|\{1\}} - \epsilon_m\Delta_1 - \bar{\epsilon}_m\Delta_1 p &\ge q_{1|\{1\}} - \epsilon_m\Delta_1 - \bar{\epsilon}_m\Delta_1 \\ &= q_{1|\{1\}} - \Delta_1 \\ &= q_{1|\{1,2\}}\end{aligned}$$

Thus, if $\eta \ge 0$, μ_2 is a non-decreasing function of p. Note, however, that having $\eta < 0$ does not necessarily mean that μ_2 is a decreasing function of p. By differentiating μ_2 once again, we have

$$\frac{\partial^2\mu_2}{\partial p^2} = \frac{2\bar{\epsilon}_m\Delta_1\eta'\lambda_1}{\left(q_{1|\{1\}} - \epsilon_m\Delta_1 - \bar{\epsilon}_m\Delta_1 p\right)^3}$$

Since the denominator is strictly positive, if $\eta' \ge 0$, μ_2 is convex with respect to p. Otherwise, it is concave with respect to p.

1) The case when $\eta \ge 0$: In this case, μ_2 is a non-decreasing function of p. Thus, maximizing p^* is the largest value satisfying both constraints in Eqs. (9) and (10), that is

$$p^* = \min\left[1, \frac{q_{1|\{1\}} - \epsilon_m\Delta_1 - \lambda_1}{\bar{\epsilon}_m\Delta_1}\right]$$

Note that the role of Eq. (9) is to impose an upper limit on p^* so that the stability of s_1 is not hampered. For $0 \le \lambda_1 \le q_{1|\{1,2\}}$, it is given by $p^* = 1$, and the corresponding maximum function value is obtained as

$$\mu^*_{2,\text{line}_1} = q_{2|\{2\}} - \frac{\Delta_2}{q_{1|\{1,2\}}}\lambda_1 \quad (11)$$

For $q_{1|\{1,2\}} < \lambda_1 \le q_{1|\{1\}} - \epsilon_m\Delta_1$, it is given by $p^* = (q_{1|\{1\}} - \epsilon_m\Delta_1 - \lambda_1)/\bar{\epsilon}_m\Delta_1$, and the corresponding maximum function value is obtained as

$$\mu^*_{2,\text{line}_2} = \frac{q_{2|\{1,2\}}}{\Delta_1}(q_{1|\{1\}} - \lambda_1) \quad (12)$$

Note that if $\lambda_1 > q_{1|\{1\}} - \epsilon_m\Delta_1$, the constraint in Eq. (9) cannot be met with any feasible $p \in [0, 1]$ and, thus, μ_2 is not defined.

2) The case when $-q_{2|\{2\}}\epsilon_f\Delta_1 \le \eta < 0$: In this case, μ_2 is concave with respect to p and, thus, equating the first derivative to zero gives the maximizing p^* as

$$p^* = \frac{q_{1|\{1\}} - \epsilon_m\Delta_1 - \sqrt{\frac{-\eta'\lambda_1}{q_{2|\{2\}}\epsilon_f}}}{\bar{\epsilon}_m\Delta_1} \tag{13}$$

and the corresponding maximum function value is obtained as

$$\mu^*_{2,\text{curve}} = \frac{(\sqrt{-\eta'} - \sqrt{q_{2|\{2\}}\epsilon_f\lambda_1})^2}{\bar{\epsilon}_m\Delta_1} + \frac{q_{2|\{1,2\}}(q_{1|\{1\}} - \lambda_1)}{\Delta_1}$$

Note that $\mu^*_{2,\text{curve}}$ is feasible when both constraints in Eqs. (9) and (10) are satisfied. For used p^*, Eq. (9) becomes

$$\lambda_1 \le \frac{-\eta'}{q_{2|\{2\}}\epsilon_f} \tag{14}$$

and Eq. (10) becomes

$$\frac{q^2_{1|\{1,2\}}q_{2|\{2\}}\epsilon_f}{-\eta'} \le \lambda_1 \le \frac{q_{2|\{2\}}\epsilon_f(q_{1|\{1\}} - \epsilon_m\Delta_1)^2}{-\eta'} \tag{15}$$

which is obtained by rearranging Eq. (13) and substituting the extreme values of p. For the considered case when $-q_{2|\{2\}}\epsilon_f\Delta_1 \le \eta < 0$, the intersection of the ranges of values of λ_1 determined by Eqs. (14) and (15) is given by

$$\frac{q^2_{1|\{1,2\}}q_{2|\{2\}}\epsilon_f}{-\eta'} \le \lambda_1 \le \frac{-\eta'}{q_{2|\{2\}}\epsilon_f} \tag{16}$$

On the other hand, if λ_1 lies on the left-hand side (LHS) of the range of Eq. (16), we observe that

$$\begin{aligned}\frac{\partial\mu_2}{\partial p} &\ge q_{2|\{2\}}\epsilon_f\left(1 - \left(\frac{q_{1|\{1,2\}}}{q_{1|\{1\}} - \epsilon_m\Delta_1 - \bar{\epsilon}_m\Delta_1 p}\right)^2\right)\\ &\ge 0\end{aligned}$$

where we used the facts that η' is negative for the considered case and $q_{1|\{1\}} - \epsilon_m\Delta_1 - \bar{\epsilon}_m\Delta_1 p \ge q_{1|\{1,2\}}$ as observed in the previous case. Since μ_2 is a non-decreasing function of p, $p^* = 1$ and the maximum function value is given by μ^*_{2,line_1} in Eq. (11). Note that the constraint in Eq. (9) is automatically satisfied when λ_1 is on the LHS of the range of Eq. (16).

Next consider the case when λ_1 lies on the right-hand side (RHS) of the range of Eq. (16). This is the case that, if p^* is set according to Eq. (13), the stability of s_1 is lost. For the stability of s_1, it is required that the multi-access probability p is bounded above as

$$p \le \frac{q_{1|\{1\}} - \epsilon_m\Delta_1 - \lambda_1}{\bar{\epsilon}_m\Delta_1} < \frac{q_{1|\{1\}} - \epsilon_m\Delta_1 + \frac{\eta'}{q_{2|\{2\}}\epsilon_f}}{\bar{\epsilon}_m\Delta_1}$$

For p satisfying the above inequality, we observe that

$$\begin{aligned}\frac{\partial\mu_2}{\partial p} &> q_{2|\{2\}}\epsilon_f\\ &\quad\times\left(1 - \left(\frac{-\eta'}{q_{2|\{2\}}\epsilon_f(q_{1|\{1\}} - \epsilon_m\Delta_1 - \bar{\epsilon}_m\Delta_1 p)}\right)^2\right)\\ &> 0\end{aligned}$$

In other words, μ_2 is an increasing function of p and, hence, we have

$$p^* = \frac{q_{1|\{1\}} - \epsilon_m\Delta_1 - \lambda_1}{\bar{\epsilon}_m\Delta_1}$$

for λ_1 on the RHS of the range of Eq. (16). The corresponding maximum function value is given by μ^*_{2,line_2} in Eq. (12). Again, if $\lambda_1 > q_{1|\{1\}} - \epsilon_m\Delta_1$, the constraint in Eq. (9) cannot be met with any feasible $p \in [0, 1]$ and, thus, μ_2 is not defined.

3) The case when $\eta < -q_{2|\{2\}}\epsilon_f\Delta_1$: In this case, μ_2 is still concave with respect to p, but the range of $\mu^*_{2,\text{curve}}$, which was the intersection of the ranges of values of λ_1 determined by Eqs. (14) and (15), would be identical with the range specified by (15). Again, for λ_1 on the LHS of the range of Eq. (15), μ_2 is a non-decreasing function of p, and the maximum function value is given by μ^*_{2,line_1} as in the previous case. On the other hand, if λ_1 lies on the RHS of Eq. (15), we observe that

$$\begin{aligned}\frac{\partial\mu_2}{\partial p} &< q_{2|\{2\}}\epsilon_f\left(1 - \left(\frac{q_{1|\{1\}} - \epsilon_m\Delta_1}{q_{1|\{1\}} - \epsilon_m\Delta_1 - \bar{\epsilon}_m\Delta_1 p}\right)^2\right)\\ &< 0\end{aligned}$$

Therefore, μ_2 is a decreasing function of p and, hence, by substituting $p^* = 0$ into (8), we have

$$\mu^*_{2,\text{line}_3} = q_{2|\{2\}}\bar{\epsilon}_f - \frac{q_{2|\{2\}}\bar{\epsilon}_f - q_{2|\{1,2\}}\epsilon_m}{q_{1|\{1\}} - \epsilon_m\Delta_1}\lambda_1$$

For $\lambda_1 > q_{1|\{1\}} - \epsilon_m\Delta_1$, μ_2 is not defined.

B. Second Dominant System: Primary User Transmits Dummy Packets

Construct a parallel dominant system in which the primary user s_1 is now transmitting dummy packets when its packet queue is empty. Since s_1 transmits with probability 1 in this dominant system, the average service rate of s_2 in (4) becomes

$$\mu_2 = q_{2|\{1,2\}}(\epsilon_m + \bar{\epsilon}_m p)$$

By Loynes' theorem, the queue at s_2 is stable if $\lambda_2 \leq \mu_2$, and it empties out with probability given by

$$\Pr[Q_2 = 0] = 1 - \frac{\lambda_2}{q_{2|\{1,2\}}(\epsilon_m + \bar{\epsilon}_m p)} \tag{17}$$

Substituting (17) into (3) and after rearranging the terms, the stability condition for the queue at s_1 is obtained as

$$\lambda_1 \leq \mu_1 = q_{1|\{1\}} - \frac{\Delta_1}{q_{2|\{1,2\}}}\lambda_2 \tag{18}$$

Observe that Eq. (18) can be rewritten as

$$\lambda_2 \leq \frac{q_{2|\{1,2\}}}{\Delta_1}(q_{1|\{1\}} - \lambda_1) \tag{19}$$

whose boundary is identical with $\mu^*_{2,\mathrm{line}_2}$ in Eq. (12) for the range of $\lambda_1 \geq q_{1|\{1\}} - \Delta_1(\epsilon_m + \bar{\epsilon}_m p)$. Since Eq. (19) does not depend on p, there is no need to optimize over p, and p only has the effect of changing the range of λ_1.

The resulting stability region of the system is the union of the stability region of the two dominant systems. From Section IV-A, the stability region of the first dominant system is defined for $0 \leq \lambda_1 \leq q_{1|\{1\}} - \epsilon_m \Delta_1$. The stability region of the second dominant system is defined by Eq. (19) for $\lambda_1 \geq q_{1|\{1\}} - \Delta_1(\epsilon_m + \bar{\epsilon}_m p)$. Finally, it is not difficult to observe that the union of the stability region of the first dominant system and that of the second dominant system is unique and complete for any value of $p \in [0, 1]$ and for all three cases of having different values of η as described in Section III.

V. Concluding Remarks

We studied the sensitivity of stable rates in cognitive radio systems to the sensing errors. Surprisingly, we observed that when there exists relatively strong capture effect, the system stability is insensitive to the sensing errors. In other words, we can achieve the identical stability region that is achieved with perfect sensing, even with positive sensing error rates. This is remarkable because the spectrum sensing itself becomes unnecessary in terms of the achieved stability region. On the other hand, when the stability is sensitive to the sensing errors, the impact of imperfect sensing on the stability region was precisely quantified. To extend the analysis to more general networks presents serious difficulties of tractability due to the complex interaction between the nodes and may require approximations or alternative approaches that go beyond the scope of this paper.

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