Relay Node Placement for Performance Enhancement with Uncertain Demand: A Robust Optimization Approach

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***Abstract*—The relay node placement problem in wireless sensor networks strongly depends on the data traffic patterns, which can be dynamically changing and not known with precision in advance. In our work, locations of the relay nodes need to be defined with the aim of improving network performance. While in current literature a uniform and constant traffic generation is usually assumed, we propose a robust optimization methodology to deal with the uncertainty in nodes' traffic generation. Solving to optimality the resulting robust optimization problem would be extremely expensive in computational terms, therefore we propose a distinctive robustness measure aiming at dramatically reducing computational costs without sacrificing robustness. The resulting optimization problem is tackled using a bio-inspired meta-heuristic, namely a genetic algorithm. The approach is validated through extensive simulation experiments. Results show that the performance degradation of the robust solutions is minimized in comparison with the solutions obtained ignoring the uncertainty.**

I. Introduction

A *wireless sensor network* (WSN) consists of a set *sensor nodes* (SNs) that are equipped with sensing and limited processing capabilities, and can locally communicate with each other through a wireless medium [1]. In typical applications, the data generated by the sensor nodes need to be transmitted to and aggregated and processed at *base stations* (BSs). The general model for the forwarding of the data from SNs to BSs is based on the definition and use of *multi-hop routing paths*.

Since a WSN can operate for relatively long times and/or it can be embedded in dynamic or hostile environments, a core issue in WSNs is the definition of effective strategies for the time maintenance of network operativity and/or for its adaptation to external or internal changes. In this direction, a wealth of research has considered the use of special nodes, referred to as *relay nodes* (RNs), that can be deployed and added to the WSN after the network has been put in place. RNs can be positioned at precise locations by hand, or they can be part of a mobile robotic unit, such that they can be deployed autonomously or on-demand.

In a previous work [2], we presented the *relay node placement for performance enhancement* problem (RNP-PE in short), defined as follows. Given a set of locations where it is feasible to deploy a restricted number of available RNs, the objective is to select from this set the locations where the RNs can be positioned in order to improve throughput and end-to-end packet delays for the data gathered at BSs. We tackled the problem using a flow-based *linear, mixed integer mathematical program* (MIP) including a number of constraints and penalty components, aimed at closely modeling the specific characteristics of the wireless environment, and a number of heuristics, aimed at speeding up the computations. The model is solved to optimality using a standard solver, finding the best locations where to position the available RNs and the paths to route data flows. The experimental evaluation presented in [2] both in simulation and in a real sensor network, showed that the provided solutions can significantly improve the overall network performance in terms of data throughput at the base stations.

One of the limitations of the proposed model is that it requires as input the data rates of all nodes in the network. That is, it assumes a full knowledge of the traffic generation rates at the nodes, which are also assumed to be constant. Such assumptions are common to most of the offline models for relay node placement presented in the current literature (see related work), since an optimal deployment of relay nodes can only be computed on the basis of both the knowledge of the network topology and the expected traffic load. However, in many real-world applications, traffic loads can be dynamically changing and/or not known with precision in advance. For instance, the sensor nodes close to a detected event will likely increase their data rate generation while responding to the event, which is a common situation in intruder detection in surveillance applications, or in the occurrence of natural events in environmental monitoring. If the issue of changing data rates is not explicitly addressed in the model, the obtained solution can either be unrealistic or not robust to the actual variations.

In this paper we propose to address this issue by taking a *robust optimization* (RO) approach for the RNP-PE problem. Robust optimization is a relatively novel approach to tackle problems affected by uncertainty in the provided input data. The goal is to provide a way to find solutions whose quality is not significantly decreased by adverse realizations of the uncertain data. In the context of RNP-PE problems, instead of considering a *fixed* traffic generation rate at a node, we consider a *discrete set* of possible data rates for each single node. This set summarizes the overall knowledge we have regarding the possible realizations of the random variables associated to the data generation rates. Since typical RO formulations reduce to NP-hard optimization problems which require massive computational resources to solve to optimality, we propose a heuristic robust formulation, based on an approximated robustness metric following a *min-max regret* criterion [3], [4]. The resulting optimization problem is solved by means of a bio-inspired meta-heuristic, namely a genetic algorithm.

The main scientific contribution of this work, compared to previous MIP models for relay node placement, is to present a robust and computationally affordable optimization approach that addresses network enhancement issues under uncertain network traffic demands, and explicitly includes most of the critical aspects of interference and congestion in wireless environments. In particular, we show through extensive simulation experiments that placements obtained using the proposed heuristic robust formulation provide a better performance, under different network traffic conditions, in comparison with the placements obtained when ignoring uncertainties on traffic demands (e.g., assuming fixed data rates).

The rest of the paper is organized as follows. In section II, we outline the relevant work on relay placement in wireless networks and robust optimization and its applications. A brief overview of the relay node placement for performance enhancement is presented in section III. In section IV, we introduce the robust optimization methodology which considers node demand uncertainty. The meta-heuristic solution approach of the resulting robust optimization problem is presented in section V. Next, in section VI, we present an extensive evaluation using network simulations in which the robust solutions are compared against solutions obtained without the consideration of uncertain node demand. Finally, we draw conclusions and discuss future work in section VII.

II. RELATED WORK

In this section, we first give a brief overview of the relay node placement problem in WSNs, pointing out the differences between our work and the other most relevant approaches. We then introduce related literature on the field of robust optimization, providing examples of its application in several areas, with a focus on networking, and discussing the relationship with methodology that we followed in our work.

In recent years, a number of studies in WSNs have considered the *relay node placement problem* (RNP) under different requirements and objectives. Most of the existing work has focused on the deployment of RNs for the provisioning of: *connectivity* [5]–[11], *extended network lifetime* [12], *energy-efficient or balanced data gathering* [13]–[15], and *survivability and fault tolerance* [9], [11], [16], [17]. The approaches focusing on performance metrics other than connectivity and survivability are reviewed in the following, where for each approach we single out some core differences, in terms of objectives and modeling, with the work which we present in this paper.

Falck *et al.* [13] have considered the RNP in the context of balanced data gathering. They presented the problem of finding an optimal routing as a linear program, but with the objective of achieving load balancing. The placement of RNs is approximated by adding relays one at a time. Instead, we directly include the placement in our mathematical model, and jointly solve both problems (i.e., optimal routing and relay placement). Patel *et al.* [14] examined the joint problem of deploying SNs, RNs, and BSs on a set of feasible locations and finding bandwidth-constrained energy-efficient routes with guaranteed coverage, connectivity, bandwidth, and robustness. These authors, which also make the use of a linear program formulation, have considered as objective maximizing network utilization when RNs can be deployed only in a set of feasible sites. Kashyap *et al.* [18] studied the placement of RNs with the goal of reducing maximum link load for a given traffic imposed on the sensors. Similarly to us, they considered flat architectures, called backbone networks. However no data-aggregation base stations are included in the network: traffic data are exchanged between pairs of nodes in the network (called profile entries). The adopted model restricts the placement of the relays to the lines joining the backbone nodes, aiming to determine the minimum number of relays needed to link two nodes.

Ergen and Varaiya [15] considered the problem of determining optimal locations for RNs together with optimal energy provisioning, such that the network operates for the desired lifetime with minimum energy expenditure. These authors considered a non-linear programming model and established a set of possible locations for the RNs based on a grid partitioning. The work mostly focuses on energy-efficiency, such that the quality of the solutions in terms of network performance is not really investigated, and considers a simplistic radio propagation model. In [19], the authors studied the joint problem of placing relay nodes and scheduling node transmissions in the presence of controlled mobility. The approach aims to maximize the lowest weighted throughput among the nodes in the network. Only star topologies are considered, in which nodes communicate to each other only through a relay node. In our work, we aim to exploit the positioning of relay nodes to maximize the overall network throughput by the creation of new data routes and the reduction of traffic congestion; moreover, we consider multi-hop topologies. Capone *et al.* [20] proposed a network flow based model for the optimal routing in wireless mesh networks addressing TDMA networks and focusing on traffic scheduling. Due to their simplicity, CSMA based MAC protocols are preferred in WSNs. Therefore, our model explicitly considers this type of network. Finally, Wang *et al.* [12] studied the deployment of RNs to maximize network lifetime in two-tiered WSNs with a single base station. We consider general flat topologies and the presence of multiple BSs, to address large-scale scenarios. None of the previous works considered uncertainty in the problem data. In this work, we tackle the relay node placement for performance enhancement in which the node traffic generation is not assumed to be fixed, but lies in a discrete set of possible data rates, representing possible scenarios.

In general terms, modeling and optimizing real-world systems usually involves dealing with uncertain data. Addressing this uncertainty, in specific for mathematical programming, has long been the focus of research of the operations research community. Probabilistic approaches, such as Stochastic Optimization [21], assume that the uncertain data can be characterized by a probability distribution, and therefore aim optimize the expected value of the objective function. However, in most of the cases, the exact probabilistic nature of the data is not known. Moreover, the resulting optimization problems are, in general, not tractable [22].

Robust Optimization [23], is a more recent approach to optimization under uncertainty. RO was originally introduced in [24] for convex optimization and in [25] for semi-definite programming. Under a RO framework, the uncertainty model is not probabilistic, but deterministic and scenario-based. A scenario is a possible realization of values of the uncertain parameters. In the interval scenario case, possible values of each uncertain parameter are continuous and bounded by a numerical interval. In the discrete scenario case, as the one we consider, the possible realizations are described by an enumerable set. There are several possible optimization criteria that can be used in RO. One of the most commonly used one is the *min-max regret* criterion. A min-max regret approach aims at minimizing the maximum deviation, over all possible scenarios, between the value of the proposed solution and the value of the optimal solution of the corresponding scenario [3].

Robust Optimization has been used recently in several applications, such as vehicle routing problems [26], and supply chain management problems [27]. In the context of networking, the RO methodology has gained a lot of attention in recent years. In [28], the authors propose a robust approach to interference management in a cellular networks. The work presented in [29], considers several optimization models in WSNs, guided by energy constraints, and subject to distance uncertainty. In [30], wireless network planning under uncertain bit rate requirements and channel conditions is tackled using a robust optimization approach. The approach, based on the seminal work of [22], enables the user to set a trade-off between robustness and profit by adjusting a robustness parameter. A robust approach for resource allocation problems in wireless relay networks is presented in [31] through a methodology that handles for uncertainties in the global channel state information. The results show that ignoring these uncertainties often leads to poor performance, highlighting the relevance of robust optimization in real-world wireless networks.

Unlike traditional RO approaches, such as the ones presented above, we propose a distinctive robustness measure aiming at dramatically reducing computational costs without sacrificing robustness. The proposed measure is a heuristic alternative to the min-max regret criterion. The solution of the resulting combinatorial optimization problem is then tackled by using a meta-heuristic algorithm.

III. THE RNP-PE MODEL

We model the WSN as a set of SNs and BSs located in a set of known positions $\mathcal{S}$ and $\mathcal{B}$, respectively. SNs both generate and forward data packets towards one of the BSs in multi-hop fashion (a data flow can be split over multiple paths). Initially, we assume that the characteristics of data generation characteristics for each SN are known. All nodes communicate with each other within the *communication range* r. A set of K RNs is also available, their role is to forward data received from other nodes. The placement of node relays is restricted to a *numerable set of candidate locations* denoted as $\mathcal{R}$. We formalize the RNP-PE by a MIP model based on a *minimum cost flow* formulation as follows.

Let $G = (V, E)$ be a connected digraph representing a WSN, where $V = \mathcal{N} = \mathcal{S} \cup \mathcal{B} \cup \mathcal{R}$ is the set of nodes, and E is the set of communication links. $\gamma : E \mapsto \Re$ is a *link cost* function, and $\tau : \mathcal{S} \mapsto \Re$ is a data generation (*traffic load*) function, expressed in the *data per second* generated by an SN In the following, we measure τ in terms of *flow units*, f_{unit}, expressed as bytes/sec. Data flows and relay positions define the two sets of *decision variables*. The *flow variable* f_{ij} denotes the amount of flow through link (i, j), expressed in *flow units*. The *binary positional variable* y_i indicates whether location $i \in \mathcal{R}$ is used to circulate flow or not. When y_i is set to 1 in a solution, an RN is to be positioned at the corresponding relay location. A full solution specifies both flows and relay positions. The SN-to-BS routes are defined in the *routing-tree* induced by the set $\{(i,\ j) \in E \mid f_{ij} > 0\}$. We formalize the RNP-PE by a *linear, mixed integer mathematical programming* (MIP) model based on a *minimum cost flow* formulation that includes a number of additional constraints and penalty components:

$$\min \ \text{RNP-PE} = \sum_{(i,\ j)\in E} \gamma_{ij} f_{ij} + \hat{R} \sum_{i \in \mathcal{R}} y_i + \alpha \sum_{i \in \mathcal{S}} p_i \hat{F} \ . \quad (1)$$

$$\text{subject to:} \quad \sum_{(i,\ j)\in E} f_{ij} - \sum_{(j,\ i)\in E} f_{ji} = \begin{cases} \tau_i & \text{if } i \in \mathcal{S}, \\ 0 & \text{if } i \in \mathcal{R} \end{cases} \quad (2)$$

$$\sum_{i \in \mathcal{B}} \sum_{(j,\ i)\in E} f_{ji} = \sum_{k \in S} \tau_k \quad (3)$$

$$\sum_{(i,\ k)\in E} f_{ik} \le y_k \sum_{j \in S} \tau_j, \quad \forall k \in \mathcal{R} \quad (4)$$

$$\left(\sum_{(i,\ k)\in E} f_{ik} \right) \ge y_k, \quad \forall k \in \mathcal{R} \quad (5)$$

$$\sum_{k \in \mathcal{R}} y_k \le K \quad (6)$$

$$\sum_{(i,\ j)\in E} f_{ij} + \sum_{(j,\ i)\in E} f_{ji} \ \le \ L_{cap}, \quad \forall i \in \mathcal{N} \quad (7)$$

$$\sum_{(i,\ j)\in E} f_{ij} \le b_{ij} L_{cap}, \quad \forall i \in \mathcal{N} \quad (8)$$

$$\delta_i^- = \sum_{(j,\ i)\in E} b_{ji} \le \delta_{max}^-, \quad \forall i \in \mathcal{S} \quad (9)$$

$$p_i = 1 \iff \sum_{(i,\ j)\in E} \sum_{(j,\ k)\in E} f_{jk} \ge F_{max}, \quad \forall i \in \mathcal{S} \quad (10)$$

Constraints (2-3) correspond to the flow definition. The number of available RNs is limited to K constraints (4-6). Since the optimal solution may be obtained using a number of RNs $k < K$, we define a penalty factor in the objective (1) to favor the use of a minimal amount of RNs: any optimal solution using k relays needs to provide a minimal gain $\hat{R}$ with respect to the solution obtained using $k-1$ relays. Parameter $\hat{R}$ can be adjusted according to the problem instance (e.g., relay node availability, economic cost).

Shared wireless channels in WSNs are necessarily *bandwidth-limited*. This condition is reflected by *link capacity* parameter L_{cap}, which is the nominal amount of data (bytes/sec) that can be transmitted by a wireless link in the network (assuming the same capacity for all links), and constraint (7).

For a node i, the *routing in-degree* (δ_i^-) is the number of i's neighbors using i to relay data. A higher value of δ_i^-, increases the chances of packet losses at i due to higher probability of simultaneous transmissions from nodes having it as next-hop. In the case of CSMA MAC protocols, commonly used in WSNs, some of these neighbor nodes may not be aware of each others' transmissions (hidden terminal problem), therefore they might initiate overlapping transmissions, thus provoking packet losses at i due to (a) interference caused by one of the transmissions, or (b) the busy state of the transceiver while engaged in the reception of data. In order overcome these problems, we provide the capability of limiting the value of δ_i^- in the solution routing trees to δ_{max}^- (set to 10 in the experiments). This is accomplished through constraints (8 - 9).

Wireless interference between different data flows is one of major factors of performance degradation [32]. In order tackle this issue, we limit the amount of flow F_i^r generated and relayed by all neighbors of an SN $i \in \mathcal{S}$, located within a disk of radius r centered in i. Since F_i^r includes all wireless transmissions in i's neighborhood, limiting it aims to reduce interference when i acts as receiver and medium contention problems when i attempts to send data. Moreover, it also prevents the formation of highly congested regions, favoring the generation of balanced routing trees, which in turn gives a balanced energy depletion. This is realized by penalizing solutions in which F_i^r exceeds a defined threshold (F_{max}), including in the objective function a penalty component.

The calculation of F_i^r requires to sum up the outgoing flows from all i's neighbors. Whenever F_i^r exceeds the predefined threshold, a binary variable p_i takes value 1. Constraints (10) formulate this condition. In order to use p for inclusion in the objective function as penalty, we derive a rough estimation, $\hat{F}$, of the optimal solution value of problem, without penalties, and we use it as a penalty score for the violation of the circulating flow limit. Using $\hat{F}$ and p, the penalty for the violation in maximum local flow is therefore included in the objective function. The parameter α weighs the penalty is set to 0.1 in the experiments.

We refer the interested reader to [2], [33], [34] for a full description of the parameters and an extensive evaluations of the RNP-PE model.

IV. THE ROBUST APPROACH

In the previous MIP model, the data rates of all SNs are given as input (parameters τ_i). Therefore, the computed solution of the RNP-PE is based on the knowledge of the traffic generation patterns of the network. We relax this limitation by assuming a possible, finite set of traffic rates for each single SN $i \in \mathcal{S}$, denoted by $\Phi_i \subseteq \Re$. We consider that during network operations, i may adopt any of the data rates specified in Φ_i with probability $\frac{1}{|\Phi_i|}$. Therefore, nodes' traffic rates are no more considered as known fixed values but random variables in the sets Φ_i. A traffic demand scenario $d : \mathcal{S} \mapsto \Re$ is a realization of Φ_i, and $\mathcal{D}$ is the set of all possible demand scenarios. In the following, we assume that, for all possible demand scenarios, there exists a feasible solution (i.e., routing trees that allow, in principle, the forwarding of data from SNs to BSs) which does not make use of RNs.

A. Measure of robustness

Let $R' \subseteq \mathcal{R}$ be a selection of positions of $|R'| \leq K$ relay nodes and d a demand scenario, we define the *relative regret* of R' under d as:

$$regret(R', d) = \frac{\text{RNP}(R', d) - \text{RNP}(\mathcal{R}, d)}{\text{RNP}(\emptyset, d) - \text{RNP}(\mathcal{R}, d)} \qquad (11)$$

where RNP(R', d) denotes the value of the optimal solution of the MIP model with R' as the restricted set of candidate locations, and d as the network demand of the SNs (i.e., replacing $\mathcal{R}$ with R' and τ with d in the model of Section III). Intuitively, $regret(R', d)$ represents the performance deviation of a fixed placement R' with respect to the optimal placement under a specific demand scenario d. In other words, if we are aware of the current data rates of the nodes, the measure says how much we will regret having chosen the placement R' in the first place, considering the actual optimal RN placement for that particular demand scenario. Note that $regret(R', d) \in [0, 1]$. A value of 0 indicates that R' is optimal for all $d \in \mathcal{D}$, while a value of 1 indicates that in at least one scenario, the best performance achieved by using relays in R' do not provide any improvement with respect to the performance achieved using no relays at all.

The *robust RNP-PE* formulation aims to determine a RN placement such that the relative regret is minimized over all possible demand scenarios. It is defined as:

$$\min_{R' \subseteq \mathcal{R} \text{ and } |R'| \leq \mathcal{K}} \max_{d \in \mathcal{D}} regret(R', d) \qquad (12)$$

Clearly, finding the maximum $regret$ of a particular placement would require to evaluate all possible, exponentially large number, of scenarios. Therefore, we propose a *heuristic approximation* of $\max_{d \in \mathcal{D}} regret(R', d)$, which serves as a basis for determining a robust RN placement.

B. Approximation of maximum $regret$

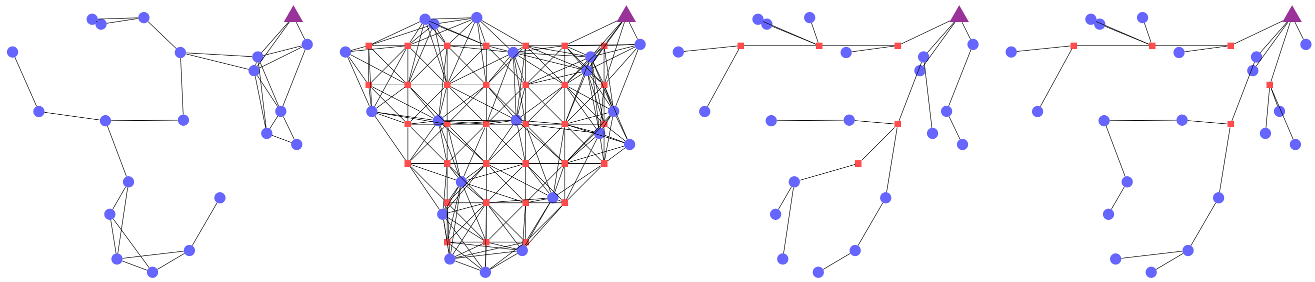
Given an RN placement R', our goal is to derive an estimation of its maximum relative regret. That is, finding a demand scenario $d \in \mathcal{D}$ such that $regret(R', d)$ is close to its maximal value over the set $\mathcal{D}$.

Definition. *Given a routing tree T, the set of* favored nodes *consists of all SNs whose routing paths to any of the BSs pass, at least, through one RN.*

In the context of the RNP-PE problem, we denote with FN(R', d) the set of favored nodes corresponding to the routing tree resulting from the solution of RNP(R', d).

Intuitively, the set FN(R', d) can be seen as the group of SNs whose data flow is (positively) affected by the placement of RNs in R'. This effect is most likely represented by a reduction of the flow cost in the objective function of the MIP model. Therefore, the higher the data rate of the favored nodes, the higher will be the cost reduction. Conversely, if all favored nodes decrease their demand, the potential benefits of the placement R' will be also reduced. This argument motivates the following method to find a demand scenario such that the associated relative regret is close to the maximal value.

Let $\bar{d} \in \mathcal{D}$ be the *median scenario*, corresponding to the demand scenario where all SNs adopt the value $m \in \Phi_i$



(a) Initial network

(b) Feasible relay locations

(c) Median placement $\bar{R}$, resulting from the optimal solution of RNP$(\mathcal{R}, \bar{d})$

(d) Optimal solution of RNP$(\mathcal{R}, d^*)$

Fig. 1. Some of the steps to solve RNP$_{rob}$. Figures (c) and (d) illustrate intermediate steps of the evaluation of $regret^*$.

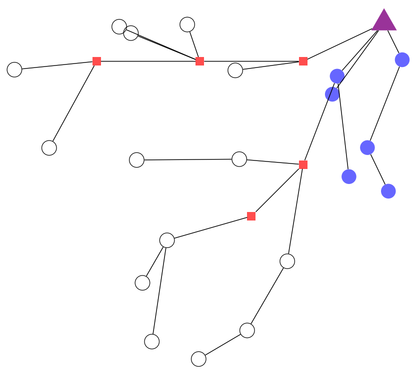


Fig. 2. Illustration of the favored nodes, represented by white circles. The squares are the RNs, circles are the SNs, and triangles the BSs.

closest to the median value of the set Φ_i. The *median placement* corresponds to the RN positions obtained by solving the problem RNP$(\mathcal{R}, \bar{d})$, that is, the optimal placement of the median scenario, and it is denoted by $\bar{R}$. Given these notions, we now formulate the heuristic approximation of $\max_{d \in \mathcal{D}} regret(R', d)$ as:

$$regret^*(R') = regret(R', d^*) \tag{13}$$

where:

$$d^*(i) = \begin{cases} \min \Phi_i & \text{if } i \in \text{ FN}(R', \bar{d}) \\ \max \Phi_i & \text{if } i \notin \text{ FN}(R', \bar{d}) \end{cases} \tag{14}$$

With the formulation described in Equation (13), we aim to find a *bad* demand scenario for R', such that the flow cost reduction, or, equivalently, the potential benefits due to the RNs in R', is minimized. This demand scenario is constructed in Equation (14) by the setting the demand of the favored nodes, corresponding to the median scenario, to their minimum value among the possible ones. The demands of the not favored nodes are set to their higher possible value. The selection of $\bar{d}$ to obtain the favored nodes is motivated by the assumption that the optimal routing tree corresponding to this scenario will very likely determine a group of favored nodes that are present in most of the other demand scenarios.

The following example can help to illustrate the meaning of the $regret^*$ measure. Let us consider a WSN consisting of 20 SNs and 1 BS, as shown in Figure 1a. The set of all feasible RN locations is defined using a uniform grid as shown in Figure 1b. Each SN can adopt ten different traffic rates $\Phi_i = \{1, \ldots, 10\}$, denoting some amount of data per second (e.g., Kbytes/sec). As median scenario $\bar{d}$ we let the nodes adopt traffic rate equal to 6. Figure 1c shows the optimal placement solution for $\bar{d}$, that is $\bar{R}$. We show the process of evaluating the value of $regret^*$ for the median placement. From the routing tree described in the solution of the median scenario RNP$(\mathcal{R}, \bar{d})$, shown in figure 1c, we compute the set of favored nodes FN$(\bar{R}, \bar{d})$, depicted in figure 2. We now configure a scenario, denoted by d^*, where the nodes in FN$(\bar{R}, \bar{d})$ adopt their higher rates (i.e., 10) while the other nodes the lower rates (i.e., 1), as described in equation (14). Solving the RNP$(\bar{R}, d^*)$, we obtain a flow cost of value 303. We now seek the optimal placement for the scenario d^*, that is RNP$(\mathcal{R}, d^*)$, depicted in figure 1d. The value of the objective function in this case happens to be equal to 288.50, less than the value of RNP$(\bar{R}, d^*)$, since we are now considering the full set of candidate locations. Interestingly, we can appreciate that the only difference between both, the median placement and the optimal for d^* is precisely one RN that is positioned to attend the larger demand of nodes not belonging to the favored node set, which has been determined before. Finally, we compute the upper bound of the objective function of the scenario d^*, by simply determining the cost of the solution without any relays. In this case, it is equal to 334. Therefore, the value of $regret^*(\bar{R})$ is:

$$\begin{aligned} regret^*(\bar{R}) &= regret(\bar{R}, d^*) &= \frac{\text{RNP}(\bar{R}, d^*) - \text{RNP}(\mathcal{R}, d^*)}{\text{RNP}(\emptyset, d^*) - \text{RNP}(\mathcal{R}, d^*)} \\ & &= \frac{303 - 288.5}{334 - 288.5} = 0.3186 \end{aligned}$$

Using the approximation introduced in equation (13), we now formulate the robust RNP-PE (denoted by RNP$_{rob}$):

$$\text{RNP}_{rob} = \min_{R' \subseteq \mathcal{R} \text{ and } |R'| \leq \mathcal{K}} regret^*(R'). \tag{15}$$

In order to compare empirically the heuristic model RNP$_{rob}$ with the formulation (12), we have considered a problem using a small network instance consisting of 7 sensor nodes and one base station, with 6 possible relay positions, and $\mathcal{K} = 2$. The small size of this example allowed us to find the optimal solution of (12) by exhaustive evaluation, and compare it to the solution obtained using the model composed by approximated metric (i.e., RNP$_{rob}$). The solutions obtained were exactly the same. Moreover, an analysis of the values obtained by the heuristic approximation $regret^*$, and $\max_{d \in \mathcal{D}} regret(R', d)$ showed that the proposed approximation is quite accurate, providing exact values for 13 out of the 21 possible placements, and a minimal difference for the remaining ones. Therefore, promising good effectiveness in larger instances.

V. META-HEURISTIC SOLUTION APPROACH

The RNP-PE problem belongs to the family of $\mathcal{NP}$-hard network design problems [35], [36]. In fact, the RNP$_{rob}$ is at least as hard as the RNP-PE since it is a particular case when the scenario set contains only one scenario. For this reason, standard approaches fail to find optimal solutions to RNP$_{rob}$, even for small size instances, in reasonably short time. Because the formulation is intractable with traditional mathematical programming methods, we propose an effective meta-heuristic method to find a robust RN placement.

Bio-inspired approaches, and more specifically genetic algorithms (GA), have been used to tackle the complexity of many optimization problems in wireless networking, including clustering [37], optimal design [38], routing and link scheduling [39], network planning [40] and node placement [41]–[43]. In a previous work [44], the GA methodology proved to be a quite effective solution approach to the RNP-PE. Given its effectiveness in dealing with RNP and similar problems arising in wireless networking, a GA is adopted for solving the robust RNP-PE. An overview of the algorithm is presented below.

A. Genetic Algorithm for solving the robust RNP-PE

The encoding of individuals (also known as *chromosome encoding*) is fundamental to the implementation of GAs in order to efficiently transmit the genetic information from parents to offsprings. In our case, an individual of the population represents a deployment of relay nodes. Since RNs can be placed at one of the set of candidate locations $\mathcal{R}$, the location of each RN can be conveniently specified as the index of the element in $\mathcal{R}$ to which it corresponds to. Accordingly, a population member is encoded as a list of index values.

Apart from the encoding, another fundamental aspect of a GA is the design of its genetic operators. We make use of traditional genetic operators designed to achieve a good trade-off between exploration and exploitation. Crossover is implemented by a random selection of RN locations from both parents. The number of RN locations to be selected is chosen randomly between the size of both parents. The mutation operator allows a controlled exploration of new regions of the solution space by inducing small perturbations to existing individuals. Mutation is implemented by displacing the relay positions, replacing it by another candidate location, within a circular area of size 2$\mathbf{r}$. Additionally, it may also modify the number of positions of relays removing existing RN positions or adding new RNs (if feasible).

B. Evaluation of individuals

Each individual R' is evaluated by computing the value of the function $regret^*$ described in Equation (13). This requires to solve four instances of the RNP MIP model using a standard solver: RNP$(R', \bar{d})$, which is needed to obtain the set FN$(R', \bar{d})$ and the scenario d^*, and the other three instances to compute $regret(R', d^*)$. Most of these problem instances are *easy* to solve since the set of RNs is restricted to R'. However, one of them requires to solve the complete model (i.e., considering the full set of candidate locations $\mathcal{R}$). In most of the cases, we may not require to find an optimal solution, as by an appropriate setting of the solver's parameters (e.g., MIP gap) we can still obtain good feasible solutions at a reduced computational cost.

To speed up the convergence of the GA we exploit valuable information obtained during the evaluation procedure. More specifically, the solutions of RNP$(\mathcal{R}, d^*)$ provide optimal placements R^* which represent potentially useful placements which can be included in the population for a further evaluation of their $regret^*$ values. Therefore, we define a solution pool containing those optimal placements. To include these individuals into the GA population, we extend the crossover operator to replace one of the offsprings by an individual from the solution pool. This replacement occurs with a probability p_{pool}, set to 0.25 in the experimental evaluation.

VI. EXPERIMENTAL EVALUATION

To evaluate the proposed robust methodology, we considered a number of randomly generated network instances. In total, we considered 50 network instances generated with different topological characteristics: *uniform, clustered*, and *small world*. Networks were embedded in an area of size 100×100 m^2, and the set of feasible relay positions was determined using a uniform grid, with the grid points separated by Δ=5 m of distance. Each sensor node has 10 possible traffic demands, from 2 $pkts/sec$ up to 20 $pkts/sec$. The network packet size was set to 96 bytes.

The evaluation consists of three steps. First, we generated 50 demand scenarios where the data rate of each node was randomly selected among the 10 possible ones. For each of these scenarios, we compute the optimal placement. Second, for each network instance, we computed a robust placement using the GA described in Section V. The population size was set to 100 individuals, and the crossover and mutation rate were set to 0.9 and 0.1 respectively. The GA was implemented using a tournament selection method. The maximum run-time allowed for this step was set to 4 hours, which enabled the GA to evolve over a hundred of generations. Finally, we computed the optimal placement under the median scenario. In case of the robust and median RN placements, we computed the optimal routing trees for each demand scenario, assuming that in a real-world situation, after being aware of the actual settings in node demands, the routing tree could be easily re-optimized in an on-line fashion. To solve the MIPs we used CPLEX®.

A. Network simulations

In order to evaluate the performance of an RN assignment, we perform network simulations using TOSSIM [45], The main reason behind the choice of TOSSIM as simulation environment for our work is because it provides accurate results due to its realistic wireless channel model. TOSSIM also profits from the component based architecture of TinyOS [46] to transparently define a hardware abstraction layer that simulates the TinyOS network stack at the processor level. The log-distance path loss model is used to compute the link gain values, which are set at the start of each simulation run. The simulations are performed using a IEEE 802.15.4 non beacon MAC implementation developed for TOSSIM and using the default parameters, defined in the IEEE 802.15.4 standard.

To account randomness, we ran 5 simulation runs for each scenario, and consider the average performance.

B. Results

For each simulation, we calculated the network packet delivery ratio (PDR), defined as the total number of packets received at the base stations, divided by the total number of packets generated by all sensor nodes. For the robust and median placements, we compute the deviation of the PDR from that of the optimal placement.This is done for each simulation instance (i.e., network topology and demand scenario).

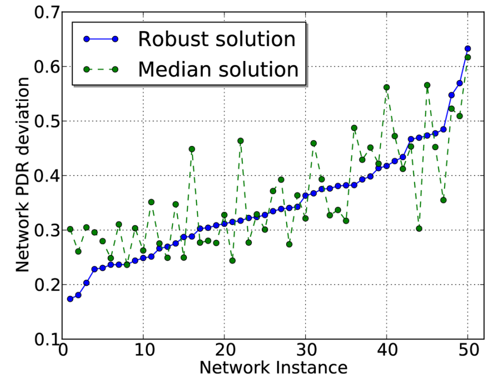
From the evaluation, we expect that the robust solutions exhibit lesser maximum deviations from the optimal, in comparison to the median scenario solutions. This condition is expected since the robust model defined in Section IV precisely has the minimization of the maximum regret (i.e., the worst deviation from optimal) as optimization criterion. Additionally, we also evaluate the average performance (in terms of deviations) of both solutions. We expect the robust solutions to be *good* in average, and significantly better compared to the median scenario solutions, which were defined completely ignoring data uncertainty.

Figures 3a and 3b show the worst and mean PDR deviation of the robust and median placements, over all demand scenarios, for each one of the topologies. To make the plots more readable, data on the x-axis (network instances and demand scenarios) have been ordered according to the increasing PDR deviation of the robust placement. We can observe that in most of the considered network topologies, the robust placement exhibits better performance compared to the median placement. In terms of worst performance, the robust placement offers lower performance degradation compared to the placement considering the median demand scenario. We also present the deviations over the demand scenarios for a particular topology, in Figure 3c. It is possible to appreciate that, although the median placement offered good performance for some of the scenarios, it also incurred in the worst performance degradation (over 40%). The degradations of the robust solution remained within 25% of the optimal.

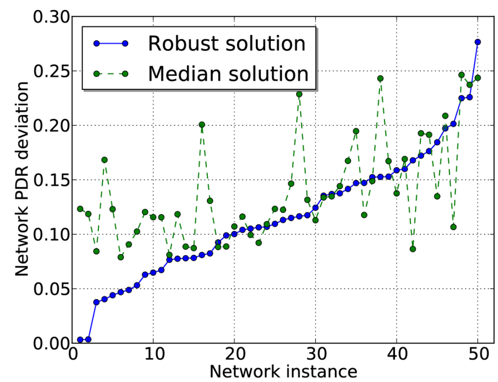
VII. CONCLUSIONS AND FUTURE WORK

In this paper, we have proposed a robust optimization (RO) methodology to the relay node placement problem for performance enhancement under uncertainty on the node traffic demands. To the best of our knowledge, this is the first attempt to use RO concepts in relay node placement problems in wireless sensor networks. Compared to other RO approaches, we developed a heuristic approach to the min-max regret problem, aiming at reducing the computational costs of the robust formulation. The resulting combinatorial optimization problem is then tackled using a meta-heuristic algorithm. Simulation experiments show that, over a number of different traffic scenarios, the performance degradation of the robust solutions is minimized in comparison with the solutions obtained ignoring data uncertainty.

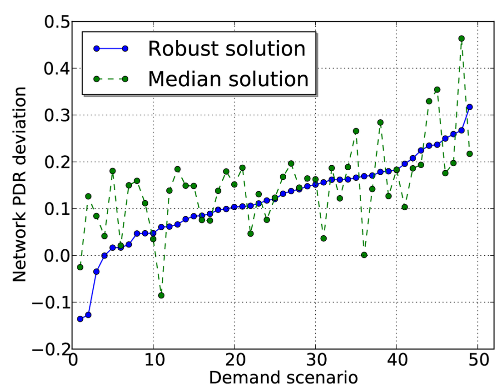
Future work includes the further evaluation of the proposed robust solutions using a real testbed with realistic application scenarios (e.g., intruder detection) with space-time correlations in data generation patterns. Additionally, we will also consider to extend our analysis in order to contemplate uncertainty in other aspects of the problem, such as the information regarding the positions of the sensor nodes.



(a) Worst performance



(b) Mean performance



(c) Deviation over all demand scenarios for one instance of the considered topologies.

Fig. 3. Summary of simulation results. Values correspond to the absolute differences between network packet delivery ratios (PDR) of optimal placements (i.e., best placement for the corresponding demand scenario) and PDRs of robust and mean placements.

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