Multi-Perspective Anomaly Detection on Bipartite Multi-Layer Social Interaction Networks

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Abstract

Anomaly detection is a prominent research direction in machine learning and complex network analysis. In this paper, we target a special type of complex networks, i. e., bipartite multi-layer networks. Here, we exploit the properties of such a complex network, i. e., the partitioning of the set of nodes into two groups, and its multi-layer characteristics. Our proposed approach includes many-objective optimization, correlation analysis and clustering – based on Eigenvector centrality – incorporated into a novel framework for identifying candidates for anomalous nodes from multiple perspectives, in a human-centered interpretable way. We exemplify the application of the proposed approach in a case study using a real-world dataset on socio-spatial interaction data.

1. Introduction

Complex networks lend themselves to the modeling of complex relationships, with many applications in science and industry. In the world of today, there is a wide range of possible application areas. Often, e. g., when considering different groups of entities like different types of actors in a social network, or different types of machines in a technical network, the resulting network – considering its set of nodes – can be partitioned into distinct groups. In the case of partitioning the set of nodes into two groups, we can then form a bipartite network. Likewise, often several relationships between the nodes can be modeled and analyzed, motivating the joint application of bipartite multi-layer network analysis [1].

In this paper, we tackle such a setting in the context of anomaly detection, for identifying *candidates* of *anomalous nodes* which indicate *deviating*, *interesting* or *exceptional* sets of nodes, i. e., which can be considered as anomalies in the network concerning their structural properties. Specifically, our proposed approach combines three methods for anomaly detection providing separate perspectives for identifying such anomalies in a human-centered way. At their core,

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these are based on the notion of centrality, specifically Eigenvector centrality for anomaly detection, forming a combined interpretable approach including many-objective optimization, as well as correlation and cluster analysis. These methods are combined into a methodological framework, for providing the different perspectives and to enable assessment by also analyzing potential commonalities and differences pointed to by the incorporated methods, respectively.

We build on our previous works [2, 3] for (1) anomaly detection using multi-objective optimization, as well as (2) a complementing approach for applying Eigenvector centrality for anomaly detection in a human-centered approach. In particular, in this paper we integrate these methods into a novel framework for recognizing and finding anomalous behavior in a complex network represented as a bipartite multi-layer network, e. g., relating to different relationships or edge types connecting the respective nodes of the network.

In short, our presented approach starts by making projections of the bipartite network. Then, from those projections and each layer, we estimate the centrality of all its contained nodes. Next, we apply many objective optimization to identify the Pareto Front, as a basis for finding a set of anomalous nodes with minimal centrality. In addition, we apply correlation analysis on the centrality properties, and can further categorize nodes using clustering into positively correlated, negatively correlated (i. e., very different) or non-correlated nodes, as complementing perspectives in assessing anomalous nodes in an interpretable way.

In more detail, our proposed approach consists of the following steps:

1. Given the network represented as a bipartite multi-layer graph, we perform many-objective optimization based on minimizing eigenvector centrality on bipartite projections of the multi-layer network. With the minimization, we aim at obtaining the set of the least important nodes according to eigenvector centrality, as candidates for anomalous nodes. This provides us with our first perspective for identifying anomalies, given by the Pareto-Front of the least important nodes according to their (minimized) eigenvector centrality.

2. Using the vector of centrality values for a node in each layer, we perform correlation analysis with respect to all other nodes, resulting in a correlation matrix and according heatmap perspective, respectively, to visually inspect anomalies.

3. Finally, we can apply clustering on the correlation matrix for obtaining clusters of nodes, as another perspective for detecting (sets of) anomalous nodes.

Overall, this enables the identification of *anomaly candidates* from multiple perspectives; this then facilitates a human-centered process for analysis and assessment with a human-in-the-loop. In particular, by making use of interpretable representations and visualizations, e. g., given by subnetwork visualizations of anomaly candidates, heatmap visualizations of clusters at the level of node vectors as well as comprehensive cluster diagrams. Then, this thus further provides for a transparent process and comprehensible approach.

It is important to note, that our approach tackles the novel problem of anomaly detection on bipartite multi-layer networks. There exist methods for anomaly detection in bipartite networks [4, 5], and multi-layer networks [2, 3], however, to the best of the authors' knowledge, there is no approach tackling the combined setting of anomaly detection on bipartite multi-layer networks. Compared to our previous work in [2, 3], we specifically extend on the integration of the methods on bipartite multi-layer networks, and present a framework which integrates different methods for anomaly detection, while providing distinctive and complementing perspectives for analysis in a human-centered approach. This also facilitiates interpretability and explainability of the whole approach and its respective results in anomaly detection.

Our contributions are summarized as follows:

1. We present a novel framework incorporating many-objective optimization and centrality-based analysis for identifying a set of anomalous nodes on bipartite multi-layers networks, using complementing distinctive perspectives.
2. We exemplify our proposed approach using a case study. Our context is given by a real-world dataset of socio-spatial interactions [6]. Applying our approach on the dataset, we illustrate the key steps providing simple to interpret perspectives on the respective network structures; altogether, this demonstrates the effectiveness of our approach in this real-world dataset.

The rest of the paper is organized as follows: Section 2 discusses related work. After that, Section 3 describes our approach in detail. Next, Section 4 presents and discusses our results. Finally, Section 5 concludes with a summary and outlines several interesting directions for future research.

2. Related Work and Background

In the following, we briefly introduce basic notation and background on the foundational concepts of bipartite and multi-layer networks, represented as graphs. After that, we summarize related work on anomaly detection, also considering methods for bipartite and multi-layer networks.

2.1. Bipartite and Multi-Layer Complex Networks

Formally, a bipartite Graph G is given by a triple $G = (U, V, E)$ with U, V being sets of vertices, where $U \cap V = \emptyset$. Furthermore, for the set of edges E it holds that for every edge $e \in E : e = (u, v)$ with $u \in U, v \in V$ or vice versa $u \in V, v \in U$.

For multi-layer (or multiplex) networks, we distinguish a set of layers – modeling sets of edges corresponding to relations, denoted by $E_l \subseteq E$, $l \in \{1...m\}$, where m indicates the number of layers. A multiplex network G_M can then be represented formally as follows: $G_M = (G_1, G_2, \ldots, G_l, \ldots, G_m)$, where $G_i = (V_i, E_i), V_i \subseteq V$. Figure 1 shows an illustration of a multi-layer network. Here, each network G_l is represented by the adjacency matrix A_l with the elements a^l_{ij}, for which $a^l_{ij} > 0$, if there is a positive weight of the link between the pair of nodes v_{il} and v_{jl}, $v_{il}, v_{jl} \in E_l$ in layer l, and $a^l_{ij} = 0$ otherwise. To simplify the formalization of weighted multiplex networks, we will consider only taking a positive integer value or zero with respect to the link between any pair of such nodes v_{il} and v_{jl} in layer l.

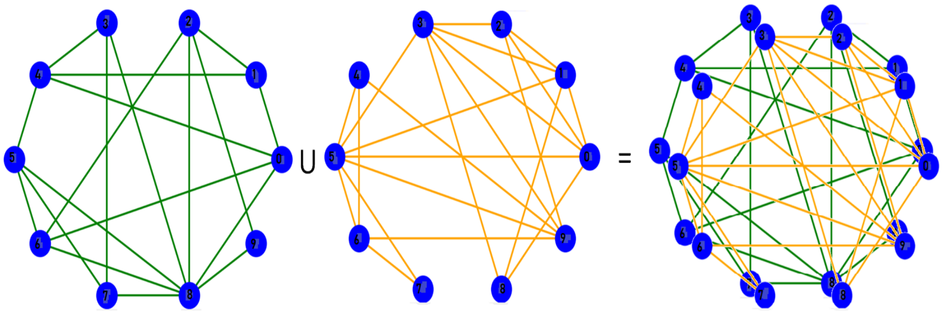

Figure 1: Illustration of a multi-layer network consisting of ten nodes, with two types of different links (see left part of the figure), as indicated by the respective different colors of the edges.

2.2. Anomaly Detection in Complex Networks

Detecting anomalies in (complex) networks data is a prominent research direction, with many practical applications. A classical definition of an anomaly [7] states it as "an outlier is an observation that differs so much from other observations as to arouse suspicion that it was generated by a different mechanism" [7]. Furthermore, for anomalies in complex networks, the general graph anomaly detection problem can be defined as follows: "Given a [...] graph database, find the graph objects [...] that are rare and that differ significantly from the majority of the reference objects in the graph" [8]. However, as we have already discussed in [2, 3, 9] in real-world networks often more complex phenomena are modeled using richer representations. For example, if there are multiple relationships between nodes, and/or multiple types of nodes, then these instantiations are difficult to capture only using simple networks/graphs.

Beyond simple graphs and multi-layer networks, we extend our view on more complex structures, i. e., towards (multi-layer) bipartite graph representations, as discussed below in more detail. In particular, our proposed approach builds on our multi-objective-optimization-based method for anomaly detection in multi-layer networks [2, 3] which we integrate for obtaining candidates for anomalous nodes – being complemented by additional methods for anomaly assessment from multiple (multi-layer) perspectives. Regarding bipartite networks, [4, 10] investigate neighborhood formation and anomaly detection in bipartite networks, for (1) identifying similar nodes (relevance) and finding anomalous ones based on their neighborhood structure. They evaluate their algorithm on synthetic data. Furthermore, [5] discuss anomaly detection on bipartite graphs in a supervised setting, exploring the bipartite structure of the networks.

We have proposed a method in [3] which employs many-objective optimization based on minimizing a given centrality measure. As already discussed, we directly integrate this method in our proposed approach. Next, [11] discuss anomaly detection in multiplex networks via a cross-layer metric indicating anomalous nodes. Furthermore, [12] focus on anomaly detection in social networks, while [13] presents a method for anomaly detection on attributed multiplex networks.

Altogether, in contrast to those approaches discussed above, we provide an unsupervised exploratory anomaly detection approach, embedded into a human-centered process, focusing on interpretable representations and visualizations. Furthermore, we focus on the novel special case of bipartite multi-layer networks, and present a novel combined approach tackling this. In a case study using a real-world dataset, we also discuss respective implications.

3. Method

Below, we first provide a bird's eye view on our proposed approach, before we discuss two of its core components, i. e., network centrality, and the applied method for many-objective optimization. Due to the limited space, we summarize correlation and k-means clustering below and refer to e. g., [14, 15] for details.

3.1. Analytical Framework – A Bird's Eye View

Below, we outline the individual steps of proposed approach:

1. We start with the bipartite multi-layer network; here, each layer is a bipartite network. We preprocess the network, constructing according bipartite projections for the individual layers of the given multi-layer network. That is, for $G = (U, V, E)$ an edge is created concerning a pair of nodes in U (V, respectively), whenever their intersection I of connected nodes in V (U, respectively), is not empty, for which we then assign $|I|$ as the new weight of that edge.

2. Given the preprocessed network, we perform many-objective optimization using minimization on the eigenvector centrality values applying the method presented in [3]. This means, that we aim to identify the Pareto-Front of the least important nodes in the network w.r.t. the nodes' eigenvector centrality.

3. Using the obtained centrality values, we perform correlation analysis on the multi-layer network for each node: We create a vector for each node consisting of the centrality values of each layer. That means, for n layers, we create a tuple $(c_1, \ldots, c_n)$ where c_i denotes the centrality value of layer i. Using these tuples, we create a correlation matrix M between all nodes, denoting the (Pearson) correlation between every pair of nodes, such that an entry m_{ij} in the matrix M indicates the correlation between node i and node j. Using a heatmap, this can then be visually inspected.

4. In addition, we perform k-means clustering on a set of nodes, e. g., the Pareto Front given the correlation matrix M. Here k is selectable by the user, e. g., in an interactive approach. For $k = 3$, for example, we can aim to cluster according to positively correlated, negatively correlated (i. e., very different) and non-correlated nodes. From each cluster, we can then calculate the average of the node centrality values. The cluster with the lowest average of the node centrality values can then be used as an indicator regarding the most anomalous set of nodes.

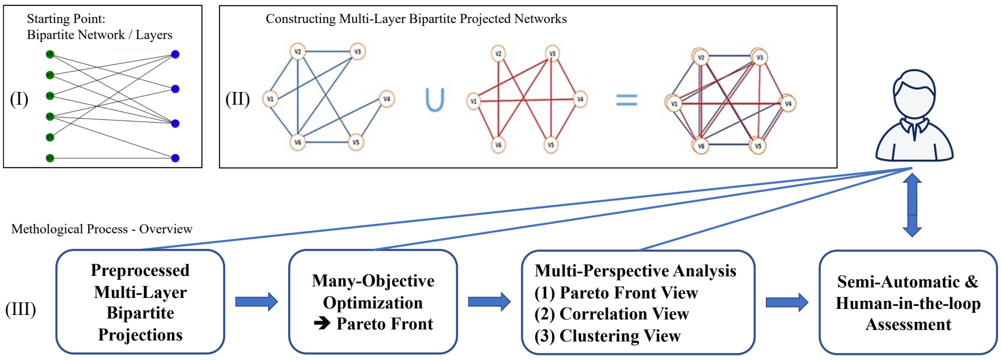
Starting Point:
Bipartite Network / Layers
(I)
Constructing Multi-Layer Bipartite Projected Networks
(II)
V1
V2
V3
V4
V5
V6
∪
=
Methological Process - Overview
(III)
Preprocessed Multi-Layer Bipartite Projections
Many-Objective Optimization ➔ Pareto Front
Multi-Perspective Analysis
(1) Pareto Front View
(2) Correlation View
(3) Clustering View
Semi-Automatic & Human-in-the-loop Assessment

Figure 2: Overview on the procedural steps of the proposed approach.

With this approach, we can identify anomaly candidates from those given multiple perspectives. First, the obtained Pareto-Front can be applied in order to find a group of nodes as candidates for anomalies – i. e., having the least importance with respect to their centrality, as we have discussed in [2, 3]. Second, the correlation analysis together with its heatmap representation provides a summarized view on the multi-layer centralities which is further condensed using the clustering approach, as the most abstracted representation. In this way, these perspectives are both complementary as well as providing different levels of abstraction. In a human-centered-approach – similar to the *Information Seeking Mantra* by Shneiderman [16] – the respective operations *overview, browse and zoom* and *details-on-demand* are then enabled by our presented perspectives.

3.2. Centrality-Based Many-Objective Optimization Approach

In network science, there are special methods for finding the most influential nodes [17] in the network using the notion of the so-called network centrality, which considers, for example, degree or the connection (structure) to other nodes. In particular, there is Eigenvector centrality, which considers the number of links from other nodes, their importance, and to how many these other nodes the respective nodes themselves point to, e. g., [18, 19]. For our proposed approach, we apply eigenvector centrality, since this precisely corresponds to our intuition for estimating the notion of connections to important nodes and/or parts of the network, which is relevant for anomaly detection, as discussed in [2, 3].

In particular, in our proposed approach, we integrate a method which we presented in [2, 3]. In summary, it estimates the centrality of all nodes on all layers of multi-layer network, followed by applying many-objective optimization with full enumeration of all layers based on a *minimation problem* to find the Pareto Front. That is, we utilize the Pareto Front as a non-dominated solution generated by many-objective optimization for *minimization* as a basis to extract a set of anomaly candidates, i. e., a set of suspected anomalous nodes from the network. For a detailed discussion, we refer to [2, 3].

4. Case Study: Results and Discussion

Below, we present the results of a case study exemplifying the presented approach in the context of a real-world socio-spatial dataset capturing human interactions [6]. Before that, we briefly summarize the applied dataset and its characteristics.

4.1. Applied Dataset: Interactions, Preferences and Perceptions

For demonstrating our approach, we provide a case study using a real-world dataset of bipartite network data. For details on the dataset, we refer to [6]. Essentially, the dataset is given by a set of bipartite networks which form a multiplex network, capturing interactions as well as preferences and perception of students attending a student career day; here, face-to-face proximity contacts between participants and companies were estimated between stationary sensors (denoting companies) and a wearable sensors worn by the participants with different signal strength thresholds, resulting in three different interaction networks. Furthermore, participants indicated *preferences* with respect to companies, as well as their *perception* which company they had really visited.

In total, for 59 participants as well as for 26 companies information is modeled. Specifically, the applied dataset [6] contains the following networks, as described in [6] in detail:

1. Socio-spatial interaction networks, taking the proximity contacts and a threshold on the received signal strength indicator (RSSI), selecting the contacts (as edges) that are stronger than the applied threshold. As individual thresholds, values of RSSI={-90, -93, -95} dBm, relating to stronger to weaker contacts were applied, resulting in the according networks. For a more detailed discussion we refer to [6].
2. A preference network [6]: An edge is created between participant p and company c whenever p selected c as a preference.
3. A perception network [6]: Here, an edge is created between participant p and company c whenever p perceived having visited c.

4.2. Case Study: Anomaly Detection in Socio-Spatial Interactions

In the following, we apply our approach and its proposed methods for identifying a set of anomalous nodes on the applied bipartite multi-layer network. Since the bipartite network consists of nodes in the *participant* as well as the *company* group, we first apply respective bipartite projections of the respective bipartite networks to those groups, respectively their nodes. The applied bipartite network data consists of five single networks, i. e., on the applied 90, 93, and 95 RSSI thresholds, as well as the perception and preference networks (corresponding to the layers $F1, \ldots F5$ in the tables below). After performing the projections, we merge the single networks into a multi-layer network. With this, we thus overall obtain two multi-layer networks, focusing on the *student* or the *company* view. With this, each multi layer network consists of the described 5 layers. In a next step, we estimate the centrality for all nodes in all layers and applying many-objective optimization through minimization. Via many-objective optimization (as our first perspective), for the student multi-layer network (59 nodes), we found 18 nodes contained in the Pareto Front as shown in Table 1; from the multi-layer company network (26 nodes), we found 6 nodes contained in the Pareto Front, as shown in Table 2.

Table 1

Pareto Front Perspective – Many Objective Centrality Optimization on the Student Multiplex Network (nodes are marked in green color). F1 is a node centrality in layer1, F2 is a node centrality in layer2, and so forth.

No	F1	F2	F3	F4	F5	Label	.level
1	0.058435	0.082584	0.336664	0.203168	0.073908	NS1	1
2	0.020032	0.008565	0.071518	0.23928	0.092498	NS11	1
3	0.00834	0.043568	0.021147	0.005369	0.204478	NS14	1
4	0.088539	0.022391	0.043383	0.227554	0.105786	NS16	1
5	0.033832	0.045463	0.044365	0.553628	0.042536	NS18	1
6	0.422805	0.008565	0.029438	0.175686	0.346587	NS19	1
7	0.108552	0.819903	0.164407	0.106101	0.050329	NS22	1
8	0.747846	0.053819	0.018351	0.635604	0.163782	NS25	1
9	0.220744	0.497118	0.153658	0.030283	0.159244	NS27	1
10	0.965811	0.333172	0.428143	0.50591	0.042415	NS28	1
11	0.179811	0.336341	0.334383	0.18536	0.111162	NS33	1
12	NA	0.711633	0.187118	0.040174	0.347735	NS57	1
13	NA	0.239367	0.991194	0.396993	0.315568	NS58	1
14	NA	0.552809	0.337983	0.542295	0.131849	NS59	1
15	NA	0.079951	0.437543	0.461424	0.175005	NS6	1
16	NA	0.017427	0.241566	0.333288	0.442596	NS7	1
17	NA	0.508283	0.015032	0.454032	0.074468	NS8	1
18	NA	NA	0.360343	0.720698	0.221801	NS9	1
19	0.239912	0.276464	0.23333	0.299679	0.393432	NS10	2
20	0.152972	0.054808	0.211162	0.409675	0.18891	NS24	2
21	0.286974	0.151257	0.093821	0.947345	0.152569	NS26	2

Table 2

Pareto Front Perspective – Many Objective Centrality Optimization on the Company Multiplex Network (nodes are marked in green color).

No	F1	F2	F3	F4	F5	Label	.level
1	0.4482975	0.1905443	0.4089364	0.3641929	0.5381588	NC1	1
2	0.6246017	0.4006532	0.2552298	0.3146327	0.5828198	NC14	1
3	0.159209	0.2224548	0.3331644	0.0195039	0.0072467	NC17	1
4	0.0625617	0.1134566	0.2490524	0.3322918	0.6890849	NC22	1
5	0.1044334	0.1031209	0.080126	0.3188913	0.6595348	NC24	1
6	0.0728712	0.0871649	0.0726725	0.4365817	0.642153	NC26	1
7	0.4553215	0.4814703	0.3724962	0.293394	0.5510949	NC12	2
8	0.6744349	0.4325141	0.382043	0.2237861	0.5357821	NC18	2
9	0.1151643	0.1267584	0.1394879	0.5209188	0.6889977	NC3	2
10	0.1979351	0.2372188	0.4171418	0.2635447	0.5853475	NC5	2
11	0.1653118	0.2630649	0.1768858	0.5340186	0.654309	NC8	2
12	0.6250001	0.679367	1	0.5199702	0.5782924	NC13	3
13	0.668332	0.5616736	0.5084357	0.4447854	0.7714854	NC19	3
14	0.7034202	1	0.8774036	0.5149992	0.5609705	NC2	3
15	0.4214642	0.4096859	0.4808022	0.5928846	0.7992488	NC21	3
16	0.2192858	0.4292074	0.7068541	0.854632	1	NC25	3
17	0.5923108	0.524016	0.4940392	1	0.6630276	N4	3
18	0.371011	0.6057899	0.9353183	0.2984418	0.7740636	NC10	3
19	0.2124341	0.4775924	0.7428361	0.4999265	0.6660691	NC23	3
20	0.4065302	0.3365142	0.216268	0.7253887	0.7394951	NC6	3
21	0.5491494	0.5007099	0.5251708	0.6648729	0.615604	NC9	3
22	0.6537326	0.7977356	0.6635758	0.8285424	0.9111193	NC11	4
23	0.5164128	0.7350974	0.8935396	0.5111472	0.6999271	NC16	4
24	0.5454632	0.6279124	0.7710405	0.6988572	0.7537753	NC20	4
25	1	0.9845064	0.9193236	0.8309337	0.8904896	NC15	5
26	0.6932629	0.9393032	0.8035952	0.8669217	0.9324355	NC7	5

Using the set of nodes in the Pareto Front as a *candidate basis* of anomalous nodes, we can apply correlation analysis as a complementing perspective (visualized as a heatmap) in order to understand the correlation and the proximity of each node compared to all other nodes in the Pareto Front better in the context of node centrality. For this, we compute the Pearson correlation values as described above. In Figure 3 we show the resulting heatmaps. The cluster perspectives are shown in Figure 4 given the respective Pareto fronts and visualization the according dimensions as discussed above.

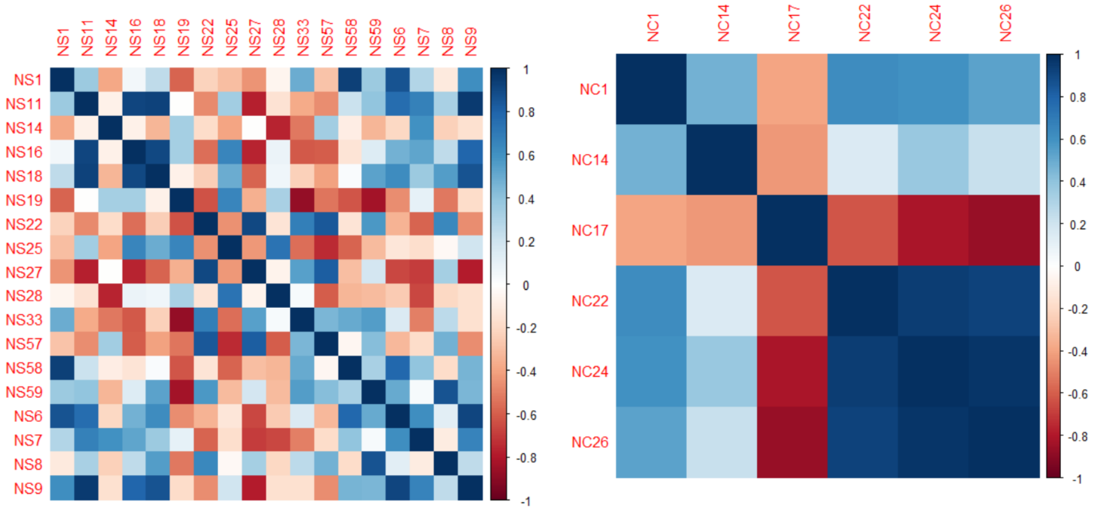

Figure 3: Correlation matrix / heatmap perspective regarding Student relations (left) as well as Company relations (right).

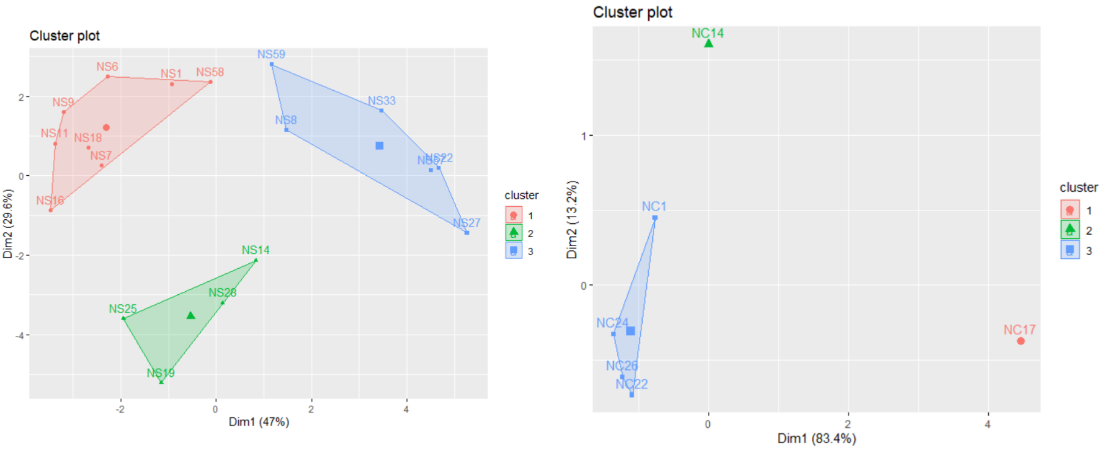

Figure 4: Cluster perspectives: Pareto front of the Student multi-layer network (left) as well as the company multi-layer network (right). Dim1 indicates the dimension of being positively correlated and Dim2 indicates the dimension being negatively correlated

As shown in Figure 3, for the correlation analysis in the student multi-layer network, we observe that the node of student 1 (NS1) is highly correlated regarding centrality (i. e., with very similar role of centrality) compared to the nodes NS58, NS6 and NS59 that are depicted in dark blue color; on the contrary, node NS1 is conflicting (i. e., with a different role of centrality) compared to nodes NS19, NS27, NS25, and NS57. Also, it is visible that NS1 has a considerable "conflict" with node NS14 (depicted in darker red color). Likewise, for the correlation analysis in the company multi-layer network, we can identify some distinctive results, regarding the set of nodes in the Pareto Front. In Figure 3, for example, we observe that the node of company 1 (NC1) is highly correlated with nodes NC14, NC22, NC24 and NC26; however, here we also observe that node NC1 is conflicting with NC17. For grouping the nodes according to their correlation, we utilize k-means clustering for further assessing interesting nodes (in the Pareto Front and/or as indicated by correlation analysis). Then, from the formed clusters, we continue by calculating the average of the centrality for each cluster and compare these centrality averages to all other clusters in order to estimate the lowest average centrality. This lowest average centrality of a cluster can then be applied in categorizing clusters of anomalous nodes in the network.

In our case, considering the nodes in the respective Pareto Fronts, for the student multi-layer network we obtained 18 nodes, consisting of 3 clusters, for which $C_1 = \{NS1, NS6, NS7, NS9, NS11, NS16, NS18 and NS58\}$, $C_2 = \{NS14, NS19, NS25, NS26\}$ and $C_3 = \{NS8, NS22, NS27, NS33, NS58, NS59\}$. From those clusters, we observe that cluster C_3 has the lowest average centrality, and therefore the nodes $NS8, NS22, NS27, NS33, NS58, NS59$ can be categorized as anomalous node candidates for the student network. Likewise, from the company multi-layer network, we obtain 3 clusters, $C_1 = \{NC17\}$, $C_2 = NC14\}$, and $C_3 = \{NC1, NC22, NC24, NC26\}$, where the lowest average centrality is found at cluster C_1.

5. Conclusions

In this paper, we proposed an approach for anomaly detection on bipartite multi-layer networks. We exemplified the approach in the context of socio-spatial interactions using a real-world dataset of human interactions. Specifically, our proposed approach integrates many-objective optimization, correlation analysis, as well as clustering for obtaining different yet complementing perspectives for anomaly detection in a human-centered way. This is facilitated, in particular, by the transparent and interpretable representations and visualizations, as we have also exemplified in our case study. For future work, we intend to extend the analysis by incorporating further methods and metrics investigating further real-world phenomena about potential anomalies [20], e. g., also including profiling [21] as well as exceptional subgraph mining techniques [22]. In addition, we aim to extend the analysis by incorporating attributed network information into the detection algorithms, e. g., [23, 24].

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