How to Handle Incomplete Knowledge Concerning Moving Objects

Nico Van de Weghe[1], Peter Bogaert[1], Anthony G. Cohn[2], Matthias Delafontaine[1], Leen De Temmerman[1], Tijs Neutens[1], Philippe De Maeyer[1], and Frank Witlox[1]

[1]Department of Geography, Ghent University, Krijgslaan 281 (S8), B-9000 Ghent, Belgium
Corresponding author: nico.vandeweghe@ugent.be
[2] School of Computing, University of Leeds, Leeds LS2 9JT, United Kingdom

Abstract. In this paper, we present a way of how to handle incomplete knowledge concerning moving objects. Our approach is based on the basic Qualitative Trajectory Calculus (QTC_B), which is a calculus for handling interactions between moving point objects (MPO's). Without elaborating on the domain of linguistics, we show that QTC_B is well-fitted to represent spatio-temporal natural language. Illustrative examples on how to deal with incomplete knowledge are presented.

1. Introduction

In the last two decades, qualitative formalisms suited to express qualitative temporal (e.g.: Freksa, C., 1992a) and spatial relationships (e.g.: Randell, D., Cui, Z., and Cohn, A.G., 1992) between entities have gained wide acceptance as a useful way of abstracting from the real world. Only in recent years, attention has been extended to applications that involve spatio-temporal data. Nevertheless, a variety of research communities have been studying movements of objects, e.g.: Wolfson, O., Xu, B., Chamberlain, S., and Jiang, L., 1998; Erwig, M., Güting, R.H., Schneider, M., and Vazirgiannis, M., 1999; Fernyhough, J.H., Cohn, A.G., and Hogg, D.C., 2000; Nabil, M., Ngu A., and Shepherd A.J., 2001; Pfoser, D., 2002. Until now, the spatio-temporal community has paid little attention to the qualitative aspects.

Apart from some limiting cases, such as a car accident and a predator catching a prey, where moving objects *meet*, mobile objects are represented by the relation *disjoint* in calculi defining topological relations, such as RCC (Randell, D., Cui, Z., and Cohn, A.G., 1992). This approach ignores some important aspects of reasoning about continuously moving physical objects. For example, given two trains on a railroad, it is of the utmost importance to know their movement with respect to each other, in order to detect whether or not they could crash in the near future. Thus, the inherent property with topological theories is that they put all *disjoint* relations into one undifferentiated set. Therefore, a challenging question remained largely unaddressed: 'How do we handle changes in movement between moving objects, if there is no change in their topological relationship?' With this in mind, and starting from the idea that the enormous complexity of interacting real world objects can be described by the relations between pairs of interacting point objects being constantly *disjoint*, the Qualitative Trajectory Calculus (QTC) was introduced by Van de Weghe Van de Weghe, N., 2004). QTC is a theory for representing and reasoning about movements of objects in a qualitative framework, able to differentiate between groups of disconnected objects. Depending on the level of detail and the number of spatial dimensions, different types of QTC were defined all belonging to QTC-Basic (QTC_B) (Van de Weghe, N., Cohn, A.G., De Tré, B., and De Maeyer, Ph., 2006) or QTC-Double Cross (QTC_C) (Van de Weghe, N., Cohn, A.G., De Maeyer, Ph., and Witlox, F., 2005). The reasoning power of QTC has been worked out, applying important reasoning techniques, such as conceptual neighbourhood diagrams (Van de Weghe, N. and De Maeyer, Ph., 2005) and composition tables (Van de Weghe, N., Kuijpers, B., Bogaert, P., and De Maeyer, Ph., 2005). In this paper, the focus is on the feasibility of QTC_B to handle incomplete knowledge[1]. Without elaborating on the domain of linguistics, we show that QTC_B is well-fitted to represent spatio-temporal natural language.

After an explanation of incomplete knowledge and how it is related to qualitative reasoning, a brief overview of QTC_B is presented. Section 4 presents illustrative examples on how to handle incomplete knowledge within the different types of QTC_B. Section 5 concludes the paper and gives some directions for further research.

2. Qualitative Reasoning and Incomplete Knowledge

Reasoning can be performed on quantitative as well as on qualitative information. According to Goyal (Goyal, R.K., 2000), a predefined unit of a quantity is used, typically when working with quantitative information. In the qualitative approach, continuous information is discretised by landmarks separating neighbouring open intervals, resulting in discrete quantity spaces (Weld, D.S. and de Kleer, J., 1990). The major idea in the qualitative approach is that only relevant distinctions are made (Clementini, E., Di Felice, P., and D. Hernandez, 1997). Thus, qualitative reasoning only studies the essence of information, represented as a small set of symbols such as the quantity space {–, 0, +} consisting of the landmark value 0 and its neighbouring open intervals $]-\infty,0[$ and $]0,\infty[$ represented respectively by the symbol – and + (Cohn, A.G. and Hazarika, S.M., 2001).

Not always everything has to be known about a situation to make inferences which are important for the specific study (Frank, A.U., 1996). Obviously in such situations sometimes information lacks for giving complete answers to queries. However, like Freksa (Freksa, C., 1992a, p.203) states, '*a partial answer may be better than no answer at all.*' By abstracting away from metrical details, qualitative representations are much more appropriate for handling such incomplete knowledge than quantitative methods (Cristani, M., Cohn, A.G., and Bennett, B., 2000).

The development of the Qualitative Trajectory Calculus (QTC) has been inspired by some important qualitative calculi in temporal and spatial reasoning, especially the temporal Semi-Interval Calculus (Freksa, C., 1992) and the spatial Double-Cross

[1] Knowledge only containing one relation in a specific calculus is called complete or fine knowledge. A union of fine relations results in incomplete knowledge.

Calculus (Freksa, C., 1992b; Zimmermann, K. and Freksa, C., 1996). Central in these theories is the specific attention to incomplete knowledge, for example produced by natural language expressions. In combination with the inherent capability of the qualitative calculi lying at the basis of QTC, one might expect that QTC ought to be able to handle incomplete knowledge.

3. The Qualitative Trajectory Calculus – Basic (QTC_B)

In this section, an informal account of the Qualitative Trajectory Calculus – Basic (QTC_B) is presented. For a formal axiomatisation, we refer to (Van de Weghe, N., 2004). Continuous time for QTC_B is assumed. In general, QTC_B compares positions of two objects at different moments in time. The movement of the first object (called *k*) with respect to the second object (called *l*) is studied by comparing the distance between *l* at the current time point (denoted *t*) and *k* during the period immediately before the current time point (denoted t^-), with the distance between *l* at *t* and *k* during the period immediately after the current time point (denoted t^+). In addition, the movement of *l* with respect to *k* is studied by comparing the distance between *k* at *t* and *l* at t^-, with the distance between *k* at *t* and *l* at t^+. QTC_{B1D} handles the qualitative movement of two constantly *disjoint* point objects restricted to 1D. Because the movement is restricted to 1D, the velocity vector of an object only has two possible directions, with the intermediate case where the object stands still. Hence, the direction of the movement of each object can be described by one single qualitative variable. Both degrees of freedom can be further subdivided according to the relative speed of the objects. This subdivision results in redundant information because the relative speed of *k* with respect to *l* is the inverse of the relative speed of *l* with respect to *k*. By reducing the continuum to the qualitative values –, 0 and +, the underlying continuous system can be described discretely. We introduce the following notation for QTC_{B1D}:

$x|t$ denotes the position of an object *x* at time *t*,
$d(u,v)$ denotes the distance between two positions *u* and *v*,
$v_x|t$ denotes the speed of *x* at time *t*,
$t_1 < t_2$ denotes that t_1 is temporally before t_2.

A movement is presented in QTC_{B1D} using the following four conditions (C):

C1. Movement of *k* with respect to the position of *l* at *t* (distance constraint):
–: *k* is moving towards *l*:

$$\exists t_1 \, (t_1 < t \wedge \forall\, t^- \, (t_1 < t^- < t \;\rightarrow\; d(k|t^-, l|t) > d(k|t, l|t))) \;\wedge$$
$$\exists t_2 \, (t < t_2 \wedge \forall\, t^+ \, (t < t^+ < t_2 \;\rightarrow\; d(k|t, l|t) > d(k|t^+, l|t)))$$

+: *k* is moving away from *l*:

$$\exists t_1 \, (t_1 < t \wedge \forall\, t^- \, (t_1 < t^- < t \;\rightarrow\; d(k|t^-, l|t) < d(k|t, l|t))) \;\wedge$$
$$\exists t_2 \, (t < t_2 \wedge \forall\, t^+ \, (t < t^+ < t_2 \;\rightarrow\; d(k|t, l|t) < d(k|t^+, l|t)))$$

0: *k* is stable with respect to *l* (all other cases):
all other cases

C2. The movement of *l* with respect to the position of *k* at *t* (distance constraint) can be described as in C1 with *k* and *l* interchanged.

C3. Relative speed of k at t with respect to l at t (which dually represents the relative speed of l at t with respect to k at t) (speed constraint):
$-$: $v_k|t < v_l|t$ $+$: $v_k|t > v_l|t$ 0: $v_k|t = v_l|t$

Accordingly, a qualitative trajectory pair can be represented by a label consisting of two or three characters, for respectively QTC_{BL1} (QTC_B of level one) only handling the changing distance between two objects and QTC_{BL2} (QTC_B of level two) also taking into account the third label representing the relative speed of both object with respect to each other. In theory, there should be 27 (3^3) *B12-relations* (QTC relations of level two in 1D). As illustrated in Fig. 1A, 10 relations are impossible (e.g. relation 2b: if object k moves towards object l and object l stands still, then $v_k < v_l$ is impossible). Therefore, we get only 17 B12-relations. Each icon in Fig. 1A represents one single relation, and therefore is called a *relation icon*, in this particular case a *B12-relation icon*. The left and the right dot of the B12-relation icon respectively represent the positions of k and l. The line segments represent whether each object can be moving towards or away from the other. A dot is filled if the object can be stationary, and open if an object cannot be stationary. The representations are no more than icons, in which we assume that k is on the left side of l.

The approach for 1D can be successfully used for higher dimensions by denoting the Euclidean distance between a pair of point objects as being the only dimension. This way 2D and even 3D movements can be reduced to 1D movements. To emphasise that we are working on 2D movements, the theory is called QTC_{B2D}. The definitions for the 2D movement are the same as the definitions for the 1D movement. In contrast with QTC_{B12}, there are 27 potential *B22-relations*, represented as 27 *B22-relation icons* in Fig. 1B. If, for example, the first character of the B22-relation is 0, then the first object stands still or can move tangentially with the second object. The icons contain line segments with the point object in the middle of it. The line segment stands for the opportunity to move to both sides of the point object. A filled dot represents the case when the object can be stationary. An open dot means that the object cannot be stationary. The icons also contain crescents with the point object in the middle of its straight border. If a crescent is used, then the movement starts in the dot and ends somewhere on the curved side of the crescent. It is important that the crescent is an open polygon: the straight boundary of a crescent is an element of another relation. Of major importance is that, in contrast to $QTC_{B1D,}$ all 27 relations are possible. The reason for this is quite straightforward. In 1D, an object can only move along a straight line. On the other hand, in 2D an object can move throughout the complete 2D space, being a higher dimension than the 1D distance. Therefore, there is a higher degree of freedom in B2D-movements compared to B1D-movements, resulting in the different number of possible relations.

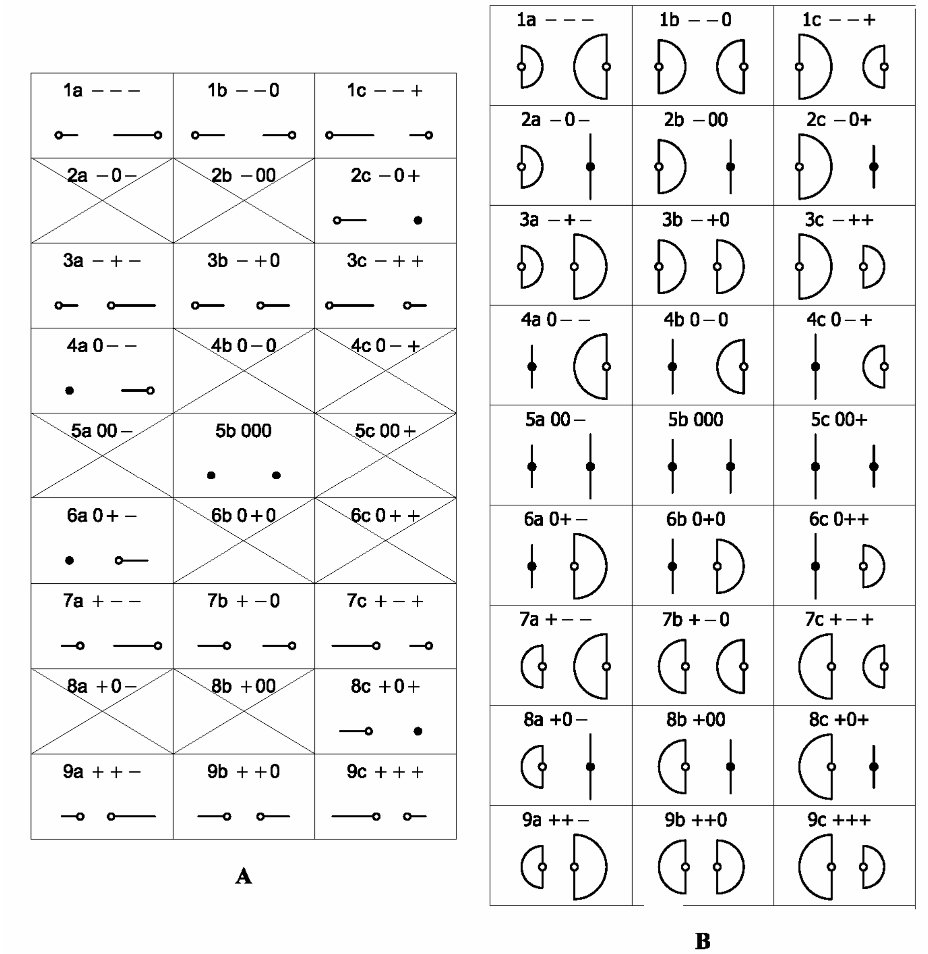

Fig. 1: (A) B12-relation icons, (B) B22-relation icons

4 Incomplete Knowledge about Moving Objects Handled Naturally

In common with qualitative spatial and temporal calculi, we need to consider that we do not always have complete knowledge about which relation holds between a pair of moving objects. In this section, illustrative examples on how to handle incomplete knowledge within QTC_B are presented. *Expressions in natural language* (*Ex*), about the movement of two objects (*k* and *l*) with respect to each other, are considered. We determine which QTC_B relations hold for each particular expression. We use the standard notation for *implication* and *equivalence*:

$a \rightarrow b$: if *a*, then *b*

$a \leftrightarrow b$: if and only if *a*, then *b*

as well as the standard notations for the following set operations:

$a \setminus b$: *a* minus *b*

$a \cap b$: intersection of *a* and *b*

4.1. From Fine to Incomplete Knowledge and Vice Versa

This example starts from *Ex1* forming fine knowledge concerning moving objects, and relaxes the constraints in order to get incomplete knowledge (*Ex2*, *Ex3* and *Ex4a*). Thereafter, the inverse approach is discussed. Starting from several incomplete constraints (*Ex4a*, *Ex4b*, *Ex4c*, and *Ex4d*), fine knowledge will be generated by the intersection of the incomplete solutions. The example is worked out for 1D and 2D.

4.1.1. From Fine to Incomplete Knowledge

Ex1: *k* is moving towards *l*, which in turn is moving away from *k*, both objects moving along the same straight line and having the same speed.

$$Ex1 \rightarrow (-\,+)_{B11} \text{ and } Ex1 \leftrightarrow (-\,+\,0)_{B12}$$

QTC_{B11} does not give full detail, because $(-\,+)_{B11}$ also contains situations where *k* and *l* have a different speed. Therefore, it is more appropriate to work at level two, which incorporates the speed variable.

$$Ex1 \rightarrow (-\,+)_{B21} \text{ and } Ex1 \rightarrow (-\,+\,0)_{B22}$$

At first sight, QTC_{B2D} and QTC_{B1D} give the same result. However, there is only an implication ($\rightarrow$) between *Ex1* and $(-\,+\,0)_{B22}$, because $(-\,+\,0)_{B22}$ does not consider the restriction in *Ex1* that both objects are moving along the same straight line, which was an implicit restriction for movements in 1D.

Ex2: *k* is moving towards *l*, which in turn is moving away from *k*, both objects moving along the same straight line.

$$Ex2 \leftrightarrow (-\,+)_{B11} \text{ and } Ex2 \leftrightarrow (-\,+\,A^2)_{B12}$$

The only difference between *Ex1* and *Ex2* is the speed constraint, which is not given in *Ex2*. In contradiction to *Ex1*, we have in *Ex2*: if $(-\,+)_{B11}$ is true, then *Ex2* must be true. *Ex2* is thus totally covered by QTC_{B11}. The difference between *Ex1* and *Ex2* has perhaps more implications for QTC_{B12}, since $(-\,+\,A)_{B12}$ consists of a disjunction of solutions:

The following statement is false: $Ex2 \rightarrow a$ (with $a \in (-\,+\,A)_{B12}$)

The following statement is true: $a \rightarrow Ex2$ (with $a \in (-\,+\,A)_{B12}$)

$$Ex2 \rightarrow (-\,+)_{B21} \text{ and } Ex2 \rightarrow (-\,+\,A)_{B22}$$

---

[2] A qualitative variable *A* (*B*, *C*, …) stands for the set {–, 0, +}

Ex2 represented in QTC_{B22} gives no extra information compared to *Ex2* represented in QTC_{B21} since the third character of QTC_{B22}, differentiating QTC_{B22} from QTC_{B21}, can have all qualitative values. Note that there is only an implication ($\rightarrow$) between *Ex2* and $(-\,+)_{B21}$, because $(-\,+)_{B21}$ does not consider the restriction in *Ex2* that both objects are moving along the same straight line, which was an implicit restriction for movements in 1D. The same applies to the implication between *Ex2* and $(-\,+\,A)_{B22}$.

Ex3: *k* is moving towards *l*, which in turn is moving away from *k*.

$$Ex3 \leftrightarrow (-\,+)_{B11} \text{ and } Ex3 \leftrightarrow (-\,+\,A)_{B12}$$

Compared to *Ex2*, the objects do not need to move along a straight line. However, this constraint is straightforward, since we are working in 1D.

$$Ex3 \leftrightarrow (-\,+)_{B21} \text{ and } Ex3 \leftrightarrow (-\,+\,A)_{B22}$$

In contrast to the 1D movement, the constraint that both objects have to move on the same straight line (or in 1D) is important in 2D. In *Ex3*, this constraint is deleted, which results in an important extension of the solution set. This extension can be seen in the formulae; on the one hand one gets an implication between *Ex2* and the B2D relations, on the other hand one gets an equivalence between *Ex3* and the B2D relations. This extension can be easily seen by comparing the relation icons for $(-\,+\,A)_{B21}$ in Fig. 1A with those for $(-\,+\,A)_{B22}$ in Fig. 1B.

Ex4a: *k* is moving towards *l*.

$$Ex4a \leftrightarrow (-\,A)_{B11} \text{ and } Ex4a \leftrightarrow (-\,A\,B)_{B12}$$

This expression does not state whether *l* is moving. Because this expression is less complete than *Ex3*, it is obvious that we cannot distinguish QTC_{B11} from QTC_{B12}. However, note in QTC_{B12} that when *l* is not moving, only $(-\,0\,+)_{B12}$ holds, because $(-\,0\,-)_{B12}$ and $(-\,0\,0)_{B12}$ are impossible in 1D.

$$Ex4a \leftrightarrow (-\,A)_{B21} \text{and } Ex4a \leftrightarrow (-\,A\,B)_{B22}$$

As could be expected, there is no difference between for QTC_{B11} and QTC_{B21}.

4.1.2. From Incomplete to Fine Knowledge

Now, let us start from four expressions (*Ex4a*, *Ex4b*, *Ex4c*, and *Ex4d*), which together form the fine compound expression *Ex1*:

Ex4a: *k* is moving towards *l*.

$$Ex4a \leftrightarrow (-\,A)_{B11}$$
$$Ex4a \leftrightarrow (-\,A\,B)_{B12}$$
$$Ex4a \leftrightarrow (-\,A)_{B21}$$

$Ex4a \leftrightarrow (- A\ B)_{B22}$

Ex4b: *l* is moving away from *k*.

$Ex4b \leftrightarrow (A\ +)_{B11}$
$Ex4b \leftrightarrow (A + B)_{B12}$
$Ex4b \leftrightarrow (A\ +)_{B21}$
$Ex4b \leftrightarrow (A + B)_{B22}$

Again, there is no difference between the representations of QTC_{B11} and QTC_{B21}, and those of QTC_{B12} and QTC_{B22}.

Ex4c: *k* and *l* are moving along the same straight line.

$Ex4c \leftrightarrow (A*$[3] $B*)_{B11}$
$Ex4c \leftrightarrow (A*\ B*\ C)_{B12}$
$Ex4c \rightarrow (A*\ B*)_{B21}$
$Ex4c \rightarrow (A*\ B*\ C)_{B22}$

Because it is specified that both objects are moving, neither of the two objects may stand still.

Ex4d: *k* and *l* have the same speed.

$Ex4d \rightarrow (A*\ B*,\ 0\ 0)_{B11}$
$Ex4d \leftrightarrow (A*\ B*\ 0,\ 0\ 0\ 0)_{B12}$

It is not specified whether the speed has to be higher than zero. Therefore, $(0\ 0)_{B11}$ and $(0\ 0\ 0)_{B12}$ are possibilities. However, since the speed of both objects has to be the same, it is impossible to have a pair of objects where only one object is moving.

$Ex4d \rightarrow (A\ B)_{B21}$ and $Ex4d \leftrightarrow (A\ B\ 0)_{B22}$

In contrast to QTC_{B11}, every relation is possible in QTC_{B21}, which is a direct result of specifications concerning the exclusive B22-relations.

4.1.3. Overall Result

The intersection of the four solution sets of the expressions *Ex4a*, *Ex4b*, *Ex4c*, and *Ex4d*, gives $(-\ +)_{B11}$ and $(-\ +\ 0)_{B12}$. One can state that the intersection of the solution sets of the components of a compound expression is the same as the solution set of the compound expression.

---

[3] A qualitative variable *A** (*B**, *C**, ...) stands for the set {−, +}

$$(-\,A)_{B11} \cap (A\,+)_{B11} \cap (A^*\,B^*)_{B11} \cap (A^*\,B^*,\,0\,0)_{B11} = (-\,+)_{B11}$$
$$(-\,A\,B)_{B12} \cap (A + B)_{B12} \cap (A^*\,B^*\,C)_{B21} \cap (A^*\,B^*\,0,\,0\,0\,0)_{B12} = (-\,+\,0)_{B12}$$

The intersection of the four solution sets for QTC_{B2D} of each expression is respectively $(-\,+)_{B21}$ and $(-\,+\,0)_{B22}$. Again (cf. QTC_{B1D}), the intersection of the solution sets of the components of a compound expression is the same as the solution set of the compound expression.

$$(-\,A)_{B21} \cap (A\,+)_{B21} \cap (A^*\,B^*)_{B21} \cap (A\,B)_{B21} = (-\,+)_{B21}$$
$$(-\,A\,B)_{B22} \cap (A + B)_{B22} \cap (A^*\,B^*\,C)_{B22} \cap (A\,B\,0)_{B22} = (-\,+\,0)_{B22}$$

4.2. How Many Objects Are Moving?

If we say that an object is moving, we can have interpretation problems; do we mean that at least one of the objects is moving, or do we mean that exactly one object is moving? This ambiguity can be overcome by QTC_B.

Ex5: At least one of the objects is moving.

$$Ex5 \leftrightarrow (A\,B) \setminus (0\,0)_{B11}$$
$$Ex5 \leftrightarrow (A\,B\,C) \setminus (0\,0\,0)_{B12}$$
$$Ex5 \rightarrow (A\,B)_{B21}$$
$$Ex5 \rightarrow (A\,B\,C)_{B22}$$

Due to this expression, it is possible that only one object is moving or it could be that both objects are moving. Note that for QTC_{B21} and QTC_{B22}, the relations where the first and the second character are zero do not need to be excluded since objects can move tangentially when both the first and the second are 0 in 2D.

Ex6: Exactly one of the objects is moving.

$$Ex6 \leftrightarrow (A^*\,0{,}0\,B^*)_{B11}$$
$$Ex6 \leftrightarrow (A^*0\,B,\,0\,A^*\,B)_{B12}$$
$$Ex6 \rightarrow (A\,0,\,0\,A)_{B21}$$
$$Ex6 \rightarrow (A\,0\,B,\,0\,A\,B)_{B22}$$

Note again the subtle difference between QTC_{B1D} and QTC_{B2D}. In QTC_{B1D}, an object can only move when a character is different from 0. In QTC_{B2D}, an object can move if a character is 0. Note that $(A^*B^*)_{B21}$ is impossible since here both objects are moving.

5. Conclusion

Based on several illustrative examples, the ability of handling incomplete knowledge and natural language expressions within QTC_B is studied. In further research, the possibilities of QTC-Double Cross (QTC_C) to handle incomplete knowledge will be discussed. Since QTC_C considers additionally the direction in which an object is moving with respect to the line segment between the two objects, this calculus is more expressive and will involve more complex reasoning. Note for example that, in contrast with $QTC_{C2D,}$ it is not possible in QTC_{B2D} to denote whether two objects are moving along the same straight line. This will be possible. In the future, we will continue to explore the bridge between natural language, perception and formal ontologies of moving objects.

References

Clementini, E., Di Felice, P., and D. Hernandez, 1997, Qualitative representation of positional information, *AI*, 95(2), 317-356.

Cohn, A.G. and Hazarika, S.M., 2001, Qualitative spatial representation and reasoning: an overview, *Fundamenta Informaticae*, 46(1-2), 1-29.

Cristani, M., Cohn, A.G., and Bennett, B., 2000, Spatial locations via morpho-mereology, *Conf. on Principles of Knowledge Representation and Reasoning*, 15-25.

Erwig, M., Güting, R.H., Schneider, M., and Vazirgiannis, M., 1999, Spatio-temporal data types: an approach to modelling objects in databases, *Geoinformatica*, 3(3), 269-296.

Fernyhough, J.H., Cohn, A.G., and Hogg, D.C., 2000, Constructing qualitative event models automatically from video input, *Image and Vision Computing*, 18(2), 81-103.

Frank, A.U., 1996, Qualitative spatial reasoning: cardinal directions as an example, *International Journal of Geographical Information Science*, 10(3), 269-290.

Freksa, C., 1992a, Temporal reasoning based on semi-intervals, *AI*, 54, 199-227.

Freksa, C., 1992b, Using orientation information for qualitative spatial reasoning, *Int. Conf. on Theories and Methods of Spatio-Temporal Reasoning in Geographic Space*, *LNCS*, (639), 162-178.

Goyal, R.K., 2000, *Similarity Assessment for Cardinal Directions between Extended Spatial Objects*, PhD thesis, USA, University of Maine, 167 pp.

Nabil, M., Ngu A., and Shepherd A.J., 2001, Modelling and retrieval of moving objects, *Multimedia Tools and Applications*, 13(1), 35-71.

Pfoser, D., 2002, Indexing the trajectories of moving objects, *IEEE Data Engineering Bulletin*, 25(2), 3-9.

Randell, D., Cui, Z., and Cohn, A.G., 1992, A spatial logic based on regions and connection, *Conf. on Knowledge Representation and Reasoning*, 165-176.

Van de Weghe, N. and De Maeyer, Ph., 2005, Conceptual neighbourhood diagrams for representing moving objects, *Int. Workshop on Conceptual Modeling for Geographic Information System*, *LNCS*, (3770), 228-238.

Van de Weghe, N., Cohn, A.G., De Tré, B., and De Maeyer, Ph., 2006, A qualitative trajectory calculus as a basis for representing moving objects in geographical information systems, *Control and Cybernetics*. (Accepted for publication)

Van de Weghe, N., 2004, *Representing and Reasoning about Moving Objects: A Qualitative Approach*, PhD thesis, Belgium, Ghent University, 268 pp.

Van de Weghe, N., Cohn, A.G., De Maeyer, Ph., and Witlox, F., 2005, Representing moving objects in computer-based expert systems: the overtake event example, *Expert Systems with Applications*, 29(4), 977-983.

Van de Weghe, N., Kuijpers, B., Bogaert, P., and De Maeyer, Ph., 2005, A qualitative trajectory calculus and the composition of its relations, *Int. Conf. on Geospatial Semantics*, *LNCS*, (3799), 60-76.

Weld, D.S. and de Kleer, J., 1990, *Readings in Qualitative Reasoning about Physical Systems*. Morgan Kaufmann, 720 pp.

Wolfson, O., Xu, B., Chamberlain, S., and Jiang, L., 1998, Moving object databases: issues and solutions, *Int. Conf. on Scientific and Statistical Database Management*, 111-122.

Zimmermann, K. and Freksa, C., 1996, Qualitative spatial reasoning using orientation, distance, and path knowledge, *Applied Intelligence*, 6(1), 49-58.