Mathematical Models of Supporting the Solution of the Algebra Tasks in Systems of Computer Mathematics for Educational Purposes

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Abstract. The article describes systems of computer mathematics for educational purposes with intelligent properties oriented to support practical activities of users - students and teachers. Systems of computer mathematics for educational purposes are oriented, first of all, in support of practical activity of users - students and teachers for solving educational mathematical tasks. The article presents mathematical models and methods for solving algebra tasks in systems of computer mathematics for educational purposes. In the work: - functional requirements for activity environments supporting the solving of learning tasks in algebra in systems of computer mathematics for educational purposes are developed; - the mathematical model of learning mathematical task is defined within the framework of the mathematical model of the training module and the construction of the corresponding algorithms of computer algebra; - mathematical models of methods for supporting the solving of learning mathematical task in systems of computer mathematics for educational purposes are constructed.

Keywords: Systems of computer mathematics for educational purposes, computer software, support of learning processes.

1 Introduction

Today there is an objective need of a society in specialists with a high level of mathematical knowledge, therefore the innovative processes in mathematical education become a catalyst for the qualitative training of specialists. One of the functions of innovation is the use of systems of computer mathematics for educational purposes (SCMEP) in the process of teaching mathematics.

SCMEP is a programmed educational system for exact and natural educational disciplines that uses mathematical models and methods of subject areas based on technologies of symbolic transformations and computer algebra methods [1-3].

Quality assimilation of mathematical knowledge largely depends on the practical mathematical activity of the student. The educational purpose of the student's practical work is to master the methods of solving tasks and construct the course of solving the educational task. Therefore, SCMEP should support the process of solving the educational mathematical problem [4-6].

The general theoretical and methodological foundations, the formulation of functional requirements for SCMEP, and the development of a model of SCMEP as a system for supporting learning processes based on the analysis of actual forms and peculiarities of learning processes in precise disciplines are described in [7].

The purpose of this work is to study mathematical models and methods for solving educational problems in algebra in SCMEP.

2 The outline of the problem

Today there is an objective need of a society in specialists with a high level of mathematical knowledge, therefore the innovative processes in mathematical education become a catalyst for the qualitative training of specialists. One of the functions of innovation is the use of systems of computer mathematics for educational purposes (SCMEP) in the process of teaching mathematics.

The researches were carried out in the process of the scientific and technical works on government contracts and state program "Information and communication technologies in education and science" and based on practical experience gained during the development of SCMEP, which were carried out under the order of the Ministry of Education and Sciences of Ukraine (copyright certificates [8-12]):

- Program-methodical complex «TerM VII» to support of practical learning mathematical activity.
- Software tool "Library of Electronic Visual Aids Algebra 7-9 grade for secondary schools in Ukraine."
- Pedagogical software tool "Algebra, Grade 7".
- Software tool for educational purposes "Algebra, Grade 8".

A systematic approach was proposed to construct SCMEP models and program implementations for the development of the SCMEP listed above, which could be used to build a broad class of educational and scientific programming systems.

The concept of SCMEP on algebra consists in the integrated automation of the support of the learning process, the main participants of which are the teacher as a professional, the carrier of knowledge both in the field of didactics and teaching methods, both in the subject area, and the student as an object of study. SCMEP's main goal is to provide comprehensive training.

The system should ensure the effective conduct of the educational process as a whole, supporting the interaction of the teacher and student.

From the set of (electronic) didactic materials on algebra you can select a group of materials that contain educational information. This is a textbook, a taskbook, a_library of visual aids, a Guide, and so on.

Each of these teaching materials is intended to provide a defined form of study, a type of educational activity and is oriented towards its user category. The functionality of these modules is described in [6, 13, 14].

Practical mathematical activity of the student is the main form of educational activity in the study of algebra. In the process of this kind of activity, the student uses the theoretical knowledge acquired at previous stages of training to solve practical tasks. In this case, the educational purpose is to build the course of solving the learning task, and not to receive an answer.

Information's support for solving process of the learning task in SCMEP is possible provided that its solution takes place in a specialized program module - the activity environment (AE). Mathematical methods of their construction are devoted [15-19]. Chart of classes of the component "Means of training" in SCMEP is presented in Fig. 1.

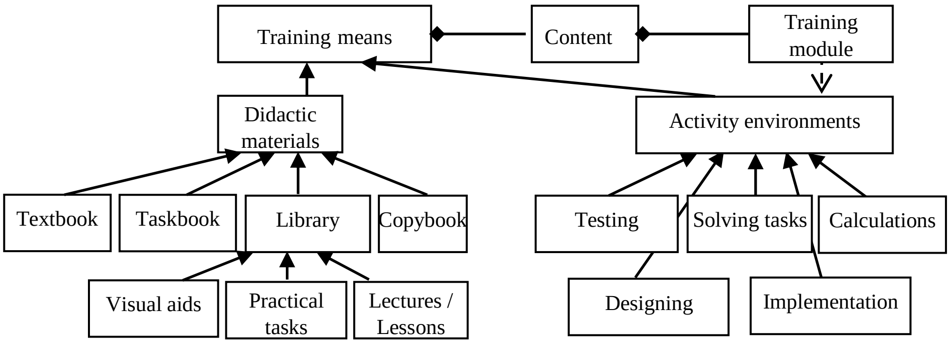

Fig. 1. Chart of classes of the component "Training means" (SCMEP)

Consider the functionality of SCMEP's activity environments for the process of solving of learning mathematical tasks.

The medium of solving tasks. The main function of the activity environment for solving tasks (MST) is support for the process of solving the LMT in various aspects of this support (different modes of work MST) (Fig. 2).

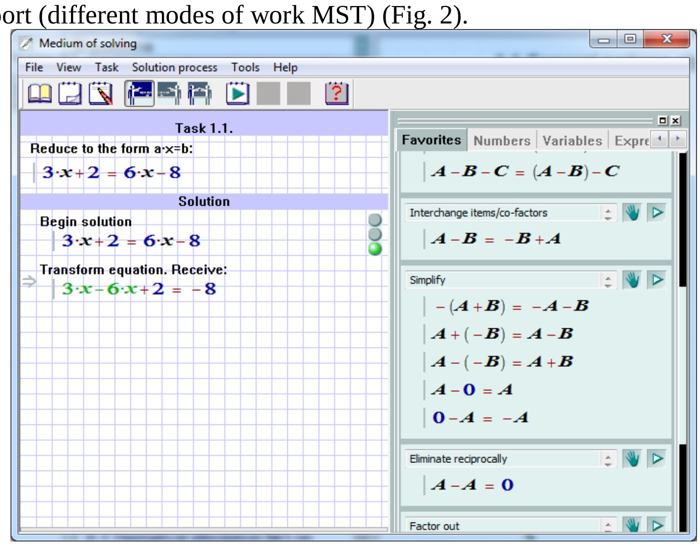

Fig. 2. View of the AE "Solving Environment" with a loaded task

List the main functional requirements for MST:

- ability to export a training task from a taskbook or notebook;
- possibility to save the progress of solving a partially or completely solved_problem in a notebook;
- possibility of entering from the keyboard the conditions of the learning task of one of the standard types;
- possibility of choosing one of the modes for solving the learning task;
- possibility to review the course of solving the learning task.

One of the most important aspects of maintaining a student's practical mathematical activity is checking the correctness of his actions at various stages of solving the task - from the stage of constructing a mathematical model to the stage of checking the correctness of the course of the solution or the answer.

Second, no less important aspect of support is automation of routine calculations-related actions.

The third aspect is to provide the student with a convenient system of prompts at various stages of solving the task in the form of generating a mathematical model of a task, the course or step of its solution, and the answer.

One of the aspects of the teacher's activity support is to check the correctness of the course of the task. The system must verify the correctness of the solution of the task previously solved by the student (test check mode).

Another aspect of supporting the teacher's activity is the automation of testing students' knowledge. Testing is the most commonly used knowledge control technology in educational systems. However, in practice, testing systems are aimed at checking declarative knowledge. The problem of testing procedural knowledge needs to be resolved.

The special activity *testing environment* in SCMEP should verify the knowledge of basic mathematical rules and formulas (special testing using mathematical tests).

A characteristic feature of the mathematical test task is that the answer should be given in the form of a mathematical expression, and the verification of the answer is to verify the semantic correctness of this expression (Fig. 3).

The main system tasks of the testing environment - the task of generating test tasks and the task of checking the correct answer. Their decision is set out in [19, 20].

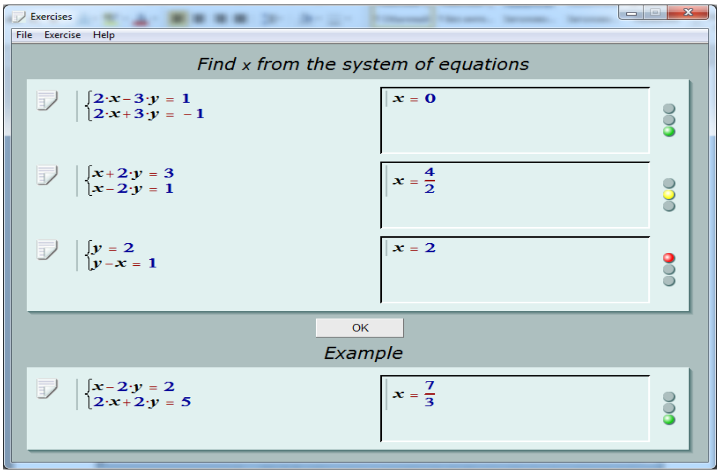

Fig. 3. View of the AE “Testing Environment” with a loaded test

Graphs. An important methodological role in the study of mathematics are graphic construction. In the course of school algebra, the topic "Charts of function" is one of the main cross-cutting themes of the course. Therefore, the activity *environment of graphic constructions* should be included in the structure of the SCMEP.

SCMEP must also provide the user with the appropriate mathematical tools (calculator, computing environment, etc.).

The main function of the *Solver environment* is the support for the solution of the learning mathematical task (LMT) - the generation of the answer or the course of the solution and the answer (Fig. 4). The environment should also support algebraic and arithmetic calculations, including approximate calculations.

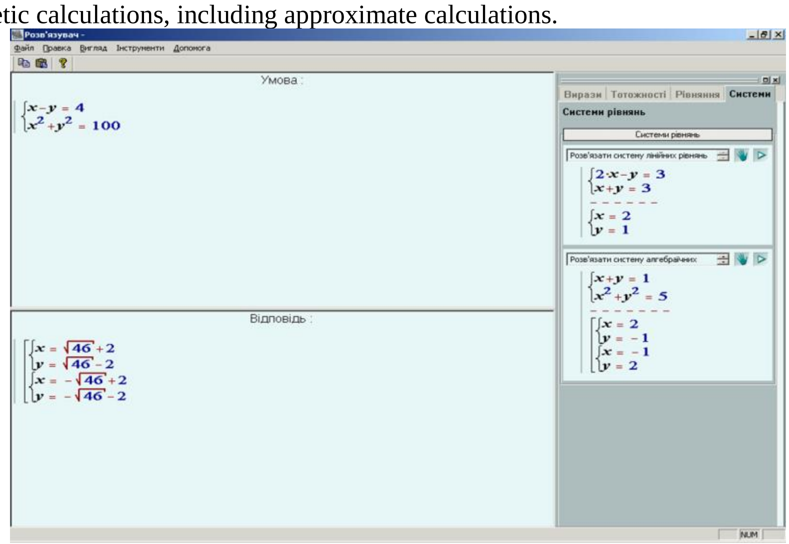

Fig. 4. General view of the window of AE “Solver”

Let's consider the main system problems of the solution environment - the problem of support for the solution of LMT. Different aspects of such support determine different ESP regimes. Let's list these tasks: - the task of generating, verifying and automatically executing transformations - the steps of the decision of LMT.

The implementation of the tasks of supporting the decision of LMT requires the definition of a mathematical model of LMT in the framework of the mathematical model of the training module and the construction of the corresponding algorithms of computer algebra.

Implementation of the automatic execution of transformations must meet the requirements of consistency, completeness and methodological correctness. To meet these requirements, a list of elementary transformations of mathematical objects - models of LMT must be carefully designed.

The problem of this study can be formulated as a study of functional requirements for activity environments supporting the resolution of educational tasks in algebra in SCMEP and the construction of mathematical models and methods for supporting the solving of algebraic tasks in SCMEP.

3 Informatics models of support the process learning of algebra

3.1 Models of the didactic content of SCMEP

Let's consider the models of presentation of procedural mathematical knowledge in SCMEP.

The main structural unit of the information provision of the learning process is the training module of discipline.

Structural logic scheme (SLS) of the module on mathematical discipline determines:

- content of teaching materials;
- signature of the training module;
- list of mathematical models of the training module;
- list of elementary transformations of the models of the training module;
- list of types of educational tasks of the training module.

Let's give a formal definition of the training module.

The training module SD is defined by the quad

$$SD = < \Sigma, MM, ET, Task >$$

where $\Sigma = \Sigma_{SD}$ is the own signature of the training module, $MM = MM_{SD}$ - the list of models of the training module, $ET = ET_{SD}$ - the list of own elementary transformations of the training module, $Task = Task_{SD}$ - the class of learning tasks, defines the content SD. In addition to the own elements of the definition of the module SD, the corresponding elements of the basic modules are used in the formulation of signatures, models, their transformations and educational tasks. If - $SD_1,..., SD_k$ the list of training modules in the SLS discipline, $SD_1,..., SD_k \to SD$, full signatures $\overline{\Sigma}_{SD}$ and complete lists of elementary transformations $\overline{ET}_{SD}$, permissible in this SD, are determined by the formulas

$$\overline{\Sigma}_{SD} = \bigcup_{j=1}^{k} \overline{\Sigma}_j \quad \overline{ET}_{SD} = \bigcup_{j=1}^{k} \overline{ET}_j$$

The mathematical models MM_{SD} of the module are determined by the constructions specific to this module, in terms of the signatures $\overline{\Sigma}_{SD}$ and the mathematical models of the modules $SD_1,..., SD_k$.

Below we consider the design of the definition of algebraic models.

Educational tasks are defined in terms of mathematical models MM_{SD}, relations of dependence φ between models and their elements (condition of the task) and the question of the task Q . Thus, the task $P \in Task_{SD}$ is determined by the triple

$$P =< MM,\varphi,Q >$$

The area of application of models SD in our work is the school algebra.

The corresponding SCMEP can be based on a single SLS as a single system based on the notion of an algebraic object (AO) and an equivalence output.

3.2 The model of the educational mathematical task

SCMEP, which is considered in this study, support the following mathematical disciplines:

Algebra, 7-9 grades of secondary schools.

Algebra and principles of analysis, 10-11 grades of secondary school.

Each of these mathematical disciplines has its own class of LMT. Of course, mathematical models of LMT, depending on the mathematical discipline, have their own peculiarities. The development of SCMEP is, in particular, in the precise definition of the training modules that are supported by the system.

Mathematical model of the LMT *of the school course of algebra*.

LMT is a quantum formula for the application logic of predicates $F(x_1,..., x_n)$. The atomic predicates of the formula $F(x_1,..., x_n)$ are predicates of equality and denial of equality $(\neq)$, strict and non-strict order, as well as other atomic predicates, the definitions of which are carried out within the framework of the corresponding training module. Logical connections - conjunction and disjunction.

For example, the task: «Find positive solutions of the equation $\frac{2 \cdot x+1}{x-2} = \frac{x+2}{x}$ » has a mathematical model: $F(x) = (\frac{2 \cdot x+1}{x-2} = \frac{x+2}{x}) \,\&\, (x > 0) \,\&\, (x = ?)$.

We give the formal definitions.

Definition 1. Domain signature Σ is called pair $< \Sigma_O, \Sigma_p >$, where Σ_o - signature of operations, Σ_p - signature of predicates.

Definition 2. A primitive AO is called the carrier element of a corresponding multistage algebraic system or a variable.

Definition 3. An atomic AO is called the term from primitive AO in the signature Σ_o.

Definition 4. A structured AO (SAO) in the signature Σ is a non-quantum formula for the applied logic of predicates $F(x_1,..., x_n)$ with equality. The set of variables $\{x_1,..., x_n\}$ is called the coordinate set of SAO.

The signature Σ_p atomic predicates AO $F(x_1,..., x_n)$, in addition to predicates $=, \neq$, may contain other atomic predicates as defined in this module. For school algebra these predicates are $\leq, <, \geq, >$. SAO $F(x_1,..., x_n)$ is interpreted, depending on the type of LMT, either as universal, or as existential.

Training modules of algebra 7-9 classes use:

- Rational numbers in the form of ordinary fractions, mixed fractions, and decimal periodic fractions (base module in SLS).
- Algebraic operations $x + y, x - y, x * y, x / y, |x|, \sqrt{x}$
- Atomic predicates $F = G, F \neq G, F < G, F \leq G, F > G, F \geq G.$

All tasks of the course of algebra 7-9 can are represented in this signature [21].

Similarly, modules of algebras of grades 10-11 can be presented. Modules of classes 7-9 of algebra are basic for them.

3.3 Algebraic types of educational mathematical tasks

The algorithms of specific SCMEP tasks, which are described below, depend on the subject domain, the type of LMT, its dimension Dim (the number of variables), as well as the algebra of the condition of the task $Cond$ and the solution algebra Sol.

For example, for the task of finding positive solutions of the equation $\frac{2 \cdot x + 1}{x - 2} = \frac{x + 2}{x}$ it is $Cond = Rat(x),\ Dim = 1,\ Sol = FiniteNumSet(Rad)$, and for the task of finding the complex solutions of the equation $x^2 + x + 1 = 0$ - $Cond = Rat[x],$ $Dim = 1,\ Sol = FiniteNumSet(Rad(Comp(Rat)))$.

We emphasize the important difference: in the first task the solution belong to the field, which is the extension of the field Rat rational numbers with square roots of positive integers, and in the second task the solution belong to the extension of the field with square roots of all integers (including from capacious). This difference leads to the use of various methods for solving these tasks.

In school algebraic tasks on computing the values of numerical expressions $Sol = Rad$ (algebra of radicals). In tasks for the simplification of rational expressions $Sol = Rad(x_1,..., x_n)$, where $(x_1,..., x_n)$ is a set of variables of the task. In the tasks on proof $Sol = Bool$. In tasks on solving equations, inequalities of one variable

$Sol = NumSet(Rad)$ (numerical set). Finally, in tasks on systems of algebraic equations from *n* variables $Cond = [Rat(x_1,..., x_n)]^n$, $Sol = FiniteNumSet(Rad)^n$

The mathematical model of the algebraic type of the system of linear inequalities is depicted in Fig. 5.

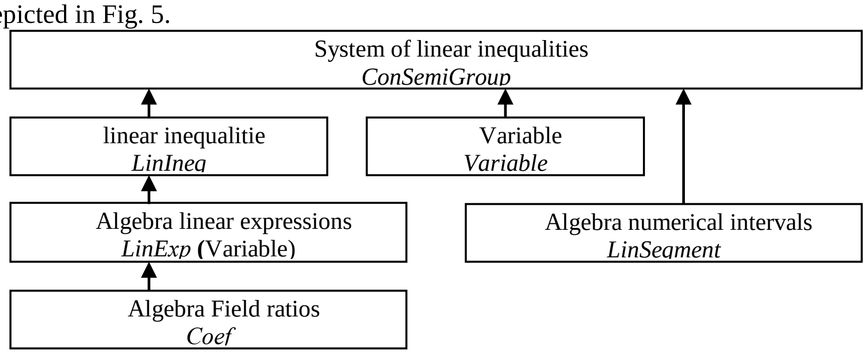

Fig. 5. Algebraic type of system of linear inequalities

Definition 5. An algebraic type of the LMT is called its algebraic model, which defines it as a structured AO with definitions of algebraic types of its elements.

Algebraic types of school algebraic tasks supported by SCMEP are determined by the algebra of the conditions $Cond$, the algebra of solutions Sol and dimensionality.

4 Specific SCMEP tasks

4.1 The concept of logical output as a model of step-by-step decision of LMT

The LMT decision in SCMEP uses an equivalence output - a derivation based on the properties of equality and the rule of replacing equal

$$\frac{F(A), A = B}{F(B)}$$

This conclusion is presented in the form of a sequence of triples (F, t, F'), where F is the ascending logical formula of the SAO, $t \in ET_{SD}$ - elementary transformation, F' - the transformation of the logical SAO formula.

Equivalent output is

$$(F_0, t_0, F_1), (F_1, t_1, F_2), ..., (F_{k-1}, t_{k-1}, F_k), \ F_{j+1} = t_j(F_j) \qquad (*)$$

In the SCMEP user interface, the output is presented as a sequence $F_0, F_1, F_2, ..., F_{k-1}, F_k$.

4.2 The tasks of supporting the step-by-step solution of LMT

The problem of supporting the LMT turnout solution is central to the implementation of the SCMEP. As we noted above, it is solved by means of activity environments. Let's list the main specific tasks that must be implemented to solve this problem.

- *The task of verifying the model of* LMT. A wide class of LMT requires a student to independently build a model of LMT. These are, for example, text problems of the school algebra course, which are solved by equations or equation systems. An incorrectly compiled mathematical model leads to an incorrect task solution, even if the model is correctly solved. The purpose of the verification is to verify the correctness of the model constructed by the student, that is, its equivalence to the correct model.
- *The task of verification of the step of the decision of* LMT. By resolving the LMT in steps, the user may make mistakes at every step of the solution. The task of verifying the step of the solution is to verify the correctness of the transformation of the model executed by the user at this step.
- *The task of verification of the course of the decision of* LMT. Having solved the LMT when carrying out the control work, the student must present the course of the decision of the LMT for verification. Verification is carried out by the teacher at his workplace. If an error has been found during the solution, this should not lead to the end of the check. The task is to identify all errors during the solution and fix them.
- *The task of generating the step of solving* LMT. In some pedagogical situations, a student who solves LMT, the system should provide a hint as a next step in the decision of LMT.
- *The task of generating the course of* LMT *resolution*. In some pedagogical situations, the system should give the student methodically correct course of the decision of LMT. We note that the solution of this problem is devoted to many SCMEP.
- *The task of automatic support for the decision of* LMT. The solution to this problem is realized in many commercial SCMEP. When solving LMT, user: 1) determines which of the elementary transformations should be performed at this step of the solution; 2) performs this transformation; 3) rewrites the result as the next step of the solution.

In our view, the phase of determining the next step of the solution is most important. It is she who has a creative character. The mode of automatic support for the LMT solution enables the user to select one of the elementary transformations, and then automatically performs phases 2, 3. As a result, the user focuses on finding a solution to a task as a logical task, rather than performing routine calculations.

Realization of the automatic support mode of the decision of the LMT (or automatic mode) requires precise formulas in the definition of SD, the exact definition of the concept of the solution of the problem as a form of logical conclusion and, most importantly, accurate, complete and methodically correct definition of the list of elementary transformations ET_{SD}.

The features of the models of subject areas do not allow to describe the algorithms of the basic specific problems, which are based on the concept of the course of the decision of LMT. Therefore, this concept must be formally defined as the concept of logical output, based on elementary transformations of models of training modules.

4.3 Equivalent output from SCMEP

Equivalent output uses elementary transformations of models of training modules. Therefore, the main problem is the problem of constructing the user-class classification of elementary transformations that are used in the decision of the LMT of the given domain. The list of elementary transformations must satisfy the requirements of correctness, completeness and "methodological correctness", that is, the correspondence with those transformations that are the subject of study in this module.

The feature of the LMT is that in their solution not only equivalent transformations are used. For example, in solving the quadratic inequality, the student initially solves the corresponding square equation, and then concludes the form of the answer to the inequality. The transition from solving the inequality to solving the corresponding equation is not equivalent to a transformation. In addition, in the process of solving LMT, various forms of presentation of a task or its solutions are often used - for example, a graphic form. All these features make it difficult to implement the logical output in the form of the course of the decision of LMT in a non-trivial design perspective.

The main differences in the concept of logical output in SCMEP from their formal-logical equivalent are expressed as a set of requirements that provide a methodically acceptable form of presentation of the output in the form of the course of the decision of LMT. In this presentation, the specific features of the course of the LMT resolution noted above should be taken into account and implemented.

In the sequence (*), the logical formula obtained in the previous step of the output is the output for the next step of the output. In the SCMEP user interface, this output is presented as a sequence of $F_0, F_1, F_2, ..., F_{k-1}, F_k$. The tasks of generating the path and generating the step of solving are the tasks of automatic generation of this sequence. The task of automatically supporting the course of the LMT solution is to support the user's activity, whose purpose is to generate this sequence. The solution to this task is to implement a special software module a Guide that contains a structured list of all elementary transformations available to the user.

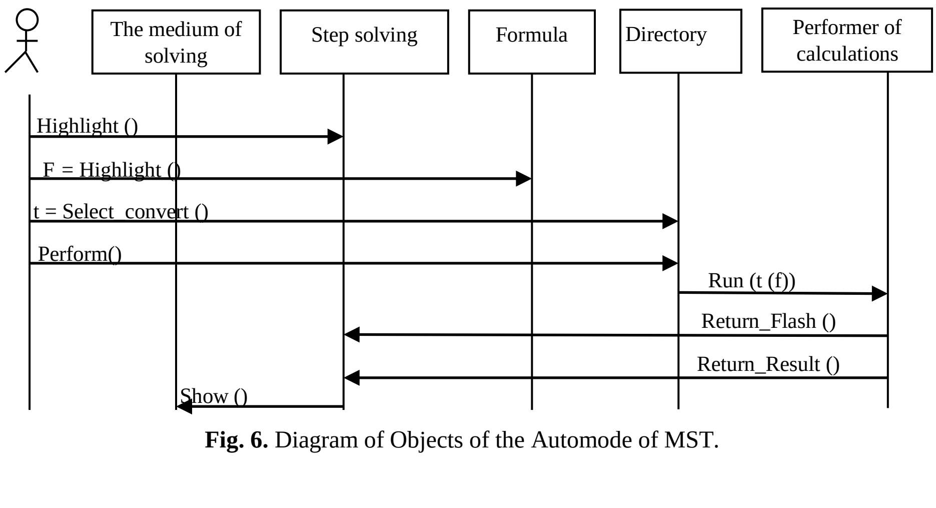

Fig. 6. Diagram of Objects of the Automode of MST.

Performing the *i*-th step of the solution in automation mode, the user selects in F_i subexpression G and selects in the Conversion Handbook t_i, applicable to G. The system applies the above transformation to $F_i(G)$, displaying the result $F_{i+1} = F_i(t_i(G))$ as the next step of the solution (Fig. 6). The main problem is to obtain the classification of elementary transformations that are used in the decision of the LMT of a given subject domain, which is adequate to the requirements of the user.

Algorithms for automatic implementation of the solution step are based on the analysis of the appropriate and adequate classification of elementary transformations in the corresponding subject area.

5 Classification of algebraic elementary transformations

Describe the principles of classification of elementary algebraic transformations on the example of the software module "Guide" of SCMEP TerM VII.

The module has a modular structure. Therefore, the classification principles outlined below can be applied to further SCMEP extensions to new training modules and subject areas by adding related elementary transformations to the Guide. That is, SCMEP Guide satisfy the condition of extensibility (Fig. 7).

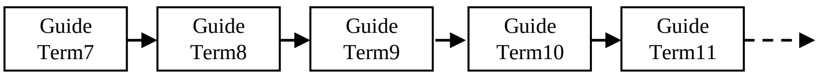

Fig. 7. The sequence of extensions of the "Guide" of TerM

The structural classification of elementary transformations of the model $F(x_1,..., x_n)$ is the basis of the presentation of the contents of the PM *Guide*. It is based on the structure of the formula $F(x_1,..., x_n)$. The contents of the Guide are determined by the list of educational modules and algebraic types of educational algebraic tasks. Here is this classification:

- Converting numbers and numeric expressions.
- Rules for using variables.
- Equivalent expression transformations.
- Equivalent transformation of equality.
- Transformation of identities.
- Methods for solving equations.
- Methods for solving inequalities.
- Algebraic transformations of systems of equations.
- Logical transformations of systems of equations and inequalities.
- Formulas of progressions.
- Analysis of task solving.

In this list are those elements of the structure of elementary transformations, which are implemented in the Guide of software module SCMEP TerM 7-9.

The list of changes in each of the sections of the structure of the Guide extends with the addition of new training modules. The structural classification of elementary transformations is natural, it reflects the structure of mathematical objects and, consequently, mathematical knowledge.

When designing it was necessary another classification - model. The model classification of elementary transformations is based on the following features: model type, model space, variety of model solutions, role of transformation. We list the following transformations:

- Converts that change the type of representation (isomorphism of models).
- Converts that extend the space of the model.
- Converts that narrow the model space (specialization).
- Transformation that extends the variety of solutions.
- Transformations that limit the variety of solutions.
- Converting a model type.

The following types of isomorphic representations are realized in SCMEP TerM:

- representation of a rational number in the form of a fraction, a mixed fraction, a decimal periodic fraction;
- representation of inequalities and double inequalities in the form of numerical intervals; $x < b \Rightarrow x \in (-\infty; b)$; $a < x < b \Rightarrow x \in (a; b)$
- graphical interpretation of intervals on a numerical axis.

In the future, this list will be expanded.

Converting the model space. Significant role in the classification of transformations is played by the objective variable formulas $F(x_1, \ldots, x_n)$. All subject variables in the TerM are defined in R_1. The model space $F(x_1, \ldots, x_n)$ is the space R^n, each coordinate of which is marked by the name of the corresponding variable.

Converts that extend the model space add one or more dimensions to the model space called the new variables. Such transformations include, for example, the replacement of variables

$$(x-a)^2 + (y-b)^2 = R^2 \sim \begin{cases} (x-a)^2 + (y-b)^2 = R^2 \\ u = x - a \\ v = y - b \end{cases}.$$

The transformation of this type is defined as a transformation that complements the formula by another equation of the type of equality, which includes the new variable $F(x_1,..., x_n) \Leftarrow F(x_1,..., x_n) \,\&\, g(x_1,..., x_n, u) = 0$.

Convergences that narrow the space of solutions (specialization) add to the formula additional equations of type of equations in the "old" coordinates. For example:

$$(x-a)^2 + (y-b)^2 = R^2 \sim \begin{cases} (x-a)^2 + (y-b)^2 = R^2 \\ x = c \end{cases}$$

This type transformation reduces the dimension of the variety of model solutions.

Transforms that extend the variety of solutions are used to apply the method of solving tasks in which the answer is first calculated, and then this answer is checked for correctness.

For example, in the task "Solve the equation $\sqrt{x} = -2$" first get an interpolation $x = 4$, and then, substituting this solution (transformation of specialization) into the original equation, we are convinced that the resulting solution is incorrect.

Transformations that limit the variety of solutions. For transformations of this type belongs, for example, the factorization of the equation, that is, the transformation of the form

$$F(x) * G(x) = 0 \sim \left[\begin{array}{l} F(x) = 0 \\ G(x) = 0 \end{array}\right.$$

The transformation of this type divides the variety of solutions into components, each of which is transformed separately (solution by case analysis).

The operational classification of elementary transformations is based on the main signs of the operations of the output and resultant conversion formulas. Let (*F*,*t*,*G*) - the step of solving the problem. Both for the formula *F* and for the formula *G* one can distinguish their main signs of operations - respectively $\otimes_F$ and $\otimes_G$. If the transformation *t* is applicable to *F*, and the result of the transformation is *G*, then $F \sim \otimes_F(X)$, $G \sim \otimes_G(X)$, and *t* has the form $\otimes_F(Y) \Rightarrow \oplus_G(Y)$, the type of transformation we call a pair of operations symbols $(\otimes_F, \otimes_G)$.

For example, the type of transformation $F \cdot G = 0 \Leftrightarrow F = 0 \vee G = 0$ is a pair $(=, \vee)$. The type of transformation $(a+b)^2 = a^2 + 2 \cdot a \cdot b + b^2$ is a pair (^2, +).

The operational classification of transformations is used in the algorithm of dynamic formation of the section of the Guide "Selected transformations". Whenever a user selects a subframe *F* that needs to be transformed, the system dynamically forms the "Selected Transformation" section, including all the transformations of type $(\otimes_F, ...)$. Practice has shown that the number of selected transformations does not exceed 10, and their average number is even smaller. We also note that both direct and inverse transformations are described in one help.

The technological classification of elementary transformations is intended to determine the arguments transmitted from the MST to the corresponding algebraic function, which carries out the selected elementary transformation. The fact is that some conversions, besides the highlighted subsection, require additional data.

For example, the transformation of $A = B \Leftrightarrow c \cdot A = c \cdot B$ requires input of the multiplier *c*. The transformation of $A = B^n$, which means representing a number or expression in the form of a degree of another number or expression, requires the introduction of B^n.

In the system for this purpose, a special input window is used for the additional expression. Defining the transformation of the requisite additional data allows implementation of transformations of this type.

6 Conclusions

The system of training for SCMEP on algebra contains relevant teaching materials and activity environments.

Practical mathematical activity of the student is the main form of educational activity in the study of algebra. It consists in solving learning mathematical tasks with the help of special computer training facilities - activities environments.

In the work:

- functional requirements for activity environments supporting the solving of learning tasks in algebra in SCMEP are developed.
- the mathematical model of LMT is defined within the framework of the mathematical model of the training module and the construction of the corresponding algorithms of computer algebra.
- mathematical models of methods for supporting the solving of LMT in SCMEP are constructed.

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