On Decidability of Persistence Notions

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Abstract. Persistence is a widely investigated fundamental property of concurrent systems, which was extended in many ways. We propose a unified characterisation of several notions considered in the literature. The main result of the paper is a detailed description of a general and extendable framework that allows to state decision problems for different persistence notions (well known as well as newly formulated) and prove their decidability.

Key words: persistence, nonviolence, step semantics, Petri nets

1 Introduction

Conflicts, in other word, situations where two components fight for resources causing mutual exclusion, are usually not desirable in concurrent systems. In the case of Petri Nets, one can exclude them on a design level by modelling systems without statical conflicts (which are certain templates in a net structure). Examples of such classes of nets are Marked Graphs, T-systems or Output-Nonbranching Nets (ON-Nets).

The next step to avoid conflicts is to study concurrent systems of arbitrary structure which work in a conflict-free manner. Such systems are called persistent, and are widely investigated [4, 6, 14, 15, 19, 21, 24, 25] due to their applications in concurrent systems, for example in net synthesis [8], or in hardware designing [9, 23] (one can use persistent nets to avoid hazards [12]).

In its standard form, persistence is stated as a property of nets operating according to the sequential semantics. The classical definition is as follows:

Definition 1 (persistent net, [19]). *A p/t-net N is* persistent *if, for all transitions $a \neq b$ and any reachable marking M of N, the enabledness of actions a and b at marking M implies that the sequence ab is also enabled at M.*

The above definition captures a property of the entire system represented by a p/t-net. We now disassemble it onto basic components. First of all, the subjects of our interest are two objects from a finite domain (e.g., transitions a and b). They should be essentially different (e.g., $a \neq b$) and initially enabled (e.g., $M[a\rangle$ and $M[b\rangle$) in a specified context (e.g., a marking M reachable from M_0; note that the set of all such markings may be infinite). We test the preservation of the terminal enabledness of the second object (e.g., $M[ab\rangle$). At the end, we wrap it by universal first-order logic quantifiers (over actions $a, b \in T$ and a reachable marking $M \in [M_0\rangle$).

We provide the following framework with parameters (degrees of freedom):

- the object type (steps or transitions as singleton steps)
- the existence of universal quantifiers
- the essential difference (especially in the case of steps) $\delta_{\alpha,\beta}$
- the initial enabledness of α (assumed to be immediate) $\zeta_{\alpha,M}$
- the initial enabledness of β (immediate, ultimate) $\xi_{\beta,M}$
- the terminal enabledness (immediate, ultimate) $\phi_{\alpha,\beta,M}$.

Example 1. Let $N = (P, T, W, M_0)$ be a p/t-net.

The classical notion of persistence in p/t-nets with sequential semantics can be obtained with the use of the following parameters:

- the object type – transitions from T
- the existence of three universal quantifiers ($\forall_{a \in T}, \forall_{b \in T}, \forall_{M \in [M_0\rangle}$)
- the essential difference – $\delta_{\alpha,\beta}$: $a \neq b$
- the initial enabledness of α – $\zeta_{\alpha,M}$: $M[\alpha\rangle$
- the initial enabledness of β – $\xi_{\beta,M}$: $M[\beta\rangle$
- the terminal enabledness – $\phi_{\alpha,\beta,M}$: $M[\alpha\beta\rangle$.

Figure 1 illustrates a persistent p/t-net, in which conflicts are not excluded structurally.

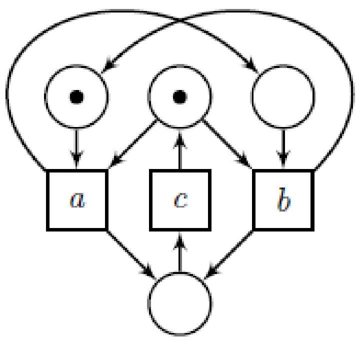

Fig. 1. A persistent p/t-net [4].

Specifying all those parameters leads to a fixed formula, which corresponds to a distinct decision problem. Many of the problems obtained this way were discussed (sometimes without answering the question about decidability) in the literature [2, 15, 18]. However, in a natural way, some new problems appeared. The presented framework allows us to answer all of them positively.

The paper is organized as follows. First, we introduce the general framework that allows to define decision problems concerning different persistence notions and specify their decidability in terms of satisfiability of first order logic formulas. After that, in Section 3, we describe in detail the technique of proving decidability of all problems formerly presented. Section 4 consists of several meaningful examples showing the applications of the presented framework. We end with a short section containing conclusions and future plans.

Part of the results was presented during PNSE workshop 2014 [1].

Due to lack of space the whole section of preliminaries concerning Petri Nets with step semantics is not a part of this version. It can be found in full version [3]. Basic definitions and notions could be also found e.g. in [4, 18].

2 General Framework

All (but the first) parameters of the framework presented in the introduction are used to specify the predicate (propositional logic formula) $\psi_{\alpha,\beta,M} = \zeta_{\alpha,M} \wedge \xi_{\beta,M} \wedge \delta_{\alpha,\beta} \Rightarrow \phi_{\alpha,\beta,M}$. Having specified ψ, we can focus on the decidability of universally quantified problems constructed this way. The diagram that presents the lattice of those problems is depicted in Figure 2.

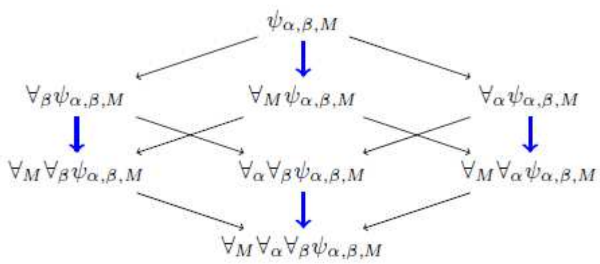

Fig. 2. Lattice of quantified formulas

Each arc of the diagram corresponds to adding one universal quantifier. The thin arcs (e.g., an arc between $\psi_{\alpha,\beta,M}$ and $\forall_\alpha \psi_{\alpha,\beta,M}$) are 'simple' as the set of interesting objects is assumed to be finite. Having established the decidability of a predecessor of a diagram arc, we obtain the decidability of its successor just by searching through a finite sequence of the relevant cases. Going through the solid thick arcs (e.g., an arc between $\forall_\alpha \forall_\beta \psi_{\alpha,\beta,M}$ and $\forall_M \forall_\alpha \forall_\beta \psi_{\alpha,\beta,M}$) is more complex. In this case, we need to check if the predicate is true for all elements M from a (possibly infinite) subset of a (possibly infinite) set of contexts.

The general idea of processing a thick arc is as follows:

(i) negate the predicate in the head node of a diagram arc;
(ii) compute the set of all contexts in which the negated predecessor is true;
(iii) intersect this set with the set of all contexts covered by a quantifier over M;
(iv) check if the resulting set is empty.

Now, we can make the following observations:

(i) The first point is easy, as a negation of a decidable predicate is obviously decidable. Note that the obtained predicate is $\neg\psi_{\alpha,\beta,M} = \zeta_{\alpha,M} \wedge \xi_{\beta,M} \wedge \delta_{\alpha,\beta} \wedge \neg\phi_{\alpha,\beta,M}$ (or a conjunction of a finite sequence of such predicates if we quantified the elements α or β).

(ii) To complete the second step we need to compute all contexts in which this negated predicate is true. We show that this is possible by proving efficient computability of the sets of contexts in which every component $\zeta_{\alpha,M}$, $\xi_{\beta,M}$, $\delta_{\alpha,\beta}$, and $\neg\phi_{\alpha,\beta,M}$ treated separately is true. We also prove that the results are rational sets, hence we can intersect them, obtaining another rational set.

(iii) In the next step we have to intersect the set of contexts obtained in (ii) with the set of all contexts under consideration. However, we do not need to do this explicitly (see (iv)).

(iv) In the case of the problems we are solving, the emptiness (or nonemptiness) of the intersection of the sets described in (iii) can be translated into a Generalised (Set) Reachability Problem.

3 Proof Technique

Let us recall that rational subsets of $\mathbb{N}^k$ are subsets built from finite subsets with finitely many operations of union $\cup$, addition $+$ and star *.

Theorem 1 (Ginsburg/Spanier [13]). *Rational subsets of $\mathbb{N}^k$ form an effective Boolean algebra (i.e. are closed under union, intersection and difference).*

Definition 2 (ω-extension). *Let $\mathbb{N}_\omega = \mathbb{N} \cup \{\omega\}$, where ω is a new symbol (of infinity). We extend, in a natural way, the addition operation: $n + \omega = \omega$, and the order: $(\forall_{n\in\mathbb{N}})n < \omega$. The set of k-dimensional vectors over $\mathbb{N}_\omega$ will be denoted by $\mathbb{N}^k_\omega$, its elements are called* ω-vectors. *The addition operation $+$ and the ordering relation $\leq$ in $\mathbb{N}^k$ are understood componentwise. For $X \subseteq \mathbb{N}^k_\omega$, we denote by $Min(X)$ the set of all minimal (w.r.t. $\leq$) members of X, and by $Max(X)$ the set of all maximal (w.r.t. $\leq$) members of X.*

Fact 1 (Dickson [10]) *Any subset of incomparable elements of $\mathbb{N}^k$ is finite.*

Definition 3 (Closures, convex sets, bottom and cover).

- *Let $x \in \mathbb{N}^k_\omega$ and $X \subseteq \mathbb{N}^k_\omega$. We denote: $\downarrow x = \{z \in \mathbb{N}^k | z \leq x\}$, $x\uparrow = \{z \in \mathbb{N}^k | x \leq z\}$, $\downarrow X = \bigcup\{\downarrow x | x \in X\}$, $X\uparrow = \bigcup\{x\uparrow\ | x \in X\}$, and call the sets left and right closures of X, respectively;*
- *A set $X \subseteq \mathbb{N}^k$ such that $X = \downarrow X (X = X\uparrow)$ is said to be left (right) closed;*
- *A set $X \subseteq \mathbb{N}^k$ such that $X = \downarrow X \cap X\uparrow$ is said to be convex;*
- *The left ω-closure of $X \subseteq \mathbb{N}^k$ is the set $\downarrow X_\omega = \{z \in \mathbb{N}^k_\omega |\ \downarrow z \subseteq \downarrow X\}$;*
- *Bottom and Cover of $X \subseteq \mathbb{N}^k$ are the sets $Bottom(X) = Min(X)$ and $Cover(X) = Max(\downarrow X_\omega)$, resp. We will write BoX and CoX, for short.*

Proposition 1 ([2]). *Any convex subset of $\mathbb{N}^k$ is rational.*

In this paper we are interested in particular sets of markings, which are needed to establish the quantifier context in the discussed formulas. Let us define the following sets of markings (for given steps α and β):

$E_\alpha = \{M \in \mathbb{N}^k | M[\alpha\rangle\}$ - markings enabling α
$E_{\alpha\beta} = \{M \in \mathbb{N}^k | M[\alpha\beta\rangle\}$ - markings enabling a step sequence $\alpha\beta$
$E_{..\alpha} = \{M \in \mathbb{N}^k | (\exists w \in (2^T)^*) M[w\alpha\rangle\}$ - mark. enabling $w\alpha$, for $w \in (2^T)^*$
$E_{\alpha..\beta} = \{M \in \mathbb{N}^k | (\exists w \in (2^T)^*) M[\alpha w\beta\rangle\}$ - mark. en. $\alpha w\beta$, for $w \in (2^T)^*$

The formulation of the expressions for some of the considered sets is immediate. Let us note the following equalities:

$E_\alpha = en\alpha + \mathbb{N}^k$
$E_{\alpha\beta} = \text{cw\_max}(en\alpha, en\alpha - ex\beta + en\beta) + \mathbb{N}^k$
$E_{\alpha(\beta\setminus\alpha)} = \text{cw\_max}(en\alpha, en\alpha - ex\alpha + en(\beta \setminus \alpha)) + \mathbb{N}^k$

where *cw_max* is componentwise maximum (i.e., the resulting vector has the largest value at every coordinate).

Fact 2 *The sets E_α, $E_{\alpha\beta}$, $E_{\alpha(\beta\setminus\alpha)}$, are rational.*

Proof. Note that all those sets are right-closed, hence convex, so, by Proposition 1, rational. □

The expressions for the rest of the sets mentioned above are more complex. That is why we take advantage of the theory of residual sets [22].

Definition 4 (Valk/Jantzen [22]). *A subset $X \subseteq \mathbb{N}^k$ has property RES if and only if the problem "Does $\downarrow v$ intersect X?" is decidable for any ω-vector $v \in \mathbb{N}^k_\omega$.*

Theorem 2 (Valk/Jantzen [22]). *Let $X \subseteq \mathbb{N}^k$ be a right-closed set. Then Bottom of X is effectively computable if and only if X has property RES.*

Lemma 1. *The following sets have the property RES:*
$E_{..\alpha}$, $E_{\alpha..\beta}$,

Proof. Clearly, the sets $E_{..\alpha}$, $E_{\alpha..\beta}$, are right-closed, by the monotonicity property. We shall prove that they have the property RES.

Let us notice that $\downarrow v$ intersects $E_{..\alpha}$ if and only if there is a path in the concurrent coverability graph of the net (P, T, W, v) containing an arc labelled by α.

Observe also that, $\downarrow v$ intersects $E_{\alpha..\beta}$ if and only if $en\alpha \leq v$ (i.e. α is enabled at v) and there is a path in the concurrent coverability graph of the net (P, T, W, v'), where v' is an ω-marking obtained from v by execution of α, containing an arc labelled by β. □

Lemma 2. *The Bottoms of the sets $E_{..\alpha}$, $E_{\alpha..\beta}$, are effectively computable.*

Proof. The sets listed above have the property *RES* (Lemma 1), and hence, by Theorem 2 their Bottom are effectively computable. □

Corollary 1. *The sets $E_{..\alpha}$, $E_{\alpha..\beta}$, are rational. Moreover, the rational expression for a set X from the above list is as follows: $X = Bottom(X) + \mathbb{N}^k$.*

In Section 2 we provided a general idea of checking the decidability of formulas depicted in Figure 2. The goal was to find a set of contexts, i.e., the set of markings for which the considered negated formula is true. The computation of such sets of markings becomes possible thanks to the sets listed above (E_α, $E_{\alpha\beta}$, $E_{..\alpha}$, $E_{\alpha..\beta}$). As all of those sets are effectively rational, we can intersect or sum them up obtaining other rational sets. We use this method for computing sets of *undesirable markings*, i.e., sets of markings at which the examined formula is false (see formulas of Figure 2). Examples of rational expressions for certain formulas are presented in the following section.

As noted in Section 2, the next step is to investigate whether any of the unwanted markings is reachable in a particular net. This task is reduced to decidability of the following problem:

Generalised (Set) Reachability Problem
Instance: A net $N = (P, T, W, M_0)$ and a set $X \subseteq \mathbb{N}^{|P|}$.
Question: Is there a marking $M \in X$, reachable in N?

Possessing a rational expression for every unwanted set and having in mind that all the sets are convex, it is enough to check whether any marking from the undesirable set connected to a distinct decision problem is reachable in a given net. The reachability of any undesirable marking gives us a negative answer for problems related to formulas of Figure 2. The following theorem yields decidability of all the problems corresponding to formulas of Figure 2.

Theorem 3 ([2]). *If $X \subseteq \mathbb{N}^{|P|}$ is a rational convex set, then the Generalised Reachability Problem for X is decidable in the class of p/t-nets.*

Remark 1. It should be noted that rational sets are exactly semi-linear sets and the reachability of semi-linear sets has already been shown in [15]. However, we refer to the proof of [2], because it fits perfectly into our applications.

4 Applications

4.1 Sequential Semantics

The first approach to the notion of persistence oriented towards weak liveness, not only enabledness of actions, appeared in [2]. One can find there three classes of persistence defined for nets with sequential semantics: the first one (corresponding to the classical notion): "no action can disable another one", and two generalizations of this notion: "no action can kill another one" and "no action can kill another enabled one". Let us recall the notions:

Definition 5 (Three kinds of persistence).
Let $N = (P, T, F, M_0)$ be a place/transition net. If $(\forall\ M \in [M_0\rangle)(\forall\ a, b \in T)$

– $M[a\rangle \wedge M[b\rangle \wedge a \neq b \Rightarrow M[ab\rangle$, *then* N *is said to be* e/e-persistent *(every enabled action stays enabled after the execution of any other action);*
– $M[a\rangle \wedge (\exists u)M[ub\rangle \wedge a \neq b \Rightarrow (\exists v \in T^*)M[avb\rangle$, *then* N *is said to be* l/l-persistent *(every weakly live action stays weakly live after the execution of any other action);*
– $M[a\rangle \wedge M[b\rangle \wedge a \neq b \Rightarrow (\exists v \in T^*)M[avb\rangle$, *then* N *is said to be* e/l-persistent *(every enabled action stays weakly live after the execution of any other action).*

In [2] the following decision problems were proved to be decidable:

Instance: A p/t-net $N = (P, T, F, M_0)$.
Questions:

EE Net Persistence Problem:	Is the net e/e-persistent?
LL Net Persistence Problem:	Is the net l/l-persistent?
EL Net Persistence Problem:	Is the net e/l-persistent?

We use an analogous proving technique in this paper. Let us notice that the problems described above correspond to the bottom vertex of the lattice depicted in Figure 2, where we can find the following formula: $\forall_M \forall_\alpha \forall_\beta \psi_{\alpha,\beta,M}$.

Example 2. For EL (LL) Net Persistence Problem we fix the framework parameters as follows:

– object type -âĂŞ transitions – sequential semantics
– difference – standard difference of elements of the set of actions
– initial enabledness of α -âĂŞ immediate standard enabledness of an action a such that $\alpha = \{a\}$
– initial enabledness of β âĂŞ- immediate standard enabledness (resp. ultimate weak liveness) of an action b such that $\beta = \{b\}$
– terminal enabledness âĂŞ- ultimate weak liveness of an action b, where $\beta = \{b\}$.

Hence, we obtain the following formulas (for EL and LL, respectively):

$$\forall_M \forall_\alpha \forall_\beta \psi_{\alpha,\beta,M} \equiv \forall_M \forall_{\alpha=\{a\}} \forall_{\beta=\{b\}} M[a\rangle \wedge M[b\rangle \wedge a \neq b \Rightarrow \exists_{v \in T^*} M[avb\rangle,$$

$$\forall_M \forall_\alpha \forall_\beta \psi_{\alpha,\beta,M} \equiv \forall_M \forall_{\alpha=\{a\}} \forall_{\beta=\{b\}} M[a\rangle \wedge \exists_{u \in T^*} M[ub\rangle \wedge a \neq b \Rightarrow \exists_{v \in T^*} M[avb\rangle.$$

In Figure 3 we present the p/t-net N together with its reachability graph. The net N is e/l-persistent but not l/l-persistent.

The rational expressions for the sets of undesirable markings are as follows: $\bigcup_{\alpha,\beta} E_\alpha \cap E_\beta \cap (\mathbb{N}^{|P|} \setminus E_{\alpha..\beta})$ for e/l-persistence, and $\bigcup_{\alpha,\beta} E_\alpha \cap E_{..\beta} \cap (\mathbb{N}^{|P|} \setminus E_{\alpha..\beta})$ for l/l-persistence.

It should be noted that the decidability of step oriented versions of the problems above (where pairs of actions were settled) is used in the proofs, which is why the formula $\forall_M \psi_{\alpha,\beta,M}$ was also the subject of interest.

Remark 2. The EE Net Persistence Problem is the old and classical Net Persistence Problem raised by Karp/Miller [16]. Later Hack [15] showed that it is reducible to the Reachability Problem, which turned out to be decidable (Mayr [20], Kosaraju [17]).

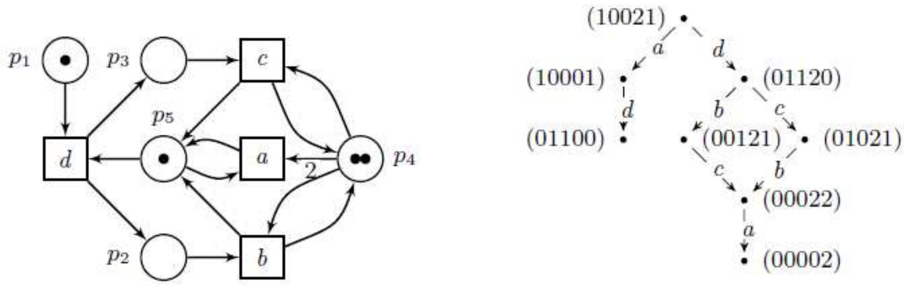

Fig. 3. A p/t-net which is e/l-persistent but not l/l-persistent, and its reachability graph (a kills b indirectly).

Let us notice that the concepts recalled above concern entire nets. A given net is persistent in one of those three meanings, if for *every* reachable marking of the net and *every* pair of *different* transitions a certain condition holds.

In our discussion, however, we would like to study the notion of persistence in a more local sense. We shall consider individual executions of actions at specified markings. This approach leads us to the analysis of two concepts: nonviolence and persistence [18].

Definition 6 (Persistence and nonviolence).
Let t be a transition enabled at a marking M of a p/t-net N. Then:

- *t is locally nonviolent at M if, for every t' enabled at M $t' \neq t \Longrightarrow M[tt'\rangle$*
- *t is locally persistent at M if, for every t' enabled at M $t' \neq t \Longrightarrow M[t't\rangle$.*

Let us notice, that local notions correspond to formulas $\forall_\beta \psi_{\alpha,\beta,M}$ (nonviolence) and $\forall_\alpha \psi_{\alpha,\beta,M}$ (persistence) in Figure 2.

Definition 7 (Globally nonviolent and persistent nets).
A transition is globally nonviolent (globally persistent) in a p/t-net N if it is locally nonviolent (locally persistent, resp.) at every reachable marking of N (at which it is enabled). A p/t-net net is globally nonviolent (globally persistent) if it contains only globally nonviolent (globally persistent, resp.) transitions.

Fact 3 *Let $N = (P, T, F, M_0)$ be a p/t-net. Then the following are equivalent:*

- *N is globally nonviolent.*
- *N is globally persistent.*
- *N is e/e-persistent.*

In the next subsection we discuss in details (in step semantics) local properties of persistence and nonviolence.

4.2 Step Semantics

Another approach to the notion of persistence appeared in [11] and [18] uses step semantics. In this case we are interested in steps (sets of transitions) instead of single actions and we specify the step difference, which turns out to be symptomatic. In [18] three significantly different notions of persistence/nonviolence based on different definitions of distinguishability of steps are defined. We start from recalling those three notions:

Definition 8 (nonviolent and persistent steps). *Let α be a step enabled at a marking M of a p/t-net N. Then:*

- α *is* locally A-nonviolent *(*β *is* locally A-persistent*) at marking M (or* LA*-nonviolent/*LA*-persistent) if, for every step β (α respectively) enabled at M,* $\beta \cap \alpha = \varnothing \implies M[\alpha\beta\rangle$ [18],
- α *is* locally B-nonviolent (β is *locally* B*-persistent*) at marking M *(or* LB*-nonviolent/*LB*-persistent) if, for every step β (α respectively) enabled at M,* $\alpha \setminus \beta \neq \varnothing \neq \beta \setminus \alpha \implies M[\alpha\beta\rangle$,
- α *is* locally C-nonviolent (β is *locally* C*-persistent*) at marking M *(or* LC*-nonviolent/*LC*-persistent) if, for every step β (α respectively) enabled at M,* $\beta \neq \alpha \implies M[\alpha\beta\rangle$ [18].

A step α (which is weakly live at M_0) is globally A/B/C-nonviolent/persistent (or GA/GB/GC-nonviolent/persistent) in N *if it is respectively* LA/LB/LC*-nonviolent/persistent at every reachable marking of N at which it is enabled.*

However, the decidability problems related to the notions defined this way were stated as future work in [11] and [18] and can be formulated as follows:
Instance: A p/t-net $N = (P, T, F, M_0)$.
Questions:

Step GA/GB/GC–**Nonviolence Problem:**
Is the step α globally A/B/C-nonviolent in N?
Step LA/LB/LC–**Nonviolence Problem:**
Is there a marking $M \in [M_0\rangle$ such that the step α is locally A/B/C-nonviolent at M?
Step GA/GB/GC–**Persistence Problem:**
Is the step β globally A/B/C-persistent in N?
Step LA/LB/LC–**Persistence Problem:**
Is there a marking $M \in [M_0\rangle$ such that the step β is locally A/B/C-persistent at M?

We rewrite two of those problems in terms of the developed framework. The formulas corresponding to the other two need to introduce existential quantifiers which is out of the scope of this paper. We obtain:

- In the case of Step GA/GB/GC–Nonviolence Problem: $\forall_M \forall_\beta \psi_{\alpha,\beta,M}$. $\exists_M \forall_\beta \theta_{\alpha,M}$.
- In the case of Step GA/GB/GC–Persistence Problem: $\forall_M \forall_\alpha \psi_{\alpha,\beta,M}$. $\exists_M \forall_\alpha \theta_{\beta,M}$.

Note that $\forall_M \forall_\alpha \psi_{\alpha,\beta,M}$ and $\forall_M \forall_\beta \psi_{\alpha,\beta,M}$ are problems depicted in Figure 2. Hence, to prove their decidability, we can use the technique described in this paper. All we need to do is fixing all the framework parameters and find the rational expressions for some sets of markings.

Example 3. For Step GA–persistence (LC–nonviolence) Problem we fix the framework parameters as follows:

- object type -âĂŞ steps (step semantics)
- difference – standard disjointness (resp. difference) of the sets of actions
- initial enabledness of α -âĂŞ immediate standard enabledness of a step
- initial enabledness of β âĂŞ- immediate standard enabledness of a step
- terminal enabledness âĂŞ- immediate standard enabledness of a step.

Hence, we obtain the following formula for GA–persistence:
$\forall_M \forall_\alpha \psi_{\alpha,\beta,M} \equiv \forall_{M \in [M_0\rangle} \forall_{\alpha \in 2^P} (M[\alpha\rangle \wedge M[\beta\rangle \wedge \beta \cap \alpha = \varnothing \Rightarrow M[\alpha\beta\rangle)$
Note that for LC–nonviolence we would deal with the following formula
$\exists_{M \in [M_0\rangle} \forall_{\beta \in 2^P} (M[\alpha\rangle \wedge \alpha \neq \beta) \wedge (M[\beta\rangle \wedge M[\alpha\rangle \wedge \alpha \neq \beta \Rightarrow M[\alpha\beta\rangle)$
See Figure 4 for a net which is LC–nonviolent but not GA–persistent. For more discriminating examples see [18].

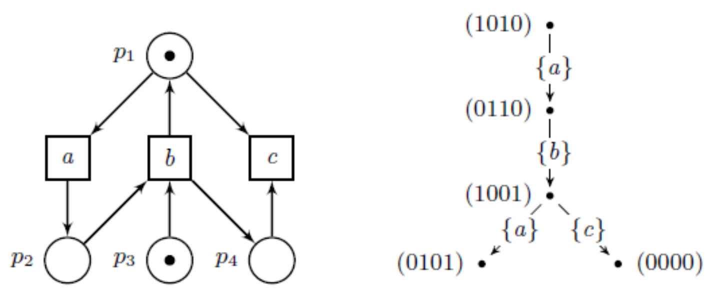

Fig. 4. A p/t-net N in which step $\{a\}$ is LC–nonviolent but not GA–persistent.

Example 4. Let us we fix the framework parameters as follows:

- object type - steps (step semantics)
- difference - standard disjointness of the sets of actions
- initial enabledness of α - immediate standard enabledness of a step
- initial enabledness of β - immediate standard enabledness of a step
- terminal enabledness - ultimate weak liveness of a step β.

Note that this is the framework for A-(e/l)-nonviolence and A-(e/l)-persistence in step semantics (which are (e/l)-nonviolence and (e/l)-persistence, where step difference means emptiness of the intersection of steps - type A). Moreover,

$\psi_{\alpha,\beta,M} \equiv M[\alpha\rangle \wedge M[\beta\rangle \wedge (\alpha \cap \beta = \varnothing) \Rightarrow \exists_{v \in (2^T)^*} M[\alpha v \beta\rangle$.

In Figure 5 we put the rational expressions for the sets of undesirable markings concerning formulas from Figure 2. For consistency, we formulate in the same manner expressions for the cases with fixed markings. To check whether the fixed marking M belongs to a distinct set (of undesirable markings) it is enough to intersect this set with a singleton $\{M\}$.

Naturally, for given steps α, β and a fixed marking M the formula $\psi_{\alpha,\beta,M}$ depicted on top of Figure 2 is easy to verify, regardless of the values of the framework parameters using concurrent coverability graphs.

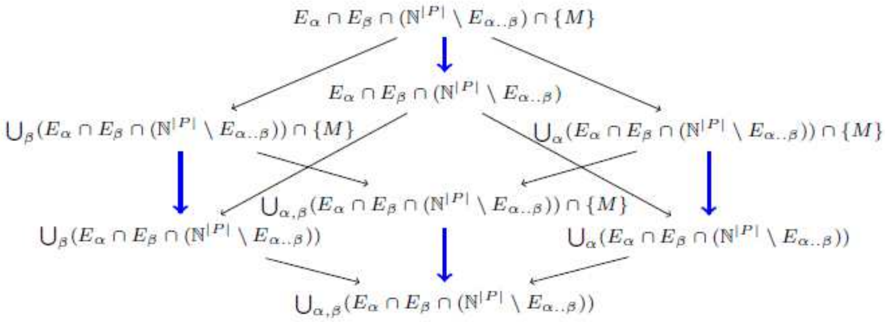

Fig. 5. Lattice of expressions for undesirable markings. Note that in the ranges of all unions we take only $\alpha \cap \beta = \varnothing$.

Remark 3. The left sides of the diagrams in Figures 2 and 5 correspond to the notion of nonviolence, the right sides are dedicated to persistence. The middle columns of vertices can be associated with the properties of markings or whole nets.

5 Conclusions and Future Work

In this paper we proved decidability of several decision problems concerning persistence notions. We split them into two classes and introduced a unified solution that allows to proceed in the same manner in many cases. The proof technique does not depend on the chosen parameters of the general framework. Besides the classical immediate persistence and nonviolence notions we covered more general weak-live oriented ones.

In particular, starting from classical notions defined in [19], we recalled and reformulated the problems from [2] and proved the decidability of the decision problems left open in [18]. In general, the presented framework allows to switch smoothly to the step semantics.

A natural way of non-trivial extending our framework would be to introduce existential quantifiers. As we already noted, this would allow to deal with local persistence and nonviolence in step semantics.

It is worth mentioning that the notion of weak persistence (see [24, 25]) does not yet fit into the introduced framework. Another notion, not yet covered, is restricted persistence oriented on weak liveness (see the notion of e/l-k-persistence from [5]). This shows the natural direction of future development.

In [2] and [18] inclusions between kinds of persistence notions defined there were investigated. It would be interesting examine the relationships between wider spectrum of persistence notions obtaining a detailed taxonomy.

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